A Note on a Competitive Lotka-Volterra Model with Lévy Noise

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Abstract. In this note, stochastic permanence for a competitive Lotka-Volterra model with Lévy noise (which can be used to describe sudden environmental perturbations) is studied by using stochastic analytical techniques. Moreover, some numerical simulations are provided to support the results.

1. Introduction

Recently, stochastic Lotka-Volterra models driven by white noise have been received great attention and have been studied extensively (see e.g. [1–8]). However, in the real world population systems often suffer sudden environmental perturbations, such as earthquakes, hurricanes, planting, harvesting, etc (see e.g. [9–11]). These phenomena cannot be described by white noise [12]. Bao et al. [13, 14] pointed out that one may use Lévy jump processes to describe these phenomena and they studied the following n-dimensional competitive Lotka-Volterra model with Lévy noise:

\begin{equation}
\frac{dX_i(t)}{dt} = a_i(t) - \sum_{j=1}^{n} b_{ij}(t)X_j(t^-) + \sigma_i(t)dW(t) + \int_{\mathbb{Z}} \gamma_i(t,\mu)\tilde{N}(dt, d\mu), \quad 1 \leq i \leq n,
\end{equation}

where $X_i(t^-)$ is the left limit of $X_i(t)$, $W(t)$ is a standard Wiener process defined on a complete probability space $(\Omega, \mathcal{F}, P)$ with a filtration $(\mathcal{F}_t)_{t \geq 0}$. $N$ is a Poisson counting measure with characteristic measure $\lambda$ on a measurable subset $\mathbb{Z}$ of $[0, +\infty)$ with $\lambda(\mathbb{Z}) < +\infty$ and $\tilde{N}(dt, d\mu) = N(dt, d\mu) - \lambda(d\mu)dt$. It is well known that permanence means the long time survival in a population dynamics and thus has its theoretical and practical significance. So, in this note we study the stochastic permanence of system (1). To the best of authors’ knowledge, this is the first attempt to investigate stochastic permanence for the general competitive Lotka-Volterra model with Lévy noise.
2. Main Results

For convenience, let

\[
A = \min_{1 \leq i \leq n} \left\{ \inf_{t \geq 0} \left[ a_i(t) - \frac{\sigma_i^2(t)}{2} - \int_{\mathbb{Z}} (\gamma_i(t, \mu) \ln(1 + \gamma_i(t, \mu))) \lambda(d\mu) \right] \right\},
\]

\[
B_\ast = \min_{1 \leq i \leq n} \left\{ \inf_{(t, \mu) \in [0, +\infty) \times \mathbb{Z}} \gamma_i(t, \mu) \right\}, \quad B^* = \max_{1 \leq i \leq n} \left\{ \sup_{t \geq 0} \sigma_i(t) \right\},
\]

\[
P_i(t) = \int_{\mathbb{Z}} \gamma_i(t, \mu) \lambda(d\mu), \quad H_i(t) = \int_{\mathbb{Z}} \ln(1 + \gamma_i(t, \mu)) \lambda(d\mu), \quad 1 \leq i \leq n,
\]

\[
X(t) = (X_1(t), ..., X_n(t))^T, \quad X_0 = (X_1(0), ..., X_n(0))^T, \quad |X(t)| = \left[ \sum_{i=1}^n X_i^2(t) \right]^{\frac{1}{2}}.
\]

In this note, we always assume that \( W(t) \) and \( N \) are independent and \((\mathcal{A})\) For \( 1 \leq i, j \leq n, i \neq j, a_i(t) > 0, b_{ij}(t) \geq 0, \sigma_i(t), \gamma_i(t, \mu) > -1 \) are bounded functions and \( \inf_{t \geq 0} b_{ii}(t) > 0. \)

(\(B\)) \( A > 0. \)

**Definition 2.1.** (see Bao et al. [13]) System (1) is said to be stochastically permanent, if, for any \( \epsilon > 0 \), there exist \( \delta_* = \delta_*(\epsilon) > 0 \) and \( \delta^* = \delta^*(\epsilon) > 0 \) such that

\[
\liminf_{t \to +\infty} P \{ \|X(t)\| \geq \delta_* \} \geq 1 - \epsilon, \quad \liminf_{t \to +\infty} P \{ \|X(t)\| \leq \delta^* \} \geq 1 - \epsilon.
\]

**Remark 2.2.** The stochastic permanence definition of multi-population systems was first proposed by Li et al. [5] and has been intensively applied (see e.g. [13, 15–17]).

**Lemma 2.3.** (see Bao et al. [13]) Under assumption \((\mathcal{A})\), for any initial value \( X_0 \in \mathbb{R}_+^n \), system (1) has a unique global solution \( X(t) \in \mathbb{R}_+^n \) for \( t \geq 0 \) a.s.

**Lemma 2.4.** (see Bao et al. [13]) Under assumption \((\mathcal{A})\), for any \( p \in [0, 1] \), there is a constant \( K \) such that

\[
\sup_{t \in \mathbb{R}_+} \mathbb{E}[\|X(t)\|^p] \leq K.
\]

**Lemma 2.5.** Assume that \( X(t) \) is the solution to system (1) with initial value \( X_0 \in \mathbb{R}_+^n \), then

\[
\sum_{i \neq j} a_i(t) \sigma_j(t) X_i(t) X_j(t) - \sum_{i \neq j} \sigma_i^2(t) X_i(t) X_j(t) \leq 0.
\]

**Proof.** We use the mathematical induction. For \( n = 2 \),

\[
\sum_{i \neq j} a_i(t) \sigma_j(t) X_i(t) X_j(t) - \sum_{i \neq j} \sigma_i^2(t) X_i(t) X_j(t) = -(\sigma_1(t) - \sigma_2(t))^2 X_1(t) X_2(t) \leq 0.
\]

Assume that for \( n = k \), (5) is true, that is

\[
\sum_{i \neq j} a_i(t) \sigma_j(t) X_i(t) X_j(t) - \sum_{i \neq j} \sigma_i^2(t) X_i(t) X_j(t) \leq 0.
\]
From (7) we have
\[
\sum_{i \neq j}^{k+1} \sigma_i(t)\sigma_j(t)X_i(t)X_j(t) - \sum_{i \neq j}^{k+1} \sigma_i^2(t)X_i(t)X_j(t)
\]
\[
= \left\{ \sum_{i \neq j}^{k} \sigma_i(t)\sigma_j(t)X_i(t)X_j(t) + 2 \sum_{i=1}^{k} \sigma_i(t)\sigma_{k+1}(t)X_i(t)X_{k+1}(t) \right\}
\]
\[
- \left\{ \sum_{i \neq j}^{k} \sigma_i^2(t)X_i(t)X_j(t) + \sum_{i=1}^{k} (\sigma_i^2(t) + \sigma_{k+1}^2(t))X_i(t)X_{k+1}(t) \right\}
\]
\[
\leq 2 \sum_{i=1}^{k} \sigma_i(t)\sigma_{k+1}(t)X_i(t)X_{k+1}(t) - \sum_{i=1}^{k} (\sigma_i^2(t) + \sigma_{k+1}^2(t))X_i(t)X_{k+1}(t)
\]
\[
= - \sum_{i=1}^{k} (\sigma_i(t) - \sigma_{k+1}(t))^2 X_i(t)X_{k+1}(t) \leq 0.
\]

In conclusion, the proof is complete. \(\square\)

**Theorem 2.6.** Under assumptions (A) and (B), system (1) is stochastically permanent.

**Proof.** Let \(V(X) = \sum_{i=1}^{n} X_i.\) Applying Itô’s formula to \(V(X)\) leads to
\[
dV = \beta(t)dt + \sigma(t)dW(t) + \int_{\mathbb{Z}} \gamma(t, \mu)\tilde{N}(dt, d\mu),
\]

where
\[
\beta(t) = \sum_{i=1}^{n} X_i \left( a_i(t) - \frac{1}{2} \sum_{j=1}^{n} b_{ij}(t)X_j \right), \quad \sigma(t) = \sum_{i=1}^{n} \sigma_i(t)X_i, \quad \gamma(t, \mu) = \sum_{i=1}^{n} \gamma_i(t, \mu)X_i.
\]

From Lemma 2.3 we have \(P(X_i(t) > 0 \text{ for } t \geq 0) = 1.\) Define \(U(X) = \frac{1}{V(X)}.\) By Itô’s formula, we obtain
\[
dU = \overline{\beta}(t)dt + \overline{\sigma}(t)dW(t) + \int_{\mathbb{Z}} \gamma(t, \mu)\tilde{N}(dt, d\mu),
\]

where
\[
\overline{\beta}(t) = - \lambda^2 \beta(t) + \lambda^3 \sigma^2(t) + \int_{\mathbb{Z}} \left\{ \frac{1}{\gamma(t, \mu)} - U + U^2 \gamma(t, \mu) \right\} \lambda(d\mu),
\]
\[
\overline{\sigma}(t) = - \lambda^2 \sigma(t), \quad \gamma(t, \mu) = \frac{1}{\gamma(t, \mu)} - U.
\]

In the light of (2), we deduce that for any \(t \geq 0\) and \(1 \leq i \leq n,
\]
\[
a_i(t) \geq A + \frac{\sigma_i(0)}{2} + P_i(t) - H_i(t).
\]

Consider the following auxiliary function:
\[
G(\theta) = A\theta - \frac{\theta^2}{2} (B^*)^2 - \int_{\mathbb{Z}} \left\{ \left( \frac{1}{1+ B} \right)^{\theta} - 1 - \theta \ln \left( \frac{1}{1+ B} \right) \right\} \lambda(d\mu), \quad 0 \leq \theta \leq 1.
\]
Then $G(0) = 0$ and

$$
G'(\theta) = A - (B')^2 \theta - \int_Z \left\{ \left( \frac{1}{1 + B_x} \right)^\theta - 1 \right\} \ln \left( \frac{1}{1 + B_x} \right) \lambda(d\mu), \ 0 \leq \theta \leq 1.
$$

(15)

According to (15) and (B), we obtain

$$
G'(0) = A > 0.
$$

(16)

Thus, there exist $\theta > 0$ and $k > 0$ such that

$$
A\theta - \frac{\theta^2}{2} (B')^2 - \int_Z \left\{ \left( \frac{1}{1 + B_x} \right)^\theta - 1 - \theta \ln \left( \frac{1}{1 + B_x} \right) \right\} \lambda(d\mu) > k > 0.
$$

(17)

Applying Itô’s formula to $[e^{\theta t}(1 + U)^{\theta}]$ yields

$$
\begin{align*}
\int d[e^{\theta t}(1 + U)^{\theta}] &= \int \left[ e^{\theta t}(1 + U)^{\theta} + e^{\theta t} \theta(1 + U)^{\theta} \sigma(t) dW(t) \right. \\
&\quad + \left. \int \left[ e^{\theta t}(1 + U)^{\theta} \right] \lambda(d\mu) \right] \\
\end{align*}
$$

(18)

where

$$
\begin{align*}
\mathcal{L} \left[ e^{\theta t}(1 + U)^{\theta} \right] &= ke^{\theta t}(1 + U)^{\theta} + e^{\theta t} \theta(1 + U)^{\theta} \sigma(t) dW(t) \\
&\quad + \int \left[ e^{\theta t}(1 + U)^{\theta} \right] \lambda(d\mu).
\end{align*}
$$

(19)

Substituting inequality (13) into (19) gives

$$
\begin{align*}
\mathcal{L} \left[ e^{\theta t}(1 + U)^{\theta} \right] &\leq ke^{\theta t}(1 + U)^{\theta} + \frac{\theta(1 + 1)}{2} e^{\theta t} U^2 (1 + U)^{\theta - 2} \left[ U \sum_{i=1}^n \sigma_i(t) X_i \right]^2 \\
&\quad + \theta e^{\theta t} (1 + U)^{\theta - 1} U \left[ U \sum_{i=1}^n \sigma_i(t) X_i \right] + \theta e^{\theta t} U (1 + U)^{\theta - 1} \left[ U \sum_{i=1}^n \gamma_i(t, \mu) X_i \right] \\
&\quad + \int \left[ e^{\theta t} \left[ 1 + V + \sum_{i=1}^n \gamma_i(t, \mu) X_i \right] \right] (1 + U)^{\theta} \lambda(d\mu) \\
&\quad - A\theta e^{\theta t} (1 + U)^{\theta - 1} - \theta e^{\theta t} U (1 + U)^{\theta - 1} U \sum_{i=1}^n \sigma_i(t) X_i \\
&\quad - \theta e^{\theta t} U (1 + U)^{\theta - 1} U \sum_{i=1}^n P_i(t) X_i + \theta e^{\theta t} U (1 + U)^{\theta - 1} U \sum_{i=1}^n H_i(t) X_i \\
&\quad + \theta e^{\theta t} (1 + U)^{\theta - 1} U^2 \sum_{i=1}^n X_i \sum_{j=1}^n b_{ij}(t) X_j \triangleq e^{\theta t} \left[ O(U^\theta) U^0 + G(U) \right],
\end{align*}
$$

(20)

where $\lim_{U \to \infty} \frac{G(U)}{U^0} = 0$. Since

$$
0 \leq U^2 \sum_{i=1}^n X_i \sum_{j=1}^n b_{ij}(t) X_j = \sum_{i=1}^n \sum_{j=1}^n b_{ij}(t) \frac{X_i X_j}{(\sum_{i=1}^n X_i)^2} \leq \frac{1}{2} \sum_{i,j=1}^n \sup_{t \geq 0} b_{ij}(t),
$$

(21)
we derive that
\[
\mathcal{O}(U^p) = k + \frac{\theta(\theta - 1)}{2} \left( \sum_{i=1}^{n} \sigma_i(t)X_i \right)^2 + \frac{\theta}{2} \int Z \left( \sum_{i=1}^{n} \sigma_i(t)X_i \right)^2 + \frac{\theta}{2} \sum_{i=1}^{n} \sigma_i(t)^2 X_i + \frac{\theta}{2} \sum_{i=1}^{n} H_i(t)X_i \]
\[
- \int Z \lambda(d\mu) - A\theta - \frac{\theta}{2} \sum_{i=1}^{n} \sigma_i(t)^2 X_i + \frac{\theta}{2} \sum_{i=1}^{n} \sigma_i(t)^2 X_i + \theta \sum_{i=1}^{n} H_i(t)X_i \]
\[
= k - A\theta - \int Z \lambda(d\mu) + \frac{\theta(\theta + 1)}{2} \left( \sum_{i=1}^{n} \sigma_i(t)^2 X_i \right)^2 - \frac{\theta}{2} \sum_{i=1}^{n} \sigma_i(t)^2 X_i + \frac{\theta}{2} \sum_{i=1}^{n} \sigma_i(t)^2 X_i \]
\[
\int Z \left( \sum_{i=1}^{n} \sigma_i(t)X_i \right)^2 \lambda(d\mu) + \theta \int Z \left( \sum_{i=1}^{n} \sigma_i(t)X_i \right)^2 \lambda(d\mu). \] (22)

On the basis of Lemma 2.5, we get
\[
\frac{\theta(\theta + 1)}{2} \left( \sum_{i=1}^{n} \sigma_i(t)X_i(t) \right)^2 - \frac{\theta}{2} \left( \sum_{i=1}^{n} \sigma_i(t)^2 X_i(t) \right)^2 \left( \sum_{i=1}^{n} X_i(t) \right) \]
\[
= \theta^2 \left( \sum_{i=1}^{n} \sigma_i(t)X_i(t) \right)^2 + \theta \left( \sum_{i\neq j} \sigma_i(t)\sigma_j(t)X_i(t)X_j(t) - \sum_{i\neq j} \sigma_i(t)^2 X_i(t)X_j(t) \right) \leq \theta^2 \left( \sum_{i=1}^{n} \sigma_i(t)X_i(t) \right)^2. \] (23)

In a view of (22), (23), Jensen’s inequality, \( e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \) and (17), we obtain
\[
\mathcal{O}(U^p) \leq k - A\theta - \int Z \lambda(d\mu) + \frac{\theta^2}{2} \left( \sum_{i=1}^{n} \sigma_i(t)X_i \right)^2 + \frac{\theta}{2} \left( \sum_{i=1}^{n} \sigma_i(t)^2 X_i \right)^2 + \frac{\theta}{2} \sum_{i=1}^{n} \sigma_i(t)^2 X_i + \theta \sum_{i=1}^{n} H_i(t)X_i \]
\[
= k - A\theta + \frac{\theta^2}{2} \left( \sum_{i=1}^{n} \sigma_i(t)X_i \right)^2 + \frac{\theta}{2} \sum_{i=1}^{n} \sigma_i(t)^2 X_i + \theta \sum_{i=1}^{n} H_i(t)X_i \]
\[
\leq k - A\theta + \frac{\theta^2}{2} (B^r)^2 + \int Z \left\{ \left( \frac{1}{1 + B^r} \right)^\theta X_i(t) \right\} \lambda(d\mu) < 0. \] (24)

On the basis of (18), (20) and (24), there exists \( K > 0 \) such that
\[
\mathbb{E}[e^{\xi_1(1 + U^p)}] - [1 + U(X_0)]^p \leq \int_0^\infty Ke^{ks}ds = \frac{K}{2}(e^k - 1). \] (25)

From (25), we have
\[
\lim_{t \to +\infty} \mathbb{E}[U^p] \leq \frac{K}{2}. \] (26)

In the light of \( \frac{1}{1 + B^r} \leq n^{\frac{r}{2}}U^p \) and (26), we get
\[
\lim_{t \to +\infty} \mathbb{E}\left[ \frac{1}{1 + B^r} \right] \leq n^{\frac{r}{2}} \frac{K}{2}. \] (27)
Based on Chebyshev’s inequality, for any $\epsilon > 0$, there exists $\delta_* = \frac{\sqrt{n}}{n} \left( \frac{1}{\delta} \right)^{\frac{1}{2}} > 0$ such that
\[
\limsup_{t \to +\infty} P \left\{ |X(t)| < \delta_* \right\} = \limsup_{t \to +\infty} P \left\{ \frac{1}{\sqrt{t}} < \frac{1}{\delta} \right\} \leq (\delta_*)^2 \limsup_{t \to +\infty} E \left\{ \frac{1}{|X(t)|^2} \right\} \leq \epsilon.
\] (28)

Therefore,
\[
\liminf_{t \to +\infty} P \left\{ |X(t)| \geq \delta_* \right\} \geq 1 - \epsilon.
\] (29)

The second part of (3) follows from combining Lemma 2.4 with Chebyshev’s inequality. That is, system (1) is stochastically permanent. \(\square\)

**Remark 2.7.** For $n = 1$, system (1) becomes
\[
dX(t) = X(t) \left[ a(t) - b(t)X(t^-) \right] dt + \sigma(t)dW(t) + \int_Z \gamma(t, \mu) \tilde{N}(dt, d\mu).
\] (30)

From Theorem 2.6, system (30) is stochastically permanent, if
\[
\inf_{t \geq 0} \left[ a(t) - \frac{c(t)}{2} - \int_Z (\gamma(t, \mu) - \ln(1 + \gamma(t, \mu))) \lambda(\mu) \right] > 0.
\] (31)

Thus, Theorem 2.6 includes Theorem 1 in [18] as a special case.

### 3. An Example

By the method in [19], for $\lambda(Z) = 1.0$ and step size $\Delta t = 0.1$ we numerically simulate the solutions of the following system to support our results:

\[
\begin{align*}
\left\{ 
& dX_1(t) = X_1(t^-) \left[ 0.81 - 0.51X_1(t^-) - 0.39X_2(t^-) \right] dt + X_1(t^-) \left\{ 0.1dW(t) + \int_Z \gamma_1(t, \mu) \tilde{N}(dt, d\mu) \right\}, \\
& dX_2(t) = X_2(t^-) \left[ 0.79 - 0.41X_1(t^-) - 0.49X_2(t^-) \right] dt + X_2(t^-) \left\{ 0.12dW(t) + \int_Z \gamma_2(t, \mu) \tilde{N}(dt, d\mu) \right\}, \\
& X_1(0) = 1.8, \ X_2(0) = 2.2.
\end{align*}
\] (32)

(a) $\gamma_1(t, \mu) = 0.48, \ \gamma_2(t, \mu) = 0.92$.

(b) $\gamma_1(t, \mu) = 1.2, \ \gamma_2(t, \mu) = 0.51.$
For system (32), we introduce some mathematical notations as follows (as in [18]):

\[
\begin{align*}
&b_i(t) = a_i(t) - \frac{1}{2}\sigma_i^2(t) - \int_Z (\gamma_i(t, \mu) - \ln(1 + \gamma_i(t, \mu)))\lambda(d\mu), \quad i = 1, 2, \\
&\Delta(t) = b_{11}(t)b_{22}(t) - b_{12}(t)b_{21}(t), \quad \Delta_1(t) = b_1(t)b_{22}(t) - b_2(t)b_{12}(t), \quad \Delta_2(t) = b_2(t)b_{11}(t) - b_1(t)b_{21}(t) .
\end{align*}
\]

(I) For \( \gamma_1(t, \mu) = 0.48, \gamma_2(t, \mu) = 0.92 \), we have (Figure 1(a)):

\[
b_1(t) = 0.71704, \quad b_2(t) = 0.51513, \quad \Delta(t) = 0.09, \quad \Delta_1(t) = 0.15045, \quad \Delta_2(t) = -0.03142 .
\]

By Theorem 2.6, system (32) is stochastically permanent. From Theorem 4(i) in [18], \( X_1(t) \) is persistent in mean while \( X_2(t) \) is extinctive a.s.

(II) For \( \gamma_1(t, \mu) = 1.2, \gamma_2(t, \mu) = 0.51 \), we have (Figure 1(b)):

\[
b_1(t) = 0.39346, \quad b_2(t) = 0.68491, \quad \Delta(t) = 0.09, \quad \Delta_1(t) = -0.07432, \quad \Delta_2(t) = 0.18799 .
\]

In view of Theorem 2.6, system (32) is stochastically permanent. Based on Theorem 4(ii) in [18], \( X_1(t) \) is extinctive while \( X_2(t) \) is persistent in mean a.s.

(III) For \( \gamma_1(t, \mu) = 0.25, \gamma_2(t, \mu) = 0.2 \), we have (Figure 1(c)):

\[
b_1(t) = 0.77814, \quad b_2(t) = 0.76512, \quad \Delta(t) = 0.09, \quad \Delta_1(t) = 0.08289, \quad \Delta_2(t) = 0.07117 .
\]

According to Theorem 2.6, system (32) is stochastically permanent. By Theorem 4(iii) in [18], both \( X_1(t) \) and \( X_2(t) \) are persistent in mean a.s.

(IV) For \( \gamma_1(t, \mu) = 2.3, \gamma_2(t, \mu) = 2.5 \), we have (Figure 1(d)):

\[
b_1(t) = -0.30108, \quad b_2(t) = -0.46444 .
\]

On the basis of Theorem 4.6 in [13], both \( X_1(t) \) and \( X_2(t) \) are extinctive a.s. Hence system (32) is not stochastically permanent. In other words, if (3) is false, then system (1) may be not stochastically permanent.

All mentioned above can be confirmed by Figure 1.

**Remark 3.1.** (see Liu et al. [18]) Consider the autonomous case of system (1). For \( n = 2 \) and \( \Delta(t) > 0 \),

(i) if \( \Delta_1(t) > 0, \Delta_2(t) < 0 \), then \( X_1(t) \) is persistent in mean while \( X_2(t) \) is extinctive a.s.

(ii) if \( \Delta_1(t) < 0, \Delta_2(t) > 0 \), then \( X_1(t) \) is extinctive while \( X_2(t) \) is persistent in mean a.s.

(iii) if \( \Delta_i(t) > 0, \Delta_i(t) < 0 \), then \( X_i(t) \) is persistent in mean a.s., \( 1 \leq i \leq 2 \).
4. Conclusions and Future Directions

In this note, sufficient conditions for stochastic permanence of a competitive Lotka-Volterra model with Lévy noise are established.

Some interesting topics deserve further investigation. To begin with, it is interesting to study “stochastic persistence in probability” (see e.g. [20, 21]) of system (1). The motivation is that multi-population systems may remain stochastically permanent, although some species are extinctive (see Figure 1(a) and Figure 1(b)).

Next, we could investigate more realistic and complex systems in lieu of system (1), for instance, hybrid population systems with Lévy noise. The motivation is that parameters in population systems may suffer abrupt changes (see e.g. [1, 22]). One can use a continuous-time Markov chain with a finite state space to describe these abrupt changes (see e.g. [9, 23]).

Motivated by the works in [6, 11, 21, 24–26], we may also study the optimization problem of harvesting for stochastic delay population systems with Lévy noise. We leave these investigations for future work.

References