A Projection Analysis of $C^2R$ Models Applied in Grey Prediction on OHASA18001 Safety Investment and Benefit of a Mining Industry

Jiangdong Bao$^a$, Jingdong Zhang$^b$, Shui-ping Shi$^c$

$^a$Research Center for Environment and Health, Zhongnan University of Economics and Law, Wuhan430073, China
$^b$Research Center for Environment and Health, Zhongnan University of Economics and Law, Wuhan430073, China
$^c$Beijing World Standard Certification Center Co., Ltd, Beijing100053, China

Abstract. A projection analysis of $C^2R$ model is adopted to calculate the safety investment, and benefit. On this basis, GM(1,1) model (Grey prediction model) is established to predict the safety benefit and safety investment values of 2017. This integrated method of $C^2R$ model and GM(1,1) model provides a direction and theoretical reference for optimizing the investment and a structure for the effective operation of mining industry occupational health and safety management system.

1. Introduction

OHASA18001 is an effective way which is recognized by the international organization and proved effective to the occupational health and safety management of the enterprises. It has put forward the PDCA requirement of system operation planning, personnel management, and operational control. Although it has introduced into mining industries for many years, the safety accidents occurring frequently result in great economic loss and passive safety investment which are contrary to the purpose of continuous improvement of OHASA18001.

In terms of safety investment and safety benefit, Alexander Guzman Urbina et al. introduced a framework to evaluate the benefits of investing in safety measures for pipelines using fuzzy logic as a tool to deal with uncertainty in 2017 [1]. Wang et al. put forward the quantitative definition of safety benefit, and established safety impairment output and safety value-added output based on nonlinear optimization model in 2009[2]. Shi et al. analyzed social and economic benefits of safety investment and enhanced the safety awareness of coal mining workers to guide them to correct the concept of safety investment in 2010[3]. Yan et al. chose the panel data of 18 listed companies in China from 2000 to 2011 and analyzed the safety investment and safety benefit empirically to verify the nonlinear effect of safety investment on safety benefit in 2015[4]. Feng et al. explored the interactive effects of safety investments, safety culture and project hazard on construction safety performance in 2014 [5]. From deeper perspective of safety investment and benefit, this study aims to explore a prediction of OHASA18001 (occupational health and safety management system) safety investment and benefit of a mining industry based on $C^2R$ model (projection...
analysis of data envelopment analysis) and GM(1,1) model (grey system) to provide a reasonable basis for
distribution and decision-making for the investment and benefit, and strong support for the healthy and
sustainable development of OHSAS18001.

2. Safety Investment and Benefit Indicators of Mining Industry OHSAS18001

According to the OHSAS18001 standard terms (National standards of the People’s Republic of China, 2011): 4.4.6-Operational control, 4.4.1-Resources, roles, responsibility, accountability and authority, 4.4.2-
Competence, training and awareness, 4.5.1-Performance measurement and monitoring and 4.4.7-Emergency
preparedness response. Four safety investment indicators of mining OHSAS18001 are selected as shown in
the following: safety technical measures, safety management and training, industrial hygiene measures,
and labor protection products [6].

3. Grey Prediction Analysis of Safety Investment and Benefits of Mining Industry OHSAS18001

The grey system theory is created by Chinese Professor Deng Julong in 1880s [7]. The grey system
theory has been successfully introduced to agricultural, industrial, economic and other science fields for
over 20 years. Grey system is not fully known with information, that is, some information is known and
some other is unknown with small sample of information, and poor information systems for the study
of uncertainty. GM(1,1) is an important part of grey system theory whose method is used to establish a
mathematical model to predict values through poor information and small samples.

3.1. Determining the Analysis Sequence

Select reference series and let \( X = \{x_0, x_1, \cdots, x_m\} \) be grey relation factor set, \( x_0 \) be a reference sequence,
\( x_i \) be a comparison sequence, and \( x_0(k)' \) \( \quad i = 1, 2, \cdots, m; \quad x_1 = (x_1(1), x_1(2), \cdots, x_1(n)), \)
\( x_2 = (x_2(1), x_2(2), \cdots, x_2(n)) \cdots \cdots \), \( x_m = (x_m(1), x_m(2), \cdots, x_m(n)) \),
where \( x_1(i) = \sum_{k=1}^{i} x_1(k), \quad i = 1, 2, \cdots, m, \) \( x_1 \) can satisfy the first order linear differential equation:
\[
\frac{dx_1}{dt} + ax_1 = u,
\]
where \( a \) is a development parameter of the model which reflects the development trend of \( x_1 \) and \( x_0; \) \( u \) is
coordination coefficient of the model which reflects the relationship between data.

According to the least squares method [8], it can be calculated as:
\[
A = (a, u)^T = (B^T B)^{-1} B^T Y_N
\]
Where
\[
B = \begin{bmatrix}
-x_1(1) + x_1(2)/2 & 1 \\
-x_1(2) + x_1(3)/2 & 1 \\
\vdots & \vdots \\
-x_1(n-1) + x_1(n)/2 & 1 \\
\end{bmatrix}, \quad Y_N = \begin{bmatrix}
x_0(2) \\
x_0(3) \\
\vdots \\
x_0(n) \\
\end{bmatrix}
\]
The result of \( a \) and \( u \) values are brought into formula (3) and the calculated result is
\[
\bar{x}_i(k+1) = [x_0(1) - u/a] e^{-ak} + u/a
\]
Reduce down formula (5) to the original shape and the grey prediction model of original sequence \( x_0 \) can be obtained as
\[
\bar{x}_0(k+1) = \bar{x}_i(k+1) - \bar{x}_i(k), \quad k = 1, 2, 3, \ldots
\]
3.2. Feasibility Judgment of the Model

Not all the models of GM(1,1) are effective, only the sequence which satisfies the conditions of grey modeling can be used to establish the GM(1,1) model as shown below [6].

In the formula \(x_0 = \{x_0(1), x_0(2), \ldots, x_0(n)\}\), let \(\sigma_0(k)\) be the class-compare of \(x_0\), where

\[
\sigma_0 = \frac{x_0(k-1)}{x_0k}, \quad k \geq 3.
\]

Only when \(\sigma_0(k)\) satisfies the condition of \(\sigma_0(k) \in (0.1353, 7.383)\) can \(x_0\) be one of the basic conditions of establish the GM(1,1) model.

3.3. The Posterior Difference Test of GM(1,1) Model

Suppose the variance of \(x_0 = \{x_0(1), x_0(2), \ldots, x_0(n)\}\) and residual sequence (E) are \(S_1^2\) and \(S_2^2\) such that

\[
S_1^2 = \frac{1}{n} \sum_{k=1}^{n} [x_0(k) - \bar{x}]^2, \quad S_2^2 = \frac{1}{n} \sum_{k=1}^{n} [x_0(k) - \bar{x}]^2, \quad \bar{x} = \frac{1}{n} \sum_{k=1}^{n} x_0(k), \quad \bar{e} = \frac{1}{n} \sum_{k=1}^{n} e(k)
\]

The posterior difference ratio can be calculated as

\[
c = S_2/S_1
\]

The small error probability can be calculated as

\[
p = P \left( |e(k) - \bar{e}| < 0.67455S_1 \right)
\]

The criterion [8] of passing the test is that the smaller the accuracy is, the better it will be. And the accuracy grade is shown in Table 1.

<table>
<thead>
<tr>
<th>Accuracy grade</th>
<th>Good</th>
<th>Qualified</th>
<th>Barely qualified</th>
<th>Unqualified</th>
</tr>
</thead>
<tbody>
<tr>
<td>small error probability (p)</td>
<td>(p \geq 0.95)</td>
<td>(0.8 \leq p \leq 0.95)</td>
<td>(0.7 \leq p &lt; 0.8)</td>
<td>(p &lt; 0.7)</td>
</tr>
<tr>
<td>Mean variance ratio(c)</td>
<td>(c \leq 0.35)</td>
<td>(0.35 &lt; c \leq 0.5)</td>
<td>(0.5 &lt; c \leq 0.65)</td>
<td>(c \geq 0.65)</td>
</tr>
</tbody>
</table>

4. Empirical Analysis

The mining lies in the Southwest of the Hubei Province, China with nice mining resources. OHSAS18001 has been one of the management methods for many years with a good reputation in the local community as well as the society. Taking the mining industry as an example, this paper evaluates the effective safety investments and safety benefits by C2R model and grey relational analysis.

4.1. Safety Investment and Loss Statistics

As Jiangdong Bao et al. [9] described, the 4 first-grade indicators of the mine industry safety investment including safety technical measures etc. and 23 second-grade indicators including ventilation system etc. are chosen to analyze. Statistical information of safety investment is selected from 2011 to 2015. The accident loss indicators of the comparison sequence of safety investment are selected including the direct accident loss of the first-grade indicator etc. from 2011 to 2015. Common total accident loss algorithm includes the one-to-four direct and indirect ratio method of Heinrich [10] from USA and total loss method of Symonds (Economic Loss Estimation, 2009) also from USA which can be calculated by the formula:

\[
\text{Total loss} = \text{Covered losses} + A \times \text{Laying-off injury times} + B \times \text{Hospitalization times} + \text{Emergency medical injuries times} + D \times \text{No accident times}
\]

In the formula, A, B, C and D separately refer to the average amount of non insurance cost which stands for varieties of different accidents degree. Per capita direct loss of accidents need to be compared with the loss of previous year, so per capita direct loss of accidents was collected as 0.204 yuan/person of 2010, and the indirect loss of decrease loss output is 4 times of direct loss. And the key point of calculating safety benefits is to calculate the safety output as shown in Table 2[6]:
programming which corresponds to decision making units (which corresponds to Theorem 4.1. Let $x$ as well.

information for the improvement of production and management especially when “production department” are “effective scale” and “technical efficiency” at the same time.

The target value of the improvement work can be achieved with the aid of “projection” analysis” when the decision making unit ($DMU_{0}$) of non DEA efficiency is changed into DEA efficiency. Reference information for the improvement of production and management efficiency in the future can be provided as well.

Theorem 4.1. Let $x_{i0} = \theta \cdot x_{i0} - S_{i0}^{-0}$, $y_{r0}^{-0} + S_{r0}^0$. In the formula, $\theta$, $S_{i0}^{-0}$ and $S_{r0}^0$ are the optimal solution for linear programming which corresponds to decision making units ($j_0$). $(x_0, y_0)$ of the relative efficiency “projection” of DEA which corresponds to $(x_{i0}, y_{r0})$ is determined to be DEA efficiency. According to the above theorem, the before and after adjustment results of the “projection” analysis” are shown in table 3.

<table>
<thead>
<tr>
<th>Year</th>
<th>Safety technical measures</th>
<th>Industrial hygiene measures</th>
<th>Safety management and training</th>
<th>Labor protection products</th>
<th>Safety benefits</th>
</tr>
</thead>
<tbody>
<tr>
<td>2011</td>
<td>133.400</td>
<td>8.700</td>
<td>78.700</td>
<td>67.600</td>
<td>4.301</td>
</tr>
<tr>
<td>2012</td>
<td>157.200</td>
<td>9.500</td>
<td>89.100</td>
<td>76.800</td>
<td>4.427</td>
</tr>
<tr>
<td>2013</td>
<td>145.100</td>
<td>7.300</td>
<td>78.900</td>
<td>70.200</td>
<td>4.192</td>
</tr>
<tr>
<td>2014</td>
<td>189.800</td>
<td>8.100</td>
<td>92.300</td>
<td>88.600</td>
<td>4.774</td>
</tr>
<tr>
<td>2015</td>
<td>197.400</td>
<td>8.900</td>
<td>95.800</td>
<td>95.100</td>
<td>5.406</td>
</tr>
</tbody>
</table>

Due to the structural adjustment of the company, the author only collected data from 2011 to 2015 to predict the relevant data of the year of 2017.

4.2. Projection Analysis of C²R Model

Data envelopment analysis (DEA) was first introduced by famous operational research experts: A. Charnes, W.W. Cooper and E. Rhodes in 1978 [11]. Data envelopment analysis (DEA) can be regarded as a new method of statistical analysis, which is based on a set of observations about the input and output to estimate the effective production frontier. It is a very good and effective way to study several inputs especially when “production department” are “effective scale” and “technical efficiency” at the same time.

The results of DEA “projection analysis” of reference sequence and comparison sequence are shown as below:

Table 3: Before and after adjustment results of safety benefits “projection” analysis” of the mining industry OHSAS18001

<table>
<thead>
<tr>
<th>DMU</th>
<th>Before adjustment results</th>
<th>After adjustment results</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$X_1$</td>
<td>$X_2$</td>
</tr>
<tr>
<td>2011</td>
<td>133.400</td>
<td>8.700</td>
</tr>
<tr>
<td>2012</td>
<td>157.200</td>
<td>9.500</td>
</tr>
<tr>
<td>2013</td>
<td>145.100</td>
<td>7.300</td>
</tr>
<tr>
<td>2014</td>
<td>189.800</td>
<td>8.100</td>
</tr>
<tr>
<td>2015</td>
<td>197.400</td>
<td>8.900</td>
</tr>
</tbody>
</table>

Note: $Y_2 = Y_1 \times$ Total safety investment

4.3. Reference Sequence and Comparison Sequence Establishing

The results of DEA “projection analysis” of reference sequence and comparison sequence are shown as below:

$x_{(total safety benefits)}(0) = (1262.774, 1503.037, 1322.344, 1815.551, 2136.734),$

$x_{(safety technical measures)}(0) = (113.712, 135.997, 166.543, 166.084, 197.400),$

$x_{(industrial hygiene measures)}(0) = (4.734, 5.807, 5.105, 7.521, 8.900),$

$x_{(safety management and training)}(0) = (53.160, 64.273, 64.890, 80.589, 95.800),$ and

$x_{(labor protection products)}(0) = (54.321, 60.374, 56.088, 80.216, 95.100).$
4.4. The Calculating of a and u

According to formula (2), (3) and (4), the corresponding values of $a$ and $u$ of $x_{\text{total safety benefit}}$, $x_{\text{safety technical measures}}$, $x_{\text{industrial hygiene measures}}$, $x_{\text{safety management and training}}$ and $x_{\text{labor protection products}}$ as shown in table 4.

<table>
<thead>
<tr>
<th>Total safety benefit</th>
<th>Safety technical measures</th>
<th>Industrial hygiene measures</th>
<th>Safety management and training</th>
<th>Labor protection products</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>$u$</td>
<td>$a$</td>
<td>$u$</td>
<td>$a$</td>
</tr>
<tr>
<td>−0.149</td>
<td>1045.257</td>
<td>−0.161</td>
<td>90.748</td>
<td>−0.180</td>
</tr>
</tbody>
</table>

4.5. The Calculating of Grey Prediction Model of Original Series $x_0$

According to formula (5) and (6), the grey prediction model of $x_0$ can be calculated as shown below.

$\hat{x}_{(\text{total safety benefit})0}(k + 1) = 8270.746e^{0.149k} - 7007.971,$

$\hat{x}_{(\text{safety technical measures})0}(k + 1) = 676.93e^{0.161k} - 563.158,$

$\hat{x}_{(\text{industrial hygiene measures})0}(k + 1) = 25.658e^{0.110k} - 20.924,$

and $\hat{x}_{(\text{safety management and training})0}(k + 1) = 372.679e^{0.150k} - 319.518.$

4.6. The Predicted Value Calculating of 2017

The safety benefit and safety investment values of 2017 can be calculated based on the grey prediction model as shown in table 5.

<table>
<thead>
<tr>
<th>Total safety benefit</th>
<th>Safety technical measures</th>
<th>Industrial hygiene measures</th>
<th>Safety management and training</th>
<th>Labor protection products</th>
</tr>
</thead>
<tbody>
<tr>
<td>Predicted value of 2017</td>
<td>2798.691</td>
<td>264.4365</td>
<td>12.50894</td>
<td>126.5579</td>
</tr>
</tbody>
</table>

4.7. Feasibility Judgment Result of the Model

According to formula (7), the value of $\sigma_0(k)$ can be calculated as shown in table 6.

<table>
<thead>
<tr>
<th>Total safety benefit</th>
<th>Safety technical measures</th>
<th>Industrial hygiene measures</th>
<th>Safety management and training</th>
<th>Labor protection products</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.862</td>
<td>0.851</td>
<td>0.835</td>
<td>0.861</td>
<td>0.832</td>
</tr>
</tbody>
</table>

Concluded from table 6, the class-compare values can satisfy $\sigma_0(k) \in (0.1353, 7.383)$, which indicates the model is feasible.
4.8. The Posterior Difference Test Result of GM(1,1) Model

According to formula (9), p value and accuracy grade of the safety benefit can be calculated as shown below.

\[ |e(1) - \bar{e}| = |0 + 0.008| = 0.008 < 0.267, \]
\[ |e(2) - \bar{e}| = |0.235 + 0.008| = 0.243 < 0.267, \]
\[ |e(3) - \bar{e}| = |-0.236 + 0.008| = 0.228 < 0.267, \]
\[ |e(4) - \bar{e}| = |-0.106 + 0.008| = 0.098 < 0.267, \]
\[ |e(5) - \bar{e}| = |0.135 + 0.008| = 0.143 < 0.267. \]

Such that \( p = 1 \)

According to formula (8), the mean variance ratio \( c \) of the safety benefit can be calculated as \( c = 0.426 \), which belongs to “qualified”. In the same way the mean variance ratio \( c \) of each investment can be calculated as shown in table 7 together with \( p \) value and the grade.

<table>
<thead>
<tr>
<th>Safety benefit</th>
<th>Safety technical measures</th>
<th>Industrial hygiene measures</th>
<th>Safety management and training</th>
<th>Labor protection products</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C )</td>
<td>0.426</td>
<td>0.381</td>
<td>0.205</td>
<td>0.331</td>
</tr>
<tr>
<td>( P )</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>Result</td>
<td>qualified</td>
<td>qualified</td>
<td>good</td>
<td>good</td>
</tr>
</tbody>
</table>

5. Discussion and Suggestion

Compared with time series analysis method, this study deals with the data which appears to be irrelevant to the data sequence with a small amount of data. Additionally, the GM(1,1) model can get a higher prediction accuracy and a long-term prediction.

Concluded from table 3, the DEA analysis result of the mining industry in 2012 and 2014 belonged to non DEA efficiency, while table 3 gives a set of effective target values of DEA efficiency after adjustment, which provides a theoretical reference for the safety investment.

Concluded from table 8, the accuracy grades of safety management and training and labor protection products belong to “good”, while those of safety benefit safety technical measures, and industrial hygiene measures belong to “qualified”, which needs to further improve the accuracy of the indicators.

Predicted values of the safety benefit and safety investment of 2017 have provided an improved direction for the effective safety investment and a guidance for the strategic development of enterprises.

6. Conclusions

The safety benefits of the mine industry are result of the comprehensive function of the internal and external factors. The relative benefits of mine industry are closely related to the national macro regulation, the market situation, the quality of the products and the state of internal management of the mine industry, which are confirmed by the DEA analysis results. The steel market of Hubei Province falling into an all-time low in 2012 resulted in low relative benefits of that year. Additionally, internal reform of the mine industry in 2014 led to low relative benefits of that year.

This mathematical integrated model provides a theoretical direction for the management and investment for any enterprises. Although projection analysis can provide managers with the goal of improving the work, this is only theoretical. In the actual work, some indicators value may not be reduced. Therefore, the improvement measures should be made according to the actual situation to achieve transition to DEA efficiency combined with the increase of output.

Of the numerous grey models the classical GM(1,1) model is fitted to predict the exponential growth of the sequence, that is, the growth rate fluctuates steadily near a value. Thus, this limits the scope of the GM(1,1) model which puts forward an optimized and improved model demands according to actual data feature of the indicators.
References