Multiple robots motion control to transport an object

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Abstract. This paper presents transporting algorithm for multiple robots to transport a concave or convex object. The object transporting includes three processes: calculating proper points process; approaching proper points process; and transporting an object process. Using fuzzy sliding mode control algorithm, we design a kinematic velocity controller. We also propose a dynamic torque controller by adaptive sliding mode control algorithm. Finally, simulations and experiment show good performance of proposed methods.

1. Introduction

Object transportation has received considerable attention, over the past decade [1]. In [2], an object is pushed from a single rotational contact point. In [3], a robot pushes a disk shaped object. It allows compliance of the manipulated object against the obstacle. Kube et al. [4] presents swarm robots push a large box, and the system could be emergency. A great number of robots are applied to move an object [5]. Suaro et al. [6] described a heuristic method to calculate formation. Since the system should first calculate a series of candidate points, it increases computation time within the system. Considering pros and cons of above object transportation methods, this paper combines object pushing and object caging to transport concave and convex objects. In this paper, multiple robots transport a concave or convex object.

There are some method for robot motion control, such as EKF [7], optimal feedback control [8], robust control [9], intelligent control [10], decentralized control [11], and Sliding mode control. EKF requires a long time to update the desired states. Intelligent control requires high computation power. In robust control, a priori information about the bounds of the uncertain or time-varying parameters is necessary. About the drawback of decentralized control, it is difficult to coordinate overall system. Using fuzzy sliding mode control algorithm, the kinematic velocity controller is designed. Since it is difficult to produce the perfect velocity for robot dynamics as the kinematic controller, the torque controller is proposed, based on adaptive integral sliding mode control algorithm.

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2. Robot model

In Fig. 1, robot $R_i$ has a passive wheel and two actuated wheels to achieve the motion and orientation. $r_i$ is the radius of wheels. $2b_i$ is the distance between two wheels. The mass center of the robot is $M_i$, and $O_i$ is located in the middle point between two wheels. $a, x, y$ is a global reference frame, and $O_x, X, Y$ is the local reference frame. The motion of robot $R_i$ is as $q_i = (x_i, y_i, \theta_i)^T$, where $x_i, y_i$, and $\theta_i$ are $x_i$ coordinate, $y_i$ coordinate, and the bearing angle, respectively. The constraint of the robot is as:

$$\dot{y}_i \cos \theta_i - \dot{x}_i \sin \theta_i = 0 \quad (1)$$

$V_i = (v_i, \omega_i)^T$ is defined as linear and angular velocity of robot. The surface friction is ignored. The model of the nonholonomic robot is defined as:

$$\dot{q}_i = S(q_i)V_i(t) \quad (2)$$

$$\bar{M}(q_i)\dot{V}_i + C(q_i, \dot{q}_i)V_i + \tau_{id} = \bar{B}\tau_i \quad (3)$$

where $s(q_i) = \begin{bmatrix} \cos \theta_i & 0 \\ \sin \theta_i & 0 \end{bmatrix}$, $\bar{M} = \begin{bmatrix} M_i & 0 \\ 0 & I_i \end{bmatrix}$, $\bar{C} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, and $\bar{B} = \frac{1}{r_i} \begin{bmatrix} 1 & 0 \\ b & -b \end{bmatrix}$.

$\tau_i = (\tau_{i1}, \tau_{i2})^T$ is the torque on left and right wheels. $m_i$ is the mass of the robot. $I_i$ is the moment of inertia of the robot.

3. Object transporting

3.1. Calculating proper points process

In this section, robots calculate their proper points, based on the shape of an object. All robots are assumed to be the same size and model. $R_{\text{robot}}$ is the radius of the robot. The robot can measure the distance and the direction toward the object. Then, the object corner vectors is calculated as $X_j = (x_j, y_j)$, where $j (j = 1, 2, ..., n)$ is the number of the corners of the object counter-clockwise, $X_1 = X_{n+1}$. The center vector $C_j(x'_j, y'_j)$ for the object edge between $X_jX_{j+1}$ can be given by Eqs. (4) and (5). The height from center point $C_j$ of the object is defined as $d_j$.

$$x'_j = \frac{1}{2}(x_j + x_{j+1}) \quad (4)$$

$$y'_j = \frac{1}{2}(y_j + y_{j+1}) \quad (5)$$

The angle $\alpha_j$ is calculated as

$$\alpha_j = a \tan 2((y_{j+1} - y_j), (x_{j+1} - x_j)) \quad (6)$$
In Fig. 2, concave point is $X_{j+2}$, and the concave angle $\lambda_{j+2}$ can be calculated as

$$\lambda_{j+2} = \alpha_{j+2} - (\alpha_{j+2} + \alpha_{j+1} - \pi)/2$$  (7)

The distance connecting the corner point $X_{j+2}$ and the center point of the robot can be calculated as

$$d_{j+2}^R = R_{\text{robot}}/\sin((\alpha_{j+2} + \alpha_{j+1} - \pi)/2)$$  (8)

If one edge is much longer than the other edges, two follower robots contact with the same long edge of the object. Three robots form an isosceles triangle formation as in Fig. 2. $q_0$ describes the proper point of leader robot. $q_1$ and $q_2$ denote proper points for follower robots 1 and 2.

Two follower robots are symmetrically located by the center point of the edge. The Euclidean distance between two follower robots’ centers is defined as $k + 2R_{\text{robot}}$, where $k$ is a positive constant. $\chi(m)(m = 1, 2)$ is the Euclidean distance between the contact point of one follower robot and edge center point.

$$\chi(m) = \begin{cases} -\frac{1}{2}d_{rr} & m = 1 \\ \frac{1}{2}d_{rr} & m = 2 \end{cases}$$  (9)

Based on (4)-(6), the contact point vector $P_m(x_m^P, y_m^P)$ ($m = 1, 2$) for the follower robot is:

$$x_m^P = x_j^P + \chi(m) \cos \alpha_j$$  (10)

$$y_m^P = y_j^P + \chi(m) \sin \alpha_j$$  (11)

The angle $\beta_j$ is as Eq. (12). If the direction of $\beta_j$ is obtained from the direction of $\alpha_j$ in counter-clock wise, $\beta_j = \alpha_j + \pi/2$; otherwise, $\beta_j = \alpha_j - \pi/2$.

$$\beta_j = \alpha_j \pm \pi/2$$  (12)

The proper points $q_m$ ($m = 1, 2$) for two follower robots can be calculated as

$$x_m = x_m^P + R_{\text{robot}} \cos \beta_j$$  (13)

$$y_m = y_m^P + R_{\text{robot}} \sin \beta_j$$  (14)

Three robots form an isosceles triangle transportation formation. The height of the transportation formation $d_f$ is calculated as Eq. (15), where $\delta$ is a compensation constant.

$$d_f = d_j + 2R_{\text{robot}} + \delta$$  (15)
The proper point \( q_0 \) for the leader robot is:

\[
\begin{align*}
x_0 &= \frac{1}{2}(x_1 + x_2) + d_f \cos(\alpha_j + \pi/2) \\
y_0 &= \frac{1}{2}(y_1 + y_2) + d_f \sin(\alpha_j + \pi/2)
\end{align*}
\] (16)

To extend the object transportation direction range, two follower robots contact with different edges of the object. If the concave object is a triangle shape object, two follower robots contact two centers of two adjacent edges to form an isosceles triangle formation as in Fig. 3. \( q_0 \) describes the proper point of leader robot. \( q_1 \) and \( q_2 \) are proper points for follower robot 1 and 2, respectively. Two follower robots are positioned symmetrically the center of two different edges. In Fig. 3, two follower robots contact with \( C_j \) and \( C_{j+2} \).

The angle \( \beta_j \) is as Eq. (12). The proper point \( q_1 \) for follower robot \( R_1 \) can be calculated as

\[
\begin{align*}
x_1 &= x_j^* + R_{\text{robot}} \cos \beta_j \\
y_1 &= y_j^* + R_{\text{robot}} \sin \beta_j
\end{align*}
\] (18)

The proper point \( q_2 \) for follower robot \( R_2 \) is:

\[
\begin{align*}
x_2 &= x_{j+2}^* + R_{\text{robot}} \cos \beta_{j+2} \\
y_2 &= y_{j+2}^* + R_{\text{robot}} \sin \beta_{j+2}
\end{align*}
\] (19)

The angle \( \varphi_{12} \) from \( x \)-axis to the connecting line between follower robot 1 and 2 is given by

\[
\varphi_{12} = \tan 2((y_2 - y_1), (x_2 - x_1))
\] (22)

The leader robot and two follower robots form a triangle transportation formation. The height \( d_f \) of the transportation formation is calculated as (23), where \( \delta \) is a compensation constant.

\[
d_f = d_{j+1} + 2R_{\text{robot}} + \delta
\] (23)

The proper point \( q_0 \) for leader robot is:

\[
\begin{align*}
x_0 &= \frac{1}{2}(x_1 + x_2) + d_f \cos(\varphi_{12} + \pi/2) \\
y_0 &= \frac{1}{2}(y_1 + y_2) + d_f \sin(\varphi_{12} + \pi/2)
\end{align*}
\] (24)

3.2. Approaching proper points process

To approach robots’ proper points, safe paths are designed. The shortest safe paths are calculated, depending on the theta* algorithm [12]. As an obstacle, the object boundary is extended by robot radius.

3.3. Transporting an object process

If there is no obstacle, three robots form the transportation formation and move to the goal. If there is an obstacle, robots should avoid the obstacle first, and then move to their goals. During obstacle avoidance, the object and robots are recognized as a rigid body, which uses theta* algorithm to avoid the obstacle. According to the width of the rigid body, the boundary of the obstacle is extended.

4. Robot Motion Control

In this section, based on robots kinematic model (2), using sliding mode control algorithm, we first design a kinematic controller to bound the error posture \( E_i = (x_{IE}, y_{IE}, \theta_{IE})^T \), and \( \lim_{t \to \infty} \| (x_{IE}, y_{IE}, \theta_{IE})^T \| = 0 \). Secondly, based on robots dynamic model (3), we use adaptive integral sliding mode control algorithm to design a torque controller. Finally, we present the structure of the presented control scheme.
4.1. Kinematic controller

Based on waypoint state $q_{iw} = (x_{iw}, y_{iw}, \theta_{iw})^T$ and current state $q_{ic} = (x_{ic}, y_{ic}, \theta_{ic})^T$, the state errors defined as $e_i = (x_{ie}, y_{ie}, \theta_{ie})^T = q_{iw} - q_{ic}$. Add a Jacobian, we transfer the state error $e_i$ as $E_i$.

$$E_i = \begin{bmatrix} x_{ie} \\
 y_{ie} \\
 \theta_{ie} \end{bmatrix} = \begin{bmatrix} \cos \theta_{ic} & \sin \theta_{ic} & 0 \\
 -\sin \theta_{ic} & \cos \theta_{ic} & 0 \\
 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{ic} \\
 y_{ic} \\
 \theta_{ic} \end{bmatrix}$$

Using Eq. (26) and $x_{iw} \sin \theta_{iw} - y_{iw} \cos \theta_{iw} = 0$, $E_i$ is:

$$\dot{E}_i = \begin{bmatrix} \dot{x}_{ie} \\
 \dot{y}_{ie} \\
 \dot{\theta}_{ie} \end{bmatrix} = \begin{bmatrix} \cos \theta_{ic} & \sin \theta_{ic} & 0 \\
 -\sin \theta_{ic} & \cos \theta_{ic} & 0 \\
 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{v}_{iw} \\
 \omega_{iw} \\
 \dot{\theta}_{ic} \end{bmatrix} + \begin{bmatrix} 0 \\
 1 \\
 -1 \end{bmatrix} \begin{bmatrix} y_{ie} \\
 \dot{v}_{ic} \\
 -x_{ic} \end{bmatrix}$$

where $\dot{v}_{iw}$ and $\omega_{iw}$ are the reference linear and angular velocities.

According to the back-stepping algorithm, switching function is designed. When $x_{ie} = 0$ and assuming $\theta_{ie} = -\arctan(v_{iw}y_{ie})$, the Lyapunov function is

$$V_y = \frac{1}{2}y_{ie}^2$$

The derivative of the Lyapunov function is

$$\dot{V}_y = y_{ie}\dot{y}_{ie} = y_{ie}(v_{iw}\sin\theta_{ie} - x_{ie}\omega_{ie}) = -x_{ie}y_{ie}\omega_{ie} - y_{ie}\dot{v}_{iw}\sin(\arctan(v_{iw}y_{ie}))$$

Lemma: For any $x \in R$ and $|x| < \infty$, there is $\phi(x) = xsin(\arctan x) \geq 0$, iff $x = 0$, $\Rightarrow \Leftarrow$ is tenable.

Proof: when $x = 0$, then $\phi(0) = 0$; when $x \in (0, \infty)$, there is $\arctan x \in (0, \pi/2)$, then $\sin(\arctan x) > 0$, namely $\phi(x) > 0$; when $x \in (-\infty, 0)$, there is $\arctan x \in (-\pi/2, 0)$, then $\sin(\arctan x) < 0$, namely $\phi(x) > 0$.

Based on Lemma, $y_{ie}\dot{v}_{iw}\sin(\arctan(v_{iw}y_{ie})) \geq 0$. When $y_{ie}\dot{v}_{iw} = 0$, $y_{ie}\dot{v}_{iw}\sin(\arctan(v_{iw}y_{ie})) = 0$. When $x_{ie} = 0$ and $\theta_{ie} = -\arctan(v_{iw}y_{ie})$, Eq. (29) is:

$$\dot{V}_y = -x_{ie}y_{ie}\omega_{ie} - y_{ie}\dot{v}_{iw}\sin(\arctan(v_{iw}y_{ie})) \leq 0$$

The switching function $s_i$ for robot $R_i$ is as:
\[
\begin{align*}
s_i &= \begin{bmatrix} s_{i1} \\ s_{i2} \end{bmatrix} = \begin{bmatrix} x_{iE} \\ \theta_{iE} + \arctan(v_{ir}y_{ir}) \end{bmatrix} \\
\end{align*}
\]

(31)

The controller is designed to converge \( s_{i1} \) and \( s_{i2} \) to zero. Then \( x_{iE} \) converges to zero, and \( \theta_{iE} \) converges to \(-\arctan(v_{ir}y_{ir})\). The system state \( y_{iE} \) converges to zero.

To make \( s_i \) to converge, \( \dot{s}_i < 0 \), \( \dot{s}_i \) is defined as

\[
\begin{align*}
\dot{s}_i &= \begin{bmatrix} \dot{s}_{i1} \\ \dot{s}_{i2} \end{bmatrix} = \begin{bmatrix} \dot{x}_{iE} + \frac{\partial}{\partial x_{iE}} \theta_{iE} + \frac{\partial}{\partial y_{iE}} y_{iE} \\ \dot{\theta}_{iE} + \frac{\partial}{\partial \theta_{iE}} v_{ir} + \frac{\partial}{\partial y_{iE}} y_{iE} \end{bmatrix} \\
&= \begin{bmatrix} v_{ir} \cos \theta_{iE} - v_{i1} + y_{iE} \omega_{i1} \\ \omega_{i1} + \frac{\partial}{\partial \omega_{i1}} v_{i1} + \frac{\partial}{\partial \omega_{i1}} (v_{ir} \sin \theta_{iE} - x_{iE} \omega_{i1}) \end{bmatrix} \\
&= \begin{bmatrix} \omega_{i1} + \frac{\partial}{\partial \omega_{i1}} v_{i1} + \frac{\partial}{\partial \omega_{i1}} (v_{ir} \sin \theta_{iE} + \epsilon_{i1} \text{sgn}(s_{i1}) + k_{i1}s_{i1}) \\ \omega_{i2} + \frac{\partial}{\partial \omega_{i2}} v_{i2} + \frac{\partial}{\partial \omega_{i2}} (v_{ir} \sin \theta_{iE} + \epsilon_{i2} \text{sgn}(s_{i2}) + k_{i2}s_{i2}) \end{bmatrix}
\end{align*}
\]

(32)

The kinematic controller for robot \( R_i \) is:

\[
\begin{align*}
\dot{v}_i &= \frac{v_{ir} \cos \theta_{iE} + y_{iE} \omega_{i1} - \dot{s}_{i1}}{1 + \frac{\partial}{\partial x_{iE}} \theta_{iE}} \\
&= \frac{v_{ir} \cos \theta_{iE} + y_{iE} \omega_{i1} + \epsilon_{i1} \text{sgn}(s_{i1}) + k_{i1}s_{i1}}{1 + \frac{\partial}{\partial x_{iE}} \theta_{iE}} \\
&= \frac{v_{ir} \cos \theta_{iE} + y_{iE} \omega_{i1} + \epsilon_{i2} \text{sgn}(s_{i2}) + k_{i2}s_{i2}}{1 + \frac{\partial}{\partial x_{iE}} \theta_{iE}}
\end{align*}
\]

(33)

(34)

where \( \epsilon_{i1} \), \( \epsilon_{i2} \), \( k_{i1} \) and \( k_{i2} \) are positive constants. Setting \( \alpha_i = \arctan(v_{ir}y_{ir}) \), \( \dot{s}_i \) for robot \( R_i \) is obtained.

Proof: consider the Lyapunov function \( V_1 = \frac{1}{2} s_{i1}^2 + \frac{1}{2} s_{i2}^2 \). Differential \( V_1 \) about time \( t \) as

\[
\dot{V}_1 = -\epsilon_{i1} |s_{i1}| - k_{i1}s_{i1}^2 - \epsilon_{i2} |s_{i2}| - k_{i2}s_{i2}^2 \leq 0
\]

(35)

When \( x_{iE} = 0 \) and \( \theta_{iE} = -\arctan(v_{ir}y_{ir}) \), \( \dot{V}_1 = 0 \). When \( x_{iE} \neq 0 \) and \( \theta_{iE} \neq -\arctan(v_{ir}y_{ir}) \), \( \dot{V}_1 < 0 \). The system is asymptotically stable in the sense of Lyapunov.

To weaken the charting, two fuzzy sliding mode controllers are designed. In controller 1, \( |s_{i1}| \) and \( \epsilon_{i1} \) are the input and output variables, respectively. \( h_{i1} \) and \( h_{i10} \) are input and output scaling. Define \( |s_{i1}| = h_{i1} |s_{i1}| \) and \( \epsilon_{i1} = h_{i10} \epsilon_{i1} \).

Five fuzzy sets describe the fuzzy partitions, such as very big (VB), big (B), medium (M), small (S), and very small (VS). The triangular functions are applied for input variable and output variable, respectively. Fig. 4 shows the input and output membership functions. The fuzzy rules of the fuzzy sliding model 1 are as: IF \( s_{i1} \) is VS, THEN \( \epsilon_{i1} \) is VS.

The output is

\[
\epsilon_{i1} = \frac{\Sigma_{i=1}^{n} \mu_{i}(c_{i1})}{\Sigma_{i=1}^{n} \mu_{i}(c_{i})}
\]

(36)

Based on \( \epsilon_{i1} \), \( \epsilon_{i1} \) is used in Eq. (34) to obtain \( v_{i1} \). In controller 2, \( |s_{i2}| \) and \( \epsilon_{i2} \) are the input and output variables. \( \epsilon_{i2} \) is used to obtain \( \omega_{i2} \).
4.2. Dynamic controller

Define the kinematic controller in Eq. (34) as $V_{id} = [v_{id}, \omega_{id}]^T$, and the velocity tracking error is as

$$V_{id} = V_{id} - V_{ic} = \begin{bmatrix} v_{id} - v_c \\ \omega_{id} - \omega_c \end{bmatrix}$$

(37)

The integral sliding surface is defined as

$$s_i = \begin{bmatrix} s_{i1} \\ s_{i2} \end{bmatrix} = V_{ic} + \eta \int_0^\tau V_{ic}(\tau)d\tau$$

(38)

where $\eta$ is a positive integral constant.

Ignoring the unknown disturbance $\tau_{id}$, based on (3), the following equation can be obtained:

$$\dot{s}_i(t) = V_{ic}(t) + \eta V_{ic}(t) = V_{id} - V_{ic} + \eta V_{ic}(t) = V_{id} - A_i \tau_i + \eta V_{ic}(t)$$

(40)

The dynamic torque controller is designed as

$$\tau_i = \tau_{eq} + \tau_{saw} = A_i^{-1}[V_{id} + \eta V_{ic} + K_s sgn(s_i)]$$

(41)

where $\tau_{eq}$ is equivalent control. $\tau_{saw}$ is switching control, and is able to constraint unknown disturbance. $s_{sgn} = \begin{bmatrix} s_{sgn1} \\ s_{sgn2} \end{bmatrix}$. $K_i$ is a positive definite gain. Define $K_i = \hat{c}_i b_i s_{sgn}$. $\hat{c}_i$ is the estimate of $c_i$, $\hat{c}_i$ is a positive constant, and $\hat{c}_i = \hat{c}_i - c_i$.

Proof: consider the Lyapunov function $V_2 = \frac{1}{2}s_i^T s_i + \frac{\hat{c}_i^2}{\gamma_i}$. Differential $V_2$ about time $t$ as

$$\dot{V}_2 = s_i^T \dot{s}_i + \frac{\hat{c}_i}{\gamma_i}$$

$$= s_i^T (V_{id} - A_i \tau_i + \eta V_{ic}(t)) + \frac{\hat{c}_i}{\gamma_i}$$

$$= s_i^T [V_{id} - A_i A_i^{-1}[V_{id} + \eta V_{ic} + K_s sgn(s_i)] + \eta V_{ic}(t)] + \frac{\hat{c}_i}{\gamma_i}$$

$$= -s_i^T K_s s_{sgn}(s_i) + \frac{\hat{c}_i}{\gamma_i}$$

$$= -s_i^T \hat{c}_i b_i [s_{sgn}(s_i)]^2 + \frac{\hat{c}_i}{\gamma_i}$$

$$\leq -s_i^T \hat{c}_i b_i [s_{sgn}(s_i)]^2 - \hat{c}_i (||s_i|| - \hat{c}_i)$$

(42)
The parameter is selected as

\[ c_i = \gamma_i \|s_i\| \]  

Therefore, (42) can be rewritten as

\[ \dot{V}_2 \leq -s_i^T \hat{c}_i s_{i\text{sgn}(s_i)}^2 - \hat{c}_i (\|s_i\| - \frac{\dot{c}_i}{\|s_i\|}) \]

\[ = -s_i^T \hat{c}_i s_{i\text{sgn}(s_i)}^2 \leq 0 \]  

when \( V_{ic} = 0, \dot{V}_2 = 0 \). When \( V_{ic} \neq 0, \dot{V}_2 < 0 \). The system is asymptotically stable.

The structure of the presented control scheme is illustrated in Fig. 5. The waypoint state of the robot and the reference linear and angular velocities of the robot are system input. The system output is the configuration of the robot. Using the sliding mode control algorithm, the system calculates the kinematic velocity controller. Based on adaptive integral sliding mode control algorithm, the system calculates the dynamic torque controller.

5. Simulations

The physical parameters of the robots are as: \( b_i = 0.1315m, r_i = 0.095m, m_i = 7.52kg, l_i = 2.325kgm^2 \). Controller parameters are chosen as: \( k_{i1} = k_{i2} = 0.0001, \varepsilon_{i1} = \varepsilon_{i2} = 0.001, \gamma_i = 0.01 \). In the simulations, the time step is defined as: \( dt = 0.1s \). The velocity constraints of the robot are set as: \( v_{\text{max}} = 0.08(m/s) \), and \( \omega_{\text{max}} = 0.62(\text{rad/s}) \).

Fig. 6 shows that three robots transport a convex object. The initial states of three robots are as follows: \((0.2m, 1.7m, 0)^T\), \((0.2m, 0.2m, 0)^T\), and \((0.2m, 1m, 0)^T\). Based on the calculating proper points process for a convex object, proper points for three robots are located at \((1.8m, 1.7m)\), \((1.2m, 0.2m)\), and \((1m, 1.1m)\). The object is regarded as an obstacle, three robots approach to their proper points, based on theta star algorithm. After approaching proper points, three robots maintain the transportation formation to transport the convex object. Leader robot’s trajectory is calculated by Eqs. (45)-(47).

\[
\begin{align*}
x_1(t+1) &= x_1(t) + 0.04 \\
y_1(t+1) &= y_1(t) + 0.04 \\
\theta_1(t+1) &= \text{atan2}((y_1(t+1) - y_1(t)), (x_1(t+1) - x_1(t)))
\end{align*}
\]

Figs. 7-9 show trajectory tracking errors of three robots, using the proposed control scheme. Figs. 10-12 show velocity tracking errors of three robots, using the proposed control scheme.

This experiment shows that three robots can transport a concave object, avoiding two obstacles, as in Fig. 13. The initial states of three robots are as: \((0.2m, 1.7m, 0)^T\), \((0.2m, 0.2m, 0)^T\), and \((0.2m, 1m, 0)^T\). Based
Figure 6: Three robots transport a convex object.

Figure 7: Leader robot’s trajectory tracking errors.

Figure 8: Follower robot 1’s trajectory tracking errors.

Figure 9: Follower robot 2’s trajectory tracking errors.

Figure 10: Leader robot’s velocity tracking errors.
Figure 11: Follower robot 1’s velocity tracking errors.

Figure 12: Follower robot 2’s velocity tracking errors.

on the calculating proper points process for a concave object, proper points for three robots are located at (1.9m, 1.8m), (2.5m, 1m), and (1.4m, 1m). The obstacles are on (0.7m, 2.6m) and (0.7m, 3.5m). The leader robot’s trajectory is calculated by Eqs. (48)-(50).

\[
x_1(t+1) = x_1(t) \tag{48}
\]
\[
y_1(t+1) = y_1(t) + 0.055 \tag{49}
\]
\[
\theta_1(t+1) = a \tan 2((y_1(t+1) - y_1(t)), (x_1(t+1) - x_1(t))) \tag{50}
\]

6. Experiment

Three mobile robots are used to transport an object. The robot uses the odometry platform to localize itself. Parameters in the experiment are the same as in the simulation. Fig. 14 shows snapshots of three robots transport a convex object. At t=0, three robot form a line formation. At t=55s, three robots form a transportation formation. At t=99s, the object is transported to the goal point. Fig. 15 presents trajectories of three robots. Comparing the result in Fig. 6, three robots’ trajectories are approximately same. The experiment video link is https://www.dropbox.com/s/4vo6crd0w4upq5x/video

7. Conclusion

In order to solve concave or convex object transportation problem, three transportation processes are designed. For robot motion control, depending on fuzzy sliding mode control algorithm, we designed a kinematic velocity controller; according to adaptive integral sliding control algorithm, we proposed a dynamic torque controller. The simulations and experiment show good performance of all methods.
Figure 13: Three robots transport a concave object avoiding two obstacles.

Figure 14: Snapshots of three robots (a) t=0; (b) t=55s; (c) t=99s (From left to right).

Figure 15: Three robots trajectories.
References