

Departman za računarske nauke - Prijemni ispit
iz matematike - rešenja

1.

$$\begin{aligned}\tan 4\alpha - \frac{1}{\cos 4\alpha} &= \frac{\sin 4\alpha}{\cos 4\alpha} - \frac{1}{\cos 4\alpha} = \frac{\sin 4\alpha - 1}{\cos 4\alpha} = \\ &= \frac{-(\sin^2 2\alpha + \cos^2 2\alpha - 2 \sin 2\alpha \cos 2\alpha)}{(\cos 2\alpha - \sin 2\alpha)(\cos 2\alpha + \sin 2\alpha)} = \\ &= \frac{-(\sin 2\alpha - \cos 2\alpha)^2}{(\cos 2\alpha - \sin 2\alpha)(\cos 2\alpha + \sin 2\alpha)} = \\ &= \frac{\sin 2\alpha - \cos 2\alpha}{\cos 2\alpha + \sin 2\alpha}\end{aligned}$$

2.

$$\begin{aligned}\sin^2 x + \cos 2x &> \frac{1}{4} \\ \sin^2 x + \cos^2 x - \sin^2 x &> \frac{1}{4} \\ \cos^2 x &> \frac{1}{4} \\ |\cos x| &> \frac{1}{2} \\ \cos x < -\frac{1}{2} \text{ ili } \cos x > \frac{1}{2} \\ x &\in \left(-\frac{\pi}{3} + 2k\pi, \frac{\pi}{3} + 2k\pi\right) \cup \left(\frac{2\pi}{3} + 2k\pi, \frac{4\pi}{3} + 2k\pi\right)\end{aligned}$$

3.

$$\begin{aligned}\sin^2 x + 3\sin x \cos x + 7\cos^2 x &= 6, \cos x \neq 0 \\ \sin^2 x + 3\sin x \cos x + 7\cos^2 x &= 6\sin^2 x + 6\cos^2 x \\ -5\sin^2 x + 3\sin x \cos x + \cos^2 x &= 0\end{aligned}$$

$$-5 \tan^2 x + 3 \tan x + 1 = 0$$

Uvedimo smenu $\tan x = t$

$$-5t^2 + 3t + 1 = 0$$

$$t_{1,2} = \frac{-3 \pm \sqrt{29}}{-10}$$

$$\text{Dakle, } x = \arctg\left(\frac{-3+\sqrt{29}}{-10}\right) + k\pi \vee x = \arctg\left(\frac{3+\sqrt{29}}{10}\right) + k\pi, k \in Z$$

4. Pronađi vrednost parametra a tako da je prava $(2a - 3)x + ay - 2 = 2y + 3a$ normalna na pravu $(3a - 1)y + 2(3a + 1)x = a - 1$

$$(2a - 3)x + ay - 2 = 2y + 3a$$

$$ay - 2y = (3 - 2a)x + 3a + 2$$

$$y = \frac{3 - 2a}{a - 2}x + \frac{3a + 2}{a - 2}, a \neq 2$$

koeficijent pravca je: $k_1 = \frac{3-2a}{a-2}$

$$(3a - 1)y + 2(3a + 1)x = a - 1$$

$$(3a - 1)y = -2(3a + 1)x + a - 1$$

$$y = \frac{-2(2a + 1)}{3a - 1}x + \frac{a - 1}{3a - 1}, a \neq 1/3$$

koeficijent pravca je: $k_2 = \frac{-2(3a+1)}{3a-1}$

Dalje, kako je $k_1 \cdot k_2 = -1$, važi da je

$$\frac{3 - 2a}{a - 2} \cdot \frac{-2(3a + 1)}{3a - 1} = -1$$

$$2(3 - 2a)(3a + 1) = (a - 2)(3a - 1)$$

$$14a + 6 - 12a^2 = 3a^2 - 7a + 2$$

$$-15a^2 + 21a + 4 = 0$$

$$a_{1,2} = \frac{-21 \pm \sqrt{681}}{-30}$$

5.

$$a = 12cm, b = 8cm, c = 6cm$$

$$P = \frac{a+b}{2} \cdot h = \frac{12+8}{2} \cdot 6 = 60cm^2$$

6.

$$a = 7cm, b = 24cm, c = 25cm$$

$$P = \sqrt{s(s-a)(s-b)(s-c)}$$

$$s = \frac{a+b+c}{2} = 28$$

$$P = \sqrt{28(28-7)(28-24)(28-25)}$$

$$P = 7 \cdot 34 = 84cm^2$$

Dalje, kako je $P = \frac{b \cdot h_b}{2}$ imamo da je

$$84 = 12h_b$$

Pa je visina $h_b = 7cm$.