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VISUAL COMMUNICATION THROUGH VISUAL MATHEMATICS

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Abstract

In this paper we present some possibilities how different areas of visual mathematics (symmetry in art and science, isometric symmetry groups, similarity symmetry, modularity, antisymmetry, tessellations, theory of proportions, theory of visual perception, perspective, anamorphoses, visual illusions, ethnomathematics, mirror curves, op-tiles, fractal structures) can be used as a tool of visual communication. The paper also contains (in parts) a description of the course "Visual Mathematics and Design" organized at the Faculty of Information Technologies (Belgrade).

1 Introduction

It is well known that visual communication is universal and the oldest way of communication, from the prehistory until today. From the other side, the need for multidisciplinary courses is rapidly arising in the last few years, especially in the areas of applied sciences which are linked to different types of art expression and are using a variety of software. There is a need for such kind of courses in Serbia also. But, in order to use computers as creative artistic tools, it is necessary to know what is the starting point and what is the idea in behind, can we use it for our own purposes, and how?

For the first time in this region we established the course *Visual Mathematics and Design*. It is one semester course in the first year of the undergraduate study of Graphic Design at the *Faculty of Information Technologies* (FIT) in Belgrade. The basic idea was to introduce students in various mathematical objects and their

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basic properties, their visual message and visual identity, and get them familiar with different software which can be used to construct and manipulate such objects.

The course has been established as a series of essays about visualization of natural, mathematical, geometrical and abstract structures. The initial concept was that almost everything, including the most abstract structures, can be visually presented and thus become clearly understandable. The course puts together subjects related to computer graphics, mathematics, design and some art and architecture disciplines and provides a base for designing visual presentations. During the course, students do several home-works and projects based on teaching materials and suggestions. The students were given instructions how to use software through the examples from the teaching materials. All students learned to use *Inkscape*, *KnotPlot* and *Ultrafractal*. For home-works and projects, students used any appropriate software they are familiar with.

In this work we are going to give attention only on several main topics connected with visual communication, specially Mirror curves and knot work. Copyrights of all students' works presented in the paper belong to the authors.

2 Symmetry in Art and Science

The first theme, *Symmetry in Art and Science* represents a very wide area of concern. There we can recognize some theme of particular interest for math, chemistry, biology, but also architecture, design and art. The beginning in symmetry exploration could be made through student's individual investigation *Symmetry everywhere* by making the set of photos with appropriate comments about symmetry. (what kind of symmetry is present, what kind of visual effects does it make) (Fig.1). In this way, starting with an informal concept of symmetry, we come to the formal, mathematically based concept of symmetry, isometric transformations (reflection, rotation, translation and glide reflection) and their symbolic notation [5, 6, 15, 18, 21, 25, 26, 27].

Figure 1: Set of photos from students' exploration on the theme *Symmetry everywhere* (Marko Milanovic)

The next topic, *Isometric symmetry groups*, is dedicated to the symmetry of rosettes, friezes and ornaments. Here, we introduce the concept of invariants and the concept of symmetry groups and their presentations (generators and relations), and consider the symmetry of natural structures (symmetry of crystals, regular and uniform polyhedra) and their symmetry groups (17 symmetry groups of ornaments, 230 crystallographic symmetry groups, point groups and symmetry of polyhedra). Through this topic, by visualization, students are easily taken into the concept of a subgroup of symmetry group, relations between groups and subgroups, and the meaning of the subgroup index in group.

The goal of this subject is to learn how to find out and recognize construction methods by working on the examples from ornamental art [15, 18, 21, 25, 26, 27]. During the study, students should be able to create rosettes, friezes and ornaments (Fig. 2) by using different software for exploring symmetry groups and tessellations. At the end of this topic, the concept of tessellations with the special attention to M.C.Escher's should be presented. After analyzing his artworks [19], students alone constructed different plane tessellations [12, 15].

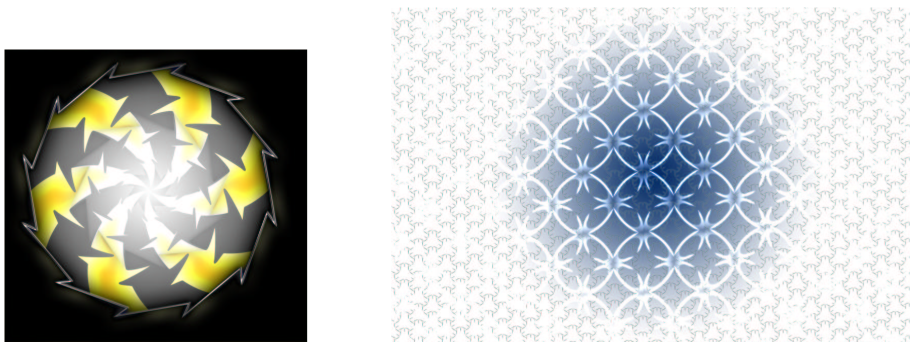


Figure 2: Students' works on rosettes and friezes (Milos Nikolic)

3 Modularity

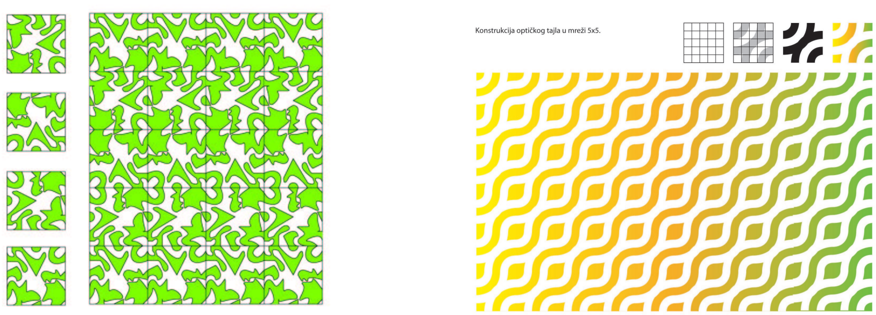
As a particular topic, we are going to set apart the *concept of Modularity*. The concept of Modularity should be introduced by recognizing modular structures in nature, art and science [13, 14, 15, 23, 24]. Modularity is treated as a generalization of symmetry and manifestation of the principle of economy: a possibility to crate a variety of structures from a few basic elements - modules.

Using the combinations of polygons from 11 uniform Archimedian tilings or prototiles producing an impression of space structures and colored prototiles, we may obtain artistic interlacing patterns, examples of modular design: the use of a few initial elements (modules - prototiles) for creating an infinite collection of designs. One of the simplest examples of module is Truchet tile and its variation - a square with black and white diagonals. One can make a series of different modular structure using the same element with different recombination. This idea can be used in any regular tessellations - by the variation of inside design of basic element and taking care that design on the border of elements would allow their further recombination.

(a) Marko Milanovic (b) Marko Milanovic (c) Milos Adamovic

Figure 3: The use of modular prototiles (Truchet tile) for construction of modular structures

Through this topic we investigate the choice of basic modules, modular construction, the level of complexity of obtained structures, modular archetypes (Truchet tile), Op-tiles, Space-tiles, Knot-tiles, and explored by individual work the principles of recombination and visual identity, economy and diversity as a result of modularity (Fig 3 and 4).



(a) Strahinja Ivkovic (b) Miroslav Zec

Figure 4: The use of optic tiles for construction of modular structures

Previous work with black and white squares can be used to present the concept related to the *Theory of Binary Codes*: bivalency as a basis of logical thinking. *Antisymmetry* is illustrated by construction of "black-white" ornaments, where students can perceive the relation between the figure and ground, the principle of duality, and visual dynamic of antisymmetric structures. In the series of lectures Antisymmetry Ornaments, students are getting familiar with 17 antisymmetry groups of friezes and with 46 "black-white" ornaments occurring in the history of ornamental art, from Neolithic until today [12, 19, 22, 23, 24]. Based on this knowledge, students experimented with antisymmetry and made their own antisymmetric constructions (Figure 5).

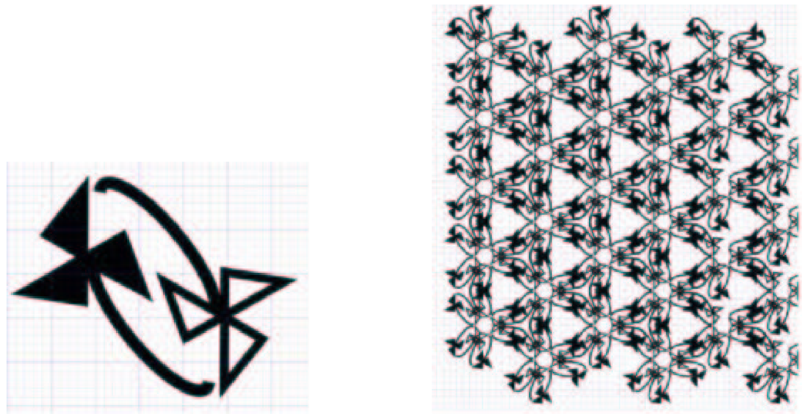
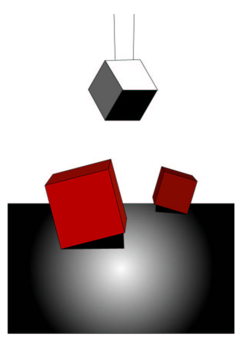


Figure 5: Antisymmetry investigation (Miroslav Gainov)

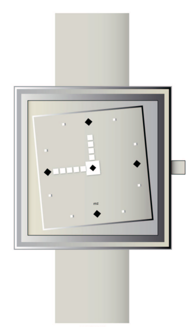
4 Visual perception

The next topic is dedicated to the *Theory of Visual Perception* and mechanisms of visual perception. We can analyze visual perception of 2D and 3D objects, perspective and its special limiting cases- anamorphoses [4]. It is possible to experiment with the structures using only one mathematical object: square, circle, or triangle (Fig 6).



(a) Filip Milovanovic

(b) Miroslav Zec



(c) Miroslav Zec

Figure 6: Different structures obtained using only one mathematical object

Through investigation of representations of 3D space in 2D plane, we can analyze different ways of representing space in the history of art (prehistoric, Egyptian, Greek and Renaissance art), static and dynamic 3D perception (e.g., different phenomena which provide the illusion of movement). In the section *Visual Illusions* are considered visual mechanisms are considered which are responsible for them, illustrated by examples of static and dynamic visual illusions (Fig. 7).

We also investigate optical properties of objects and analyze the dynamics of static visual objects. We should take special attention on some well known *Impossible objects* (tribar, Kofka's cube) [8]

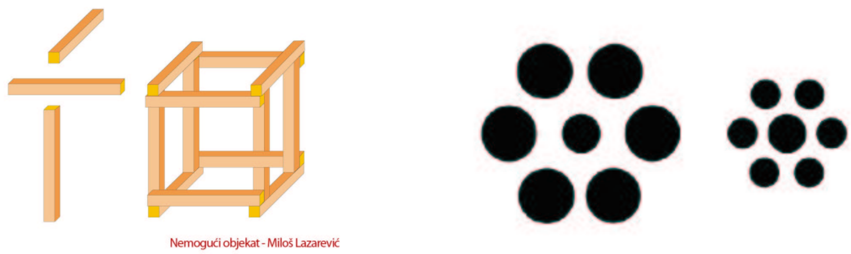


Figure 7: Visual illusions and impossible objects (Milos Lazarevic)

Within this topic, we can also pay attention to the elements of the *Theory of Proportion* : golden section, Fibonacci sequence, Similarity symmetry (dynamic symmetry) which could be recognized in natural structures and in the process of growing (logarithmic spiral, meander, maze structures) [15, 18].

5 Ethnomathematics

As a part of Ethnomathematics, students learned the basics of *Graph theory* and its applications in visual mathematics. Graphs occurring in different cultures are reviewed and explained as a universal tool for illustrating visual relations, with a special attention to the application of graphs in visual presentations of real-life models (traffic, telecommunication) [2]. In particular, students are introduced in the concept of *Mirror Curves* through the examples of mirror-curves in ornamental art (Chokwe sand drawings, Tamil "pavitram" curves, and Celtic knots) [3, 10, 11].

5.1 Mirror curves

What is a Mirror curve? Start with any connected edge-to-edge tiling of a part of a plane by polygons. Connect the midpoints of adjacent edges to obtain a 4-regular graph: every vertex is incident to four edges, called steps. Every closed path in this graph, where each step appears only once, is called a component. A mirror curve is the set of all components. Since the graph is 4-valent, at each vertex we have three choices of edges to continue the path: to choose the left, middle, or right edge. If the middle edge is chosen the vertex is called a crossing. Every mirror curve can be converted into a knotwork design by introducing the relation "over-under".

The name "mirror curves" can be justified by visualizing them on a rectangular square grid $RG[a,b]$ of dimensions a,b $(a,b \in N)$, whose sides are mirrors, and additional internal two-sided mirrors are placed between the square cells, coinciding with an edge, or perpendicular to it at its midpoint. In this grid, a ray of light, emitted from one edge-midpoint at an angle of 45^0, will close a component after a series of reflections. Beginning from a different edge-midpoint, and continuing until the whole step graph is used, we trace a mirror curve. This construction can be extended to any connected part of a regular triangular, square or hexagonal tessellation, this means to any polyiamond, polyomino or polyhexe, respectively.

The (culturally) ideal design is composed of a single continuous line. The well known fact is that for a rectangular square grid of dimensions a,b where a and b are relatively prime, the mirror curve is always a single closed curve uniformly covering the rectangle. Moreover, there is one more beautiful geometrical property: mirror curves can be obtained using only a few different prototiles. In particular, only three prototiles are sufficient for the construction of all mirror curves with internal mirrors incident to the cell-edges of a regular triangular tiling, five for square, and 11 for hexagonal regular tiling [17].

The common geometrical construction principle of Mirror curves, discovered by P. Gerdes, is the use of (two-sided) mirrors incident to the edges of a square, triangular or hexagonal regular plane tiling, or perpendicular to the edges in their midpoints [9, 10, 11]. In the ideal case, after a series of consecutive reflections, the ray of light reaches its initial point, defining a single closed curve. In other cases, the result consists of several closed curves.

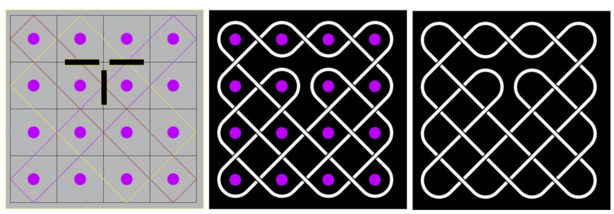


Figure 8: Construction of mirror curves (Strahinja Ivkovic)

5.2 Construction of mirror curves

Can we find a mathematical principle behind constructing a perfect curve - single line placed uniformly in a regular tiling? In principle, any polyomino (polyiamond or polyhexe) with mirrors on its border, and two-sided mirrors between cells or perpendicular to the internal cell-edges in their midpoints, can be used for creating perfect curves.

We propose the following construction from a polyomino (polyiamond or polyhexe): first, construct all different curves in a polyomino containing lines that connect different cell-edge midpoints until the polyomino is uniformly covered by k curves. In order to obtain a single curve, place internal mirrors and use ”curve surgery”, according to the following rules: any mirror placed in a crossing point of two distinct curves connects them in one curve; depending on the position of a mirror, a mirror placed into a self-crossing point of an (oriented) curve either does not change the number of curves, or breaks the curve in two closed curves (Fig. 9).

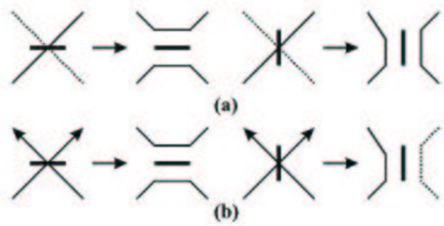


Figure 9:

Placing the minimal number of mirrors, we need to obtain a single curve, and to preserve this property when we add other mirrors. In the case of a rectangular square grid of dimensions a, b the initial number of curves, obtained without internal mirrors is $k = GCD(a, b)(GCD$ - greatest common divisor).

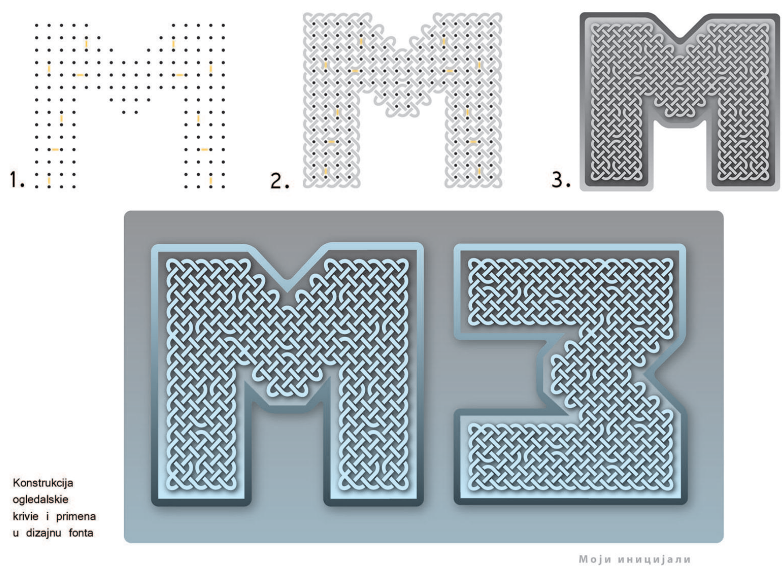


Figure 10: Construction of mirror curves and its application in font design (Miroslav Zec)

According to the rules for placing internal mirrors, we propose the following algorithm for creating mono-linear designs: in every step one of $k-1$ internal mirrors is placed at a crossing point belonging to different curves. After this, when the curves are combined and transformed into a single line, we can add other mirrors according to the rules described above, taking care of the number of curves. We can combine different patterns to obtain more complicated mirror curves and knots [10, 17]

Inspired by P. Gerdes' work, students made their own works on Mirror curves, Lunda designs, Lunda fractals and knotwork lettering (Fig. 8, 10 and 11). More about Lunda fractal and Lunda design can be found in [10].

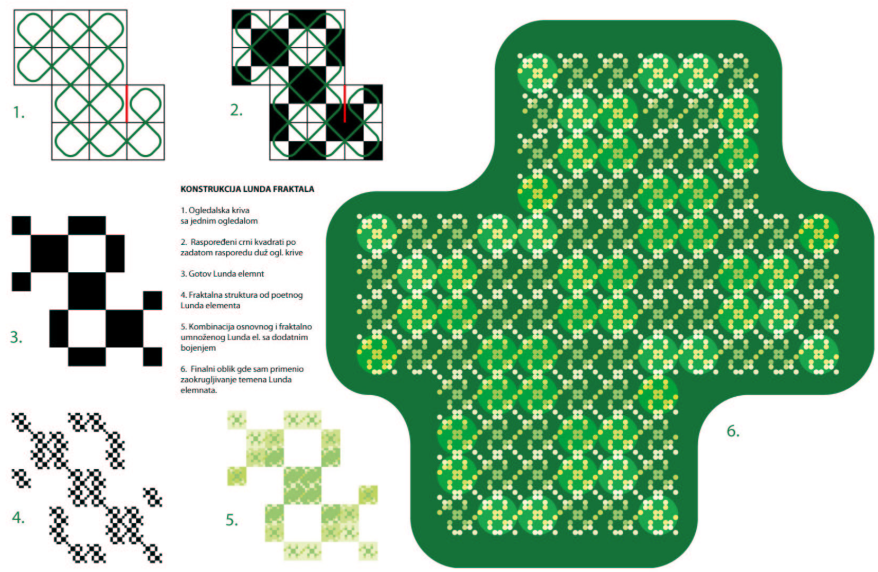
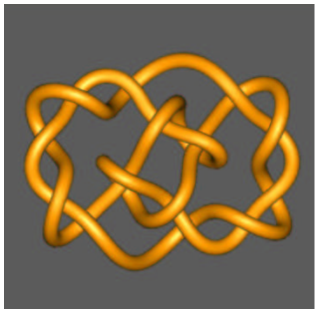


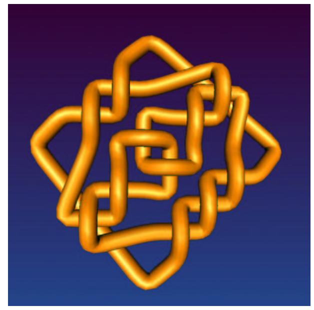
Figure 11: Mirror curves, Lunda design and construction of Lunda fractals (Miroslav Zec)

6 Elements of Knot Theory

Within the theme concert on visual mathematics, we need to mention an introduction to knot theory and should present basic elements of topology, surfaces in space, one and two-sided surfaces, minimal surfaces and their models (torus, Mbius band, Klein bottle). Decorative knots have been used from prehistory till our days as a basis of different artworks, so they still can be an inspiration for artists and artisans. Within the theme *Elements of Knot Theory* students become familiar with the mathematical concept of knots and links, basic terminology and coding of knots: isotopy, knot and link diagrams, Dowker and Gauss codes, Conway notation, Reidemeister moves, minimal diagrams, families of knots and links, tangles and basic polyhedra [1, 9, 20]. Special attention is given to the applications of knots in science and art. Students worked with real knots, their models and drawings, and explored their properties (amphichirality, unknotting, relaxation, symmetry) by using knot theory software: "KnotPlot" and "LinKnot" (Figure 12).



(a) Marko Milanovic



(b) Miroslav Zec

Figure 12: Exploration and construction of knots with KnotPlot

7 Symmetry in Architecture

The next theme presents *Symmetry in Architecture*: symmetry of 3D structures, discussing static and dynamic architectural symmetry by analyzing classic and modern construction principles in architecture. Modern architecture and design is reviewed: thanks to the usage of computers and the possibilities of computer design and usage of new materials, the construction of various new structures become possible, including organic-like structures. This is illustrated by examples of contemporary architecture (F.Gehry, M.Watanabe, H.Lalvani, and S.Calatrava). Students also learned about modular architectural design, multidimensional polytopes and their use in architectural projects (K.Mizayaki).

In this way, we come back to the *Symmetry as the Organization Principle of Art Work*. We should give an overview of different applications of symmetry in the history of art. It is pointed out that symmetry can be used as the relevant criterion for the analysis of different patterns occurring in the history of art, in cultures distant in space and time, as well as the successful method for their reconstruction and recognition of construction methods used by different cultures (D. Crowe, D.K. Washburn) [26]. This approach is not restricted only to isometric symmetry and similarity symmetry, but extended to anti-symmetry, colored symmetry, modularity (as a form of recombination), aperiodicity, "order"-"disorder" principle, and self-referential systems (e.g., fractals). Supporting the Gestalt theory approach, symmetry is recognized as an important element of visual perception. On the other hand, desymmetrization and symmetry breaking are distinguished as dynamic principles in creating art-works.

8 Fractals

There is one more topic that shouldn't be omitted. *Fractals* are included in the course as the illustration of *self-referential systems*. Through the examples of Koch, Peano and Dragon curve, students become familiar with the basic terms of the fractal theory and the concept of recursion, iterations and iteration series [20]. Moreover, students learned about *L-systems* (Lindenmayer systems), and natural recursive systems. So, they become more familiar with the concept of fractals and fractal structure. As an exercise, students generated self referent systems (Fig.11) and some fractal images using free software ("Fractint" and "Ultrafractal").

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