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CONSTRUCTIVE PROCEDURE FOR DETERMINATION OF ABSOLUTE CONIC FIGURE IN GENERAL COLLINEAR SPACES

Sonja Krasić *, Vladan Nikolić

Abstract

When they are collinear, projective spaces set with five pairs of biunivocally associated points are general. In order to map quadrics (II degree surfaces), in these spaces, the absolute conic was used. Geometrical position of all the absolute points in the infinitely distant plane of one space, i.e. an absolute conic of space cannot be graphically represented. To the infinitely distant planes are associated by the vanishing planes, and the absolute conics are associated by the conic in the vanishing planes, that is, figures of the absolute conics. Prior to mapping the quadrics, it is necessary to constructively determine the characteristics parameters such as the vanishing planes, axes and centers of space, and then the figures of the absolute conics, in the vanishing planes of both spaces. In order to constructively determine the figure of the absolute conic in the second space, a sphere in the first space was used, which maps into a rotating ellipsoid in the second space. The center of the sphere is on the axis of the first space, and the infinitely distant plane intersects it along the absolute conic. The associated rotational ellipsoid, whose center is on the axis of the seconds space is intersected by the vanishing plane of the first space along the imaginary circumference a_I, whose real representative is circumference a_z. The circumference a_I is the figure of the absolute conic of the first space. General collinear spaces are presented in a pair of Monge's projections.

1 Introduction

In order to simplify the mapping, it is necessary to determine the characteristic parameters in the general collinear (GC) spaces θ^1 and θ^2, set by five pairs of biunivocally associated points A_1, B_1, C_1, D_1, E_1 and A_2, B_2, C_2, D_2, E_2, (fig.1) which are presented in a pair of Monge's projections. The projective geometry methods, with the aid of cross ratio on the associated sequences of points, firstly determine the vanishing planes N_1 and M_2, which are associated to the infinitely distant planes

*Corresponding author
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N_2^∞ and M_1^∞, in both spaces. Then in the same way the space axes, o_1 in the space θ^1 and o_2 in the space θ^2, are determined, and they are the only two associated straight lines perpendicular on the vanishing planes and the centers of the spaces, W_1 in the space θ^1 and X_2 in the space θ_2, piercing points of the axes o_1 and o_2 through the vanishing planes N_1 and M_2, (fig.1).

Conjugate imaginary elements are defined in the projective space as real, with the aid of involutory sequences of points. The elliptic involutory sequence defines two conjugate imaginary point. Imaginary elements always occur in pairs, so on one straight line, there are ∞^2 of conjugate imaginary points, as opposed to real ∞^1 points. All circular involutory pencils of straight lines of one plane intersect an infinitely distant straight lie of that plane in elliptical involutory sequence (absolute sequence) whose conjugate imaginary double points are "**the absolute points of a plane**". "Every plane in space has its infinitely distant straight line on which lie the absolute points of this plane. The geometrical position of all absolute points in the infinitely distant plane of space is the **absolute conic** of space"/1/.

Each quadric (II degree surface) intersects infinitely distant plane of space along one conic, which can be real, imaginary or degenerated into two real or imaginary straight lines. When this conic in the infinitely distant plane is an absolute conic of space, the quadric is the sphere. In order to constructively determine the figure of the absolute conic in the vanishing plane in the second space, a sphere in the first space was selected, because its infinitely distant conic is the absolute conic of space, and the quadric associated to it in the second space is rotating ellipsoid.

2 Constructive procedure for determination of figure of absolute conic in general collinear spaces

In the paper, the constructive determination of the figure of absolute conic in the space θ^2 was presented, and the same procedure can be used for determination of the figure of absolute conic in space θ^1. Only first Monge's projections and planes of transformation for both spaces were presented.

In the space θ^1 the sphere s_1 (fig. 2) has been selected, which intersects its infinitely distant plane M_1^∞ along the absolute conic a_{i1}, and the quadric associated to it in the space θ^2 is a rotational ellipsoid s_2 which intersects the vanishing plane M_2 along the imaginary circumference a_{I2}, whose real representative is the circumference a_{Z2}, which is associated to the absolute conic of the space θ^1. In order to determine the figure of the absolute conic of the first space in the vanishing plane of the second space, it is necessary to firstly map the sphere into the rotational ellipsoid, and then determine its intersection with the vanishing plane M_2.

The sphere s_1 in the space θ^1 has been selected to map as simple as possible into the associated quadric in the space θ^2. Regarding that the points on the axes of space are mutually associated, the point O_1 has been chosen as a center of the sphere, and one diameter $1_1 2_1$, on the axis of the space θ^1, because with the aid of cross ratio on the associated axes $\lambda = (O_1 W_1^\infty 1_1 2_1) = (O_2 W_2 1_2 2_2)$, these points can be more simply mapped, an in this way one diameter of the associated quadric

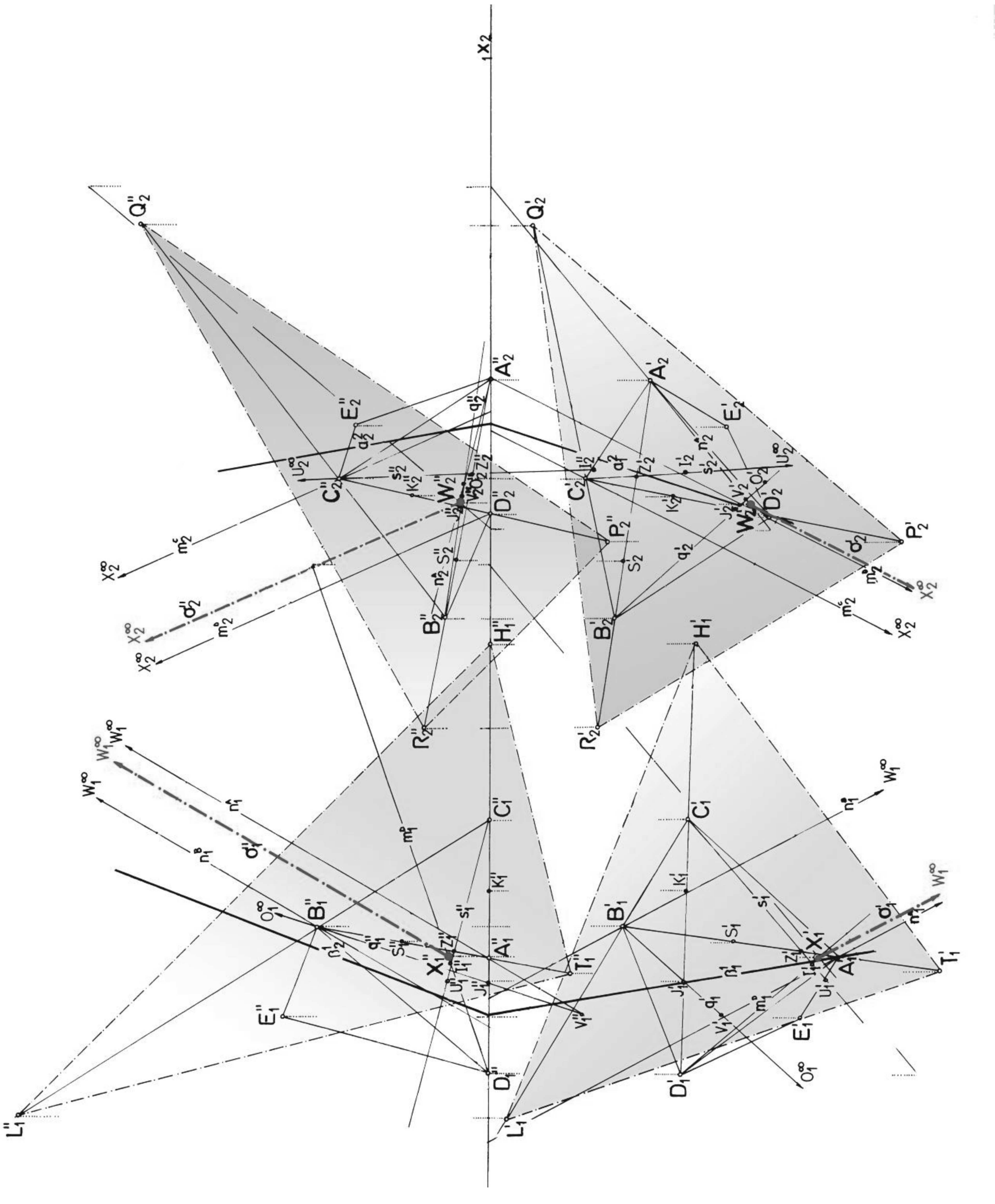

Figure 1: determination of characteristic parameters in *GC* spaces

has been determined, that is the axis of the ellipse has been determined (contours of the rotational ellipsoid), that is lying on the axis o_2 of space θ^2.

The point O_2 is the pole of the rotational ellipsoid s_2 in respect to the vanishing plane M_2 and it is associated to the center of the sphere O_1 which is a pole, in respect to the infinitely distant plane M_1^{∞}. To the point L_1 which is a pole in respect to the vanishing plane N_1 in the space θ^1 the point L_2 has been associated, which is a pole in respect to the infinitely distant plane N_2^{∞} of the space θ^2. The point L_2 is the center of the rotational ellipsoid and it is obtained with the aid of dual relationship $\lambda = (O_1 W_1^{\infty} L_1 X_1) = (O_2 W_2 L_2 X_2^{\infty})$.

In order to accomplish association of the remaining points in both spaces, which are on the axes of space, the vanishing planes are brought into a ray position through transformation, so in these projections, the quadric contours are conics, the sphere s_1 is a circumference, and the rotational ellipsoid s_2 is an ellipse. The planes parallel to the vanishing plane N_1 in the space θ^1 map in the planes parallel to the vanishing plane M_2 in space θ^2 and in these projections they appear as rays.

The intersection of the straight lines parallel to the vanishing plane N_1, with the sphere s_1 in the space θ^1 are circumferences, and for the mapping in the space θ^2 two were selected, one through the center O_1, sphere s_1 of the diameter $3_1 4_1$, and the other through the pole L_1 of the diameter $5_1 6_1$. In the space θ^2 the associated circumferences with the centers O_2 and L_2, are sought, whose diameters must be obtained by mapping the end points of the circumferences 3_1, 4_1 and 5_1, 6_1 whose centers are O_1 and L_1, and their true size is obtained by rotation.

Through the point 3_1, of the circumference $3_1 4_1$, in the space θ^1 (fig. 2) a perpendicular line to the vanishing plane N_1 has been placed, and their piercing points through the planes $A_1 B_1 E_1$ and $A_1 B_1 D_1$, have been determined, those being Lj_1 and K_1 points. With the aid of cross ratio on the associated sequences of points, the points Lj_2 and K_2 in the space θ^2 have been determined. From the dual relationship $\lambda = (3_1 W_1^{\infty} Lj_1 K_1) = (3_2 W_2 Lj_2 K_2)$, the point 3_2, has been determined, which lies on the plane through the point O_2 that is parallel to the vanishing plane M_2. The radius of the associated circumference $3_2 4_2$, $O_2 3_2^o$ has been determined in the true size, by rotation, and the point symmetrical to 3_2^o is o 4_2^o in respect to O_2. The chord of the ellipse (contours of rotational ellipsoid s_2) $3_2^o 4_2^o$ is the polar line for the pole W_2 in the vanishing plane M_2.

In the same manner the points 5_2^o and 6_2^o have been found, that determine the diameter (axis) of the ellipse s_2 (contour of rotational ellipsoid). Through the point 5_1, of the circumference $5_1 6_1$, a perpendicular line to the vanishing plane N_1 in the space θ^1 (fig. 2) has been set, and their piercing points through the planes $A_1 B_1 E_1$ and $A_1 B_1 D_1$, have been determined, those being the points $\check{\mathrm{S}}_1$ and $\acute{\mathrm{C}}_1$. With the aid of cross ratio on the associated sequences of points, the points $\check{\mathrm{S}}_2$ and $\acute{\mathrm{C}}_2$ in the space θ^2 have been determined. From the dual relationship $\lambda = (5_1 W_1^{\infty} \check{\mathrm{S}}_1 \acute{\mathrm{C}}_1) = (5_2 W_2 \check{\mathrm{S}}_2 \acute{\mathrm{C}}_2)$, the point 5_2, has been determined, lying in the plane through the point L_2 parallel to the vanishing line M_2. The radius of the associated circumference $5_2 6_2$, $L_2 5_2^o$ has been determined in the true size, by rotation, and the point symmetrical to the point 5_2^o is the point 6_2^o in respect to L_2. The axis of the ellipse s_2 (contours of rotational ellipsoid) $5_2^o 6_2^o$ is the polar line for the pole X_2^{∞} in

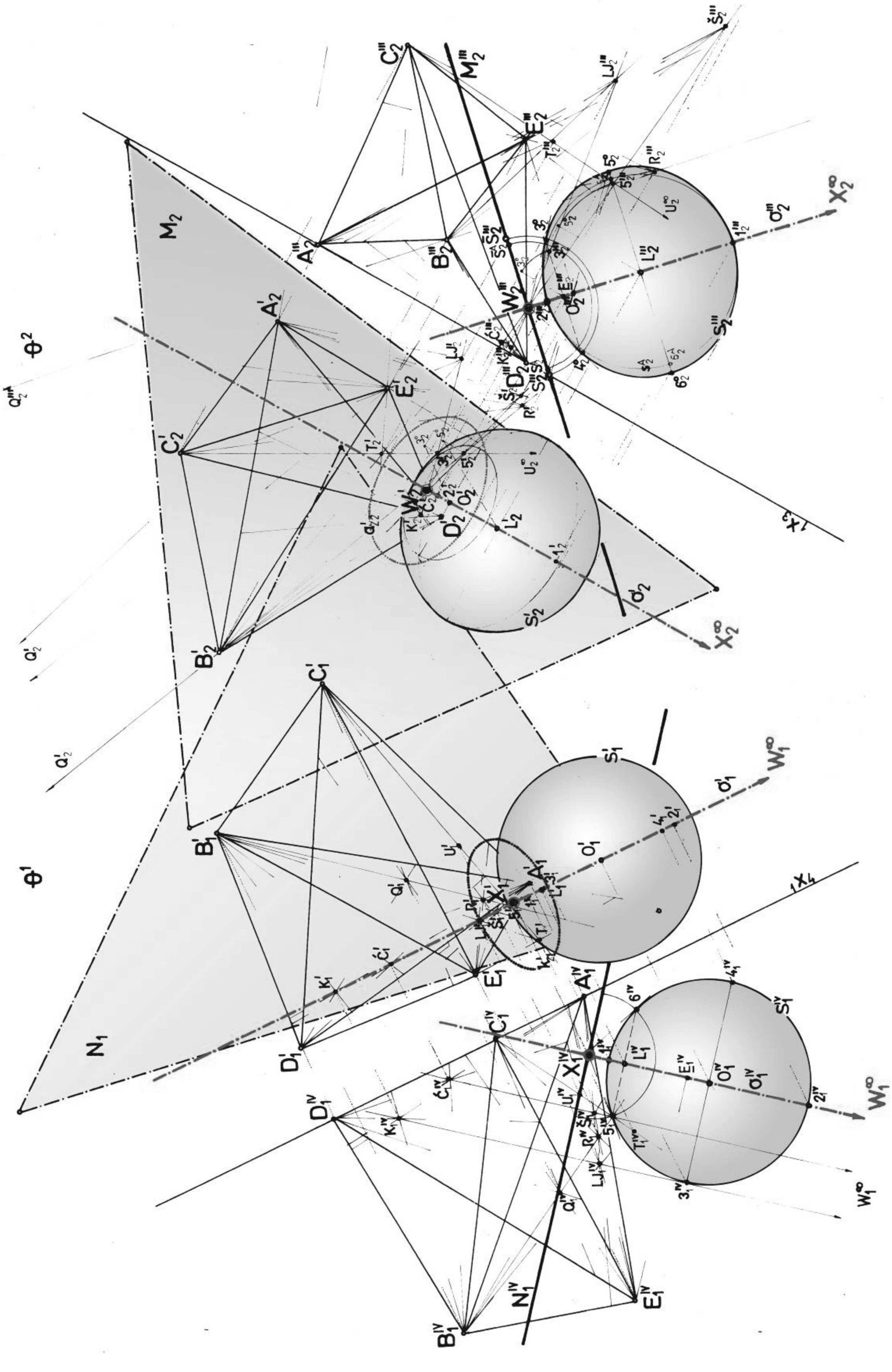

Figure 2: determination of the figure of absolute conic in space θ^2

the infinitely distant plane N_2^∞.

The contour of the rotational ellipsoid s_2 in the space θ^2 in the transformation plane, which is associated to the sphere s_1 in the space θ^1 has been determined with the aid of the axes $1_2 2_2$ and $5_2^o 6_2^o$ of the ellipse, $3_2^o 4_2^o$ being its chord. Intersection of the rotational ellipsoid s_2 with the vanishing plane M_2 is the imaginary circumference a_{I2}, whose real representative is circumference a_{Z2}, and it has been associated to the absolute conic a_{I1} of the space θ^1.

Constructive procedure for determination of the imaginary circumference a_{I2} is determination of the conjugate imaginary double points of elliptical involutory sequences of pencils of straight lines W_2 in the vanishing plane M_2. Symmetrical points of the mentioned elliptical involutory sequences determine the circumference a_{Z2}, the real representative of the imaginary circumference a_{I2}. In order to determine the symmetrical points on the straight line M_2 (ray position of the vanishing plane M_2), induced by the ellipse s_2 (contour of rotational ellipsoid), which is associated to the absolute sequence on the infinitely distant straight line of the infinitely distant plane M_1^∞, it is necessary first to determine the symmetrical points S_2^A and $\overline{S_2^A}$ of the elliptic involutory sequence induced by the affine circumference s_2^A to the given ellipse s_2. The affine circumference to the give ellipse (contour of the ellipsoid) has a diameter equal to one axis of the ellipse $1_2 2_2$ which is situated on the axis of space, and it is simultaneously the affinity axis of these two conics. With the aid of affinity, the symmetrical points of the elliptical involutory sequence of the M_2 induced by the ellipse s_2 (contour of ellipsoid) S_2 and $\overline{S_2}$ can be determined.

In this way, the diameter of the circumference a_{Z2}, $S_2\overline{S_2}$, has been determined, which is a real representative of the imaginary circumference a_{I2}, whose center is the point W_2, which is the center of the space θ^2. In the general position of the space, in a pair of Monge's projections, this imaginary circumference represented by the real representative appear as ellipse (fig.2).

3 Conclusion

In general collinear spaces, figures of the absolute conics of space can be constructively determined by the intersection of the quadrics (II degree surfaces) with the vanishing planes in other space, which are associated to the sphere in the first space which is intersected by the infinitely distant plane along the absolute conic. In the first space, the sphere is a quadric intersecting the infinitely distant plane of its space along the absolute conic. The quadric associated to a sphere in the first space, can be in the second space sphere or rotating (ellipsoid, two-sheet hyperboloid and paraboloid) or triaxial (ellipsoid, two-sheet hyperboloid and paraboloid) depending on the position of the sphere in respect to the axis and the vanishing plane in the space. Of these quadrics, rotating ellipsoid was chosen, because its intersection with the vanishing plane in the second space is imaginary circumference, which has a real representative, whose center coincides with the center of this space. This imaginary circumference is the figure of the absolute conic of the first space.

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Sonja Krasić,
University of Niš, Faculty of Civil Engineering and Architecture, Aleksandra Medvedeva 14 18000 Niš
E-mail: sonjak@gaf.ni.ac.rs

Vladan Nikolić,
University of Niš, Faculty of Civil Engineering and Architecture, Aleksandra Medvedeva 14 18000 Niš
E-mail: vladan_nikolic@yahoo.com