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LACUNARY SERIES IN MIXED NORM SPACES ON THE BALL AND THE POLYDISK

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Abstract

We characterize lacunary series in mixed norm spaces on the unit ball $\mathbb{B}^n$ in $\mathbb{C}^n$ and on the unit polydisk $\mathbb{D}^n$ in $\mathbb{C}^n$.

Introduction and main results

Let n be a positive integer. Two domains will be used in the paper: the open unit ball $\mathbb{B}^n$ in $\mathbb{C}^n$,

$$\mathbb{B}^n = \{z \in \mathbb{C}^n : |z| < 1\},$$

and the open unit polydisk $\mathbb{D}^n$ in $\mathbb{C}^n$,

$$\mathbb{D}^n = \{z = (z_1, ..., z_n) \in \mathbb{C}^n : |z_1| < 1, ..., |z_n| < 1\}.$$

We write $\mathbb{D} = \mathbb{B}^1 = \mathbb{D}^1$.

Denote by $\mathbb{T}^n$ the Shilov boundary of $\mathbb{D}^n$, by $\partial\mathbb{B}^n$ the boundary of $\mathbb{B}^n$, by $d\sigma_n$ the normalized surface measure on $\partial\mathbb{B}^n$, and define the measure $d\mu_n$ on $\mathbb{T}^n$ by

$$d\mu_n(e^{i\theta_1}, \dots, e^{i\theta_n}) = d\theta_1 \cdots d\theta_n.$$

Lacunary series on the unit ball $\mathbb{B}^n$

The mixed norm space $H^{p,q,\alpha}(\mathbb{B}^n)$, $0 < p, q \leq \infty$ $0 < \alpha < \infty$, consists of all functions f holomorphic in $\mathbb{B}^n$, $f \in H(\mathbb{B}^n)$, such that

$$||f||^q_{p,q,\alpha} = \int_0^1 (1-r)^{q\alpha-1} M_p(r,f)^q dr < \infty, \quad \text{if} \quad 0 < q < \infty,$$

and

$$||f||_{p,\infty,\alpha} = \sup_{0<r<1} (1-r)^{\alpha} M_p(r,f) < \infty.$$

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Here, as usual,

$$M_p(r,f) = \left(\int_{\partial \mathbb{B}^n} |f(r\xi)|^p d\sigma_n(\xi) \right)^{1/p}, \quad 0 < p < \infty,$$

and

$$M_\infty(r,f) = \sup_{|\xi|=1} |f(r\xi)|.$$

We write $||f||_p = \sup_{0<r<1} M_p(r,f)$.

Note that when $0 < p = q < \infty$, then $H^{p,p,(\alpha+1)/p}(\mathbb{B}^n)$, where $\alpha > -1$, coincides, as a topological linear space, with the weighted Bergman space $A^{p,\alpha}(\mathbb{B}^n)$, consisting of those $f \in H(\mathbb{B}^n)$ for which

$$\int_{\mathbb{B}^n} |f(z)|^p (1-|z|^2)^\alpha dV_n(z) \ < \infty,$$

where dV_n is the normalized volume measure on $\mathbb{B}_n$.

We say that a holomorphic function f on $\mathbb{B}^n$ has a lacunary expansion if its homogeneous expansion is of the form

$$f(z) = \sum_{k=1}^{\infty} f_{m_k}(z),$$

where m_k satisfies the condition

$$\inf_{1 \le k < \infty} \frac{m_{k+1}}{m_k} = \lambda > 1.$$

The series $\sum_{k=1}^{\infty} f_{m_k}(z)$ as well as the sequence $\{m_k\}$ are then said to be lacunary.

In this paper we characterize holomorphic functions with lacunary expansions in mixed norm spaces $H^{p,q,\alpha}(\mathbb{B}^n)$. More precisely, we prove

Theorem 1. *Let $0 < p, q \le \infty$, $0 < \alpha < \infty$ and let $f(z) = \sum_{k=1}^{\infty} f_{m_k}(z)$ be a holomorphic function on $\mathbb{B}^n$ with a lacunary expansion. Then $f \in H^{p,q,\alpha}(\mathbb{B}^n)$ if and only if*

$$\sum_{k=1}^{\infty} \frac{||f_{m_k}||_p^q}{m_k^{q\alpha}} < \infty \quad \textit{if} \quad 0 < q < \infty,$$

or

$$\sup_{1 \le k < \infty} m_k^{-\alpha} ||f_{m_k}||_p < \infty, \quad \textit{if} \quad q = \infty.$$

Lacunary series in $H^{p,q,\alpha}(\mathbb{D})$ are characterized in [MP]. (See also [JP]).

Our work was motivated by characterizations of lacunary series in weighted Bergman spaces $A^{p,\alpha}(\mathbb{B}^n)$, see [Ch], [YO], and [St]. Case $q = \infty$ of Theorem 1 also follows from [ZZ, Proposition 63]. We note that in [St] lacunary series in mixed norm spaces $H^{p,q,\alpha}(\mathbb{B}^n)$ are considered and some partial results have been obtained.

Lacunary series on the unit polydisk in $\mathbb{C}^n$

For any Lebesgue measurable function f in $\mathbb{D}^n$, we define

$$M_p(r,f) = \Big(\int_{\mathbb{T}^n} |f(r\xi)|^p d\mu_n(\xi) \Big)^{1/p}, \quad 0 < p < \infty,$$

and

$$M_\infty(r,f) = \sup_{\xi \in \mathbb{T}^n} |f(r\xi)|,$$

where $r = (r_1, ..., r_n)$.

If $0 < p \leq \infty$, $0 < q < \infty$, and $\alpha = (\alpha_1, ..., \alpha_n), \alpha_j > 0, j = 1, ..., n$, let

$$||f||^q_{p,q,\alpha} = \int_{I^n} \Big(\prod_{j=1}^n (1-r_j)^{q\alpha_j - 1} M_p(r,f)^q \Big) dr,$$

where $I^n = [0,1)^n$ and $dr = dr_1 \cdots dr_n$. The mixed norm space $H^{p,q,\alpha}(\mathbb{D}^n)$ is then defined to be the space of functions f holomorphic in $\mathbb{D}^n$, $f \in H(\mathbb{D}^n)$, such that $||f||_{p,q,\alpha} < \infty$.

The mixed norm space $H^{p,\infty,\alpha}(\mathbb{D}^n), 0 < p \leq \infty, \alpha = (\alpha_1, ..., \alpha_n), \alpha_1 > 0, ..., \alpha_n > 0$, is the set of those functions $f \in H(\mathbb{D}^n)$ for which

$$||f||_{p,\infty,\alpha} = \sup_{r \in I^n} \prod_{j=1}^n (1-r_j)^{\alpha_j} M_p(r,f)$$

is finite.

Our second result is a characterization of lacunary series in mixed norm spaces $H^{p,q,\alpha}(\mathbb{D}^n)$.

Theorem 2. *Let $0 < p \leq \infty, 0 < q \leq \infty, \alpha_j > 0$, $j = 1, ..., n$, and*

$$f(z) = \sum_{k_1,...,k_n \geq 1} a_{k_1,...,k_n} z_1^{m_{1,k_1}} ... z_n^{m_{n,k_n}}$$

be a holomorphic function on $\mathbb{D}^n$ such that there is $\lambda > 1$ satisfying the condition

$$m_{j,k_j+1}/m_{j,k_j} \geq \lambda \quad \textit{for all} \quad k_j \in \mathbb{N}, \ j = 1, \ldots n.$$

If $0 < q < \infty$, then the following statements are equivalent:

(i) $f \in H^{p,q,\alpha}(\mathbb{D}^n)$;

(ii) $\displaystyle\sum_{k_1,...,k_n \geq 1} \frac{|a_{k_1,...,k_n}|^q}{\prod_{j=1}^n m_{j,k_j}^{q\alpha_j}} < \infty.$

If $q=\infty$, then the following statements are equivalent:

(iii) $f \in H^{p,\infty,\alpha}(\mathbb{D}^n)$;

(iv) $\displaystyle\sup_{k_1,\dots,k_n\geq 1} \frac{|a_{k_1,..,k_n}|}{\prod_{j=1}^n m_{j,k_j}^{\alpha_j}} < \infty.$

We note that the equivalence (iii) and (iv) also follows from [Av, Theorem 3]. The equivalence (i) $\Longleftrightarrow$ (ii) for $0<p=q<\infty$ was proved in [St].

1 Preliminaries

In this section we gather several well-known lemmas that will be used in the proofs of our results.

LEMMA 1. *[P] Let $\alpha>-1, 0<q<\infty$ and $I_n=\{k\in\mathbb{N}: 2^n\leq k<2^{n+1}\}$ for $n\geq 1$, $I_0=\{0,1\}$. If $\{a_n\}_0^\infty$ is a sequence of non-negative numbers such that the series $G(r)=\sum_{n=0}^\infty a_n r^n$ converges for every $r\in(0,1)$, then the following two conditions are equivalent and the corresponding quantities are "proportional":*

(i) $\int_0^1 (1-r)^\alpha G(r)^q dr < \infty$;

(ii) $\sum_{n=0}^\infty 2^{-n(\alpha+1)}\left(\sum_{k\in I_n} a_k\right)^q < \infty.$

In the case of the function $G(r)=\sup_{n\geq 0} a_n r^n$ in (i) *the expression $\sum_{k\in I_n} a_k$ in* (ii) *should be replaced by $\sup_{k\in I_n} a_k$.*

LEMMA 2. *If $\{n_k\}$ is a lacunary sequence of positive integers, that is $\inf_k \frac{n_{k+1}}{n_k}=\lambda>1$, and $\{a_k\}$ is a sequence of nonnegative real numbers, then the following conditions are equivalent and the corresponding quantities are "proportional":*

(i) $\int_0^1 (1-r)^\alpha \left(\sum_{k=1}^\infty a_k r^{n_k}\right)^q dr < \infty$;

(ii) $\int_0^1 (1-r)^\alpha \left(\sup_{k\geq 1} a_k r^{n_k}\right)^q dr < \infty$;

(iii) $\sum_{k=1}^\infty \frac{|a_k|^q}{n_k^{\alpha+1}} < \infty.$

Proof. By Lemma 1,

$$\int_0^1 (1-r)^\alpha \Big(\sum_{k=1}^\infty a_k r^{n_k}\Big)^q dr \cong \sum_{k=1}^\infty 2^{-k(\alpha+1)}\Big(\sum_{n_j\in I_k} a_j\Big)^q.$$

Since $\frac{n_{j+1}}{n_j} \geq \lambda > 1$, for all $j \in N$, the number of a_j when $n_j \in I_k$ is at most $[\log_\lambda 2] + 2$. Using this and the fact that $n_j \cong 2^k$ when $n_j \in I_k$, we see that

$$\sum_{k=1}^{\infty} 2^{-k(\alpha+1)} \Big(\sum_{n_j \in I_k} a_j \Big)^q \cong \sum_{k=1}^{\infty} \frac{a_k^q}{n_k^{\alpha+1}}.$$

$\square$

LEMMA 3. *[Zy, Du, P] Let* $0 < p < \infty$. *If* $\{n_k\}$ *is an increasing sequence of positive integers satisfying* $n_{k+1}/n_k \geq \lambda > 1$ *for all* k, *then there is a positive constant* C *depending only on* p *and* λ *such that*

$$C^{-1} \Big(\sum_{k=1}^{\infty} |a_k|^2 \Big)^{1/2} \leq \Big(\frac{1}{2\pi} \int_0^{2\pi} \Big| \sum_{k=1}^{\infty} a_k e^{i n_k \theta} \Big|^p d\theta \Big)^{1/p} \leq C \Big(\sum_{k=1}^{\infty} |a_k|^2 \Big)^{1/2}.$$

These Paley's inequalities were extended to the unit polydisk $\mathbb{D}^n$ in [Av]:

LEMMA 4. *Let* $\{m_{j,k_j}\}_{j=1}^{\infty}, j = 1, ..., n$, *be arbitrary lacunary sequences and* $f(z)$ *be a holomorphic function in* $\mathbb{D}^n$ *given by*

$$f(z) = \sum_{k_1,...,k_n \geq 1} a_{k_1,...,k_n} z_1^{m_{1,k_1}} \cdots z_n^{m_{n,k_n}}, \quad z = (z_1, ..., z_n) \in \mathbb{D}^n.$$

Then for any p, $0 < p < \infty$, f *is in the Hardy space* $H^p(\mathbb{D}^n)$, *i.e.* $||f||_p = \sup_{r \in I^n} M_p(r, f) < \infty$, *if and only if* $\sum_{k_1,...,k_n \geq 1} |a_{k_1,...,k_n}|^2 < \infty$. *Moreover,*

$$C^{-1} ||f||_p \leq \Big(\sum_{k_1,...,k_n \geq 1} |a_{k_1,...,k_n}|^2 \Big)^{1/2} \leq C ||f||_p,$$

where C *is a constant independent of* f.

2 Proof of Theorem 1

Let

$$\sum_{k=1}^{\infty} \frac{||f_{n_k}||_p^q}{n_k^{q\alpha}} < \infty, \quad 0 < p \leq \infty, \quad 0 < q < \infty.$$

If $1 \leq p < \infty$, then by using Minkowski's inequality we obtain

$$M_p(r, f) \leq \sum_{k=1}^{\infty} ||f_{n_k}||_p r^{n_k}. \tag{1}$$

If $p = \infty$, then

$$M_\infty(r, f) \leq \sum_{k=1}^{\infty} ||f_{n_k}||_\infty r^{n_k}. \tag{2}$$

An application of Lemma 2 gives

$$\begin{aligned} ||f||^q_{p,q,\alpha} &\le \int_0^1 (1-r)^{q\alpha-1} \big(\sum_{k=1}^{\infty} ||f_{n_k}||_p r^{n_k}\big)^q dr \\ &\le C \sum_{k=1}^{\infty} \frac{||f_{n_k}||_p^q}{n_k^{q\alpha}}. \end{aligned}$$

If $0 < p < 1$, then

$$M_p^p(r,f) \le \sum_{k=1}^{\infty} ||f_{n_k}||_p^p r^{pn_k}. \tag{3}$$

Hence,

$$\begin{aligned} ||f||^q_{p,q,\alpha} &\le \int_0^1 (1-r)^{q\alpha-1} \big(\sum_{k=1}^{\infty} ||f_{n_k}||_p^p r^{pn_k}\big)^{q/p} dr \\ &\le C\int_0^1 (1-r)^{q\alpha-1} \big(\sum_{k=1}^{\infty} ||f_{n_k}||_p^p r^{n_k}\big)^{q/p} dr \\ &\le C \sum_{k=1}^{\infty} \frac{||f_{n_k}||_p^q}{n_k^{q\alpha}}, \end{aligned}$$

by Lemma 2.

If $\alpha > 0$ and $\{n_k\}$ is a lacunary sequence of positive integers, then

$$\sum_{k=1}^{\infty} n_k^{\alpha} r^{n_k} = O\Big(\frac{1}{(1-r)^{\alpha}}\Big), \quad \text{see [Du].}$$

Using this, (1), (2), and (3) we find that

$$||f||_{p,\infty,\alpha} = \sup_{0<r<1} (1-r)^{\alpha} M_p(r,f) \le C \sup_{k\ge 1} \frac{||f_{n_k}||_p}{n_k^{\alpha}}.$$

Conversely, let $||f||_{p,q,\alpha} < \infty$.

If $0 < p < \infty$, then by using the slice integration formula [Ru2, Proposition 1.4.7] and Lemma 3 we find that

$$\begin{aligned} M_p(r,f) &= \Big(\int_{\partial\mathbb{B}^n} \Big|\sum_{k=1}^{\infty} f_{n_k}(r\xi)\Big|^p d\sigma(\xi)\Big)^{1/p} \\ &= \Big(\int_{\partial\mathbb{B}^n} \big(\frac{1}{2\pi}\int_0^{2\pi} \Big|\sum_{k=1}^{\infty} f_{n_k}(\xi) r^{n_k} e^{in_k\theta}\Big|^p d\theta\big) d\sigma(\xi)\Big)^{1/p} \\ &\cong \Big(\int_{\partial\mathbb{B}^n} \Big(\sum_{k=1}^{\infty} |f_{n_k}(\xi)|^2 r^{2n_k}\big)^{p/2} d\sigma(\xi)\Big)\Big)^{1/p}, \end{aligned}$$

and consequently

$$M_p(r,f) \geq C||f_{n_k}||_p r^{n_k}, \quad \text{for all} \quad k \geq 1.$$

If $p=\infty$, also we have $M_\infty(r,f) \geq ||f_{n_k}||_\infty r^{n_k}$, for all $k \geq 1$.

Thus, if $0<q<\infty$, then

$$\begin{aligned} ||f||^q_{p,q,\alpha} &\geq C\int_0^1 (1-r)^{q\alpha-1}(\sup_{k\geq 1}||f_{n_k}||_p r^{n_k})^q dr \\ &\geq C\sum_{k=1}^{\infty}\frac{||f_{n_k}||_p^q}{n_k^{q\alpha}}, \end{aligned}$$

by Lemma 2.

If $q=\infty$, then

$$\begin{aligned} ||f||_{p,\infty,\alpha} &\geq \sup_{0<r<1}(1-r)^\alpha \sup_{k\geq 1}||f_{n_k}||_p r^{n_k} \\ &\geq \sup_{k\geq 1}||f_{n_k}||_p \frac{1}{n_k^\alpha}\Big(1-\frac{1}{n_k}\Big)^{n_k} \\ &\geq \frac{1}{e}\sup_{k\geq 1}\frac{||f_{n_k}||_p}{n_k^\alpha}. \end{aligned}$$

This finishes the proof of Theorem 1.

3 Proof of Theorem 2

In order to avoid too much calculations we will assume that $n=2$.

Proof of implications (ii) $\Longrightarrow$ (i) **and** (iv) $\Longrightarrow$ (iii)

Let $0<p\leq\infty, r=(r_1,r_2)$ and $\alpha=(\alpha_1,\alpha_2), \alpha_1>0$, $\alpha_2>0$. Then

$$M_p(r,f) \leq \sum_{k_1,k_2\geq 1}|a_{k_1,k_2}|r_1^{m_{1,k_1}}r_2^{m_{2,k_2}}.$$

If $0<q<\infty$ then by applying Lemma 2 twice we obtain

$$\begin{aligned} ||f||^q_{p,q,\alpha} &= \int_0^1 (1-r_2)^{q\alpha_2-1}dr_2\int_0^1(1-r_1)^{q\alpha_1-1}M_p(r,f)^q dr_1 \\ &\leq \int_0^1 (1-r_2)^{q\alpha_2-1}dr_2\int_0^1(1-r_1)^{q\alpha_1-1} \\ &\quad\times(\sum_{k_1\geq 1}(\sum_{k_2\geq 1}|a_{k_1,k_2}|r_2^{m_{2,k_2}})r_1^{m_{1,k_1}})^q dr_1 \\ &\leq C\int_0^1(1-r_2)^{q\alpha_2-1}\big(\sum_{k_1\geq 1}\frac{1}{m_{1,k_1}^{q\alpha_1}}(\sum_{k_2\geq 1}|a_{k_1,k_2}|r_2^{m_{2,k_2}})^q\big)dr_2 \end{aligned}$$

$$
\begin{aligned}
&= C\sum_{k_1\ge 1} m_{1,k_1}^{-q\alpha_1}\int_0^1 (1-r_2)^{q\alpha_2-1}\Big(\sum_{k_2\ge 1}|a_{k_1,k_2}|r_2^{m_{2,k_2}}\Big)^q dr_2\\
&\le C\sum_{k_1\ge 1}\sum_{k_2\ge 1} m_{1,k_1}^{-q\alpha_1} m_{2,k_2}^{-q\alpha_2}|a_{k_1,k_2}|^q.
\end{aligned}
$$

If $q=\infty$, then we have

$$
\begin{aligned}
||f||_{p,\infty,\alpha} &= \sup_{0<r_1<1}\sup_{0<r_2<1}(1-r_1)^{\alpha_1}(1-r_2)^{\alpha_2}M_p(r,f)\\
&\le \sup_{0<r_1<1}\sup_{0<r_2<1}(1-r_1)^{\alpha_1}(1-r_2)^{\alpha_2}\sum_{k_1,k_2\ge 1}|a_{k_1,k_2}|r_1^{m_{1,k_1}}r_2^{m_{2,k_2}}\\
&\le \sup_{0<r_1<1}(1-r_1)^{\alpha_1}\sum_{k_1\ge 1}\Big(\sup_{0<r_2<1}(1-r_2)^{\alpha_2}\sum_{k_2\ge 1}|a_{k_1,k_2}|r_2^{m_{2,k_2}}\Big)r_1^{m_{1,k_1}}\\
&\le C\sup_{0<r_1<1}(1-r_1)^{\alpha_1}\sum_{k_1\ge 1}\sup_{k_2\ge 1}\frac{|a_{k_1,k_2}|}{m_{2,k_2}^{\alpha_2}}r_1^{m_{1,k_1}}\\
&\le C\sup_{k_1\ge 1}\sup_{k_2\ge 1}\frac{|a_{k_1,k_2}|}{m_{1,k_1}^{\alpha_1}m_{2,k_2}^{\alpha_2}}.
\end{aligned}
$$

Proof of implications (i) $\implies$ (ii) **and** (iii) $\implies$ (iv)

By Lemma 4 we have

$$
M_p(r,f)\cong\Big(\sum_{k_1,k_2\ge 1}|a_{k_1,k_2}|^2 r_1^{2m_{1,k_1}}r_2^{2m_{2,k_2}}\Big)^{1/2}.
$$

Thus

$$
M_p(r,f)\ge \sup_{k_1,k_2\ge 1}|a_{k_1,k_2}|r_1^{m_{1,k_1}}r_2^{m_{2,k_2}},\quad 0<p<\infty.
$$

This holds also for $p=\infty$. Hence, if $0<q<\infty$, by applying Lemma 2 twice we get

$$
\begin{aligned}
||f||_{p,q,\alpha}^q &\ge \int_0^1 (1-r_1)^{q\alpha_1-1}dr_1\int_0^1(1-r_2)^{q\alpha_2-1}\\
&\times\Big(\sup_{k_2\ge 1}\big(\sup_{k_1\ge 1}|a_{k_1,k_2}|r_1^{m_{1,k_1}}\big)r_2^{m_{2,k_2}}\Big)^q dr_2\\
&\ge C\int_0^1(1-r_1)^{q\alpha_1-1}\sum_{k_2\ge 1}m_{2,k_2}^{-q\alpha_2}\big(\sup_{k_1\ge 1}|a_{k_1,k_2}|r_1^{m_{1,k_1}}\big)^q dr_1\\
&= C\sum_{k_2\ge 1}m_{2,k_2}^{-q\alpha_2}\int_0^1(1-r_1)^{q\alpha_1-1}\big(\sup_{k_1\ge 1}|a_{k_1,k_2}|r_1^{m_{1,k_1}}\big)^q dr_1\\
&\ge C\sum_{k_1\ge 1}\sum_{k_2\ge 1}m_{2,k_2}^{-q\alpha_2}m_{1,k_1}^{-q\alpha_1}|a_{k_1,k_2}|^q.
\end{aligned}
$$

If $q = \infty$, then

$$\begin{aligned} ||f||_{p,\infty,\alpha} &= \sup_{0<r_1<1} \sup_{0<r_2<1} (1-r_1)^{\alpha_1}(1-r_2)^{\alpha_2} M_p(r,f) \\ &\geq \sup_{0<r_1<1} \sup_{0<r_2<1} (1-r_1)^{\alpha_1}(1-r_2)^{\alpha_2} \sup_{k_1,k_2\geq 1} |a_{k_1,k_2}| r_1^{m_{1,k_1}} r_2^{m_{2,k_2}} \\ &\geq C \sup_{k_1,k_2\geq 1} \frac{|a_{k_1,k_2}|}{m_{1,k_1}^{\alpha_1} m_{2,k_2}^{\alpha_1}}. \end{aligned}$$

This finishes the proof of Theorem 2.

References

[Av] K. L. Avetisyan, Hardy-Bloch type spaces and lacunary series on the polydisk, *Glasgow Math.J.* **49**(2007),345-356.

[Ch] J. S. Choa, Some properties of analytic functions on the unit ball with Hadamard gaps, *Complex Var. Theory Appl.* **29**(1996), no. 3, 277–285.

[Du] P. L. Duren, *Theory of H^p Spaces*, Academic Press, New York 1970; reprinted by Dover, Mineola, NY 2000.

[JP] M. Jevtić, M. Pavlović, Coefficient multipliers on spaces of analytic functions, *Acta Sci. Math. (Szeged)* **64**(1998), 531-545.

[MP] M. Mateljević, M. Pavlović, Duality and multipliers in Lipschitz spaces, *Proc. International Conference on Complex Analysis, Varna* (1983), 151-161.

[P] M. Pavlović, *Introduction to function spaces in the unit ball of C^n*, Mat. Inst. SANU, Belgrade, 2004.

[Ru1] W. Rudin, *Function theory in polydiscs,* Benjamin, New York, 1969.

[Ru2] W. Rudin, *Function theory in the unit ball of C^n,* Springer-Verlag, New York, 1980.

[St] S. Stević, A generalization of a result of Choa on analytic functions with Hadamard gaps, *J. Korean Math. Soc.* **43**(2006), 579-591.

[YO] W. Yang, C. Oyang, Exact location of α-Bloch spaces in L^p_a and H^p of a complex unit ball, *Rocky Mountain J. Math.* **30**(2000), 1151-1169.

[ZZ] R. Zhao, K. Zhu, *Theory of Bergman spaces in the unit ball of C^n,* arxiv:math. CVI0611093v1 3 nov 2006.

[Zy] A. Zygmund, *Trigonometric series,* Cambridge University Press, 1959.

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