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Remarks on neighborhood star-Lindelöf spaces II

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Abstract. A space X is said to be *neighborhood star-Lindelöf* if for every open cover $\mathcal{U}$ of X there exists a countable subset A of X such that for every open $O \supseteq A$, $X = St(O, \mathcal{U})$. In this paper, we continue to investigate the relationship between neighborhood star-Lindelöf spaces and related spaces, and study topological properties of neighborhood star-Lindelöf spaces in the classes of normal and pseudocompact spaces.

1. Introduction

By a space, we mean a topological space. In the rest of this section, we give definitions of terms which are used in this paper. Let X be a space and $\mathcal{U}$ a collection of subsets of X. For $A \subseteq X$, let $St(A, \mathcal{U}) = \bigcup\{U \in \mathcal{U} : U \cap A \neq \emptyset\}$. As usual, we write $St(x, \mathcal{U})$ for $St(\{x\}, \mathcal{U})$.

Recall that a space X is *strongly starcompact* (see [6, 8, 9] under different name) if for every open cover $\mathcal{U}$ of X there exists a finite subset A of X such that $X = St(A, \mathcal{U})$; A space X is *strongly star-Lindelöf* (see [2, 3, 6, 9, 10] under different name) if for every open cover $\mathcal{U}$ of X there exists a countable subset A of X such that $X = St(A, \mathcal{U})$; A space X is *starcompact* (resp., *star-Lindelöf*) (see [6, 9] under different name) if for every open cover $\mathcal{U}$ of X there exists a finite (resp., countable) subset $\mathcal{V}$ of $\mathcal{U}$ such that $X = St(\bigcup \mathcal{V}, \mathcal{U})$. Clearly, every strongly starcompact space is strongly star-Lindelöf, every strongly starcompact space starcompact, every strongly star-Lindelöf space is star-Lindelöf and every strongly star-Lindelöf space is star-Lindelöf. It is known that every countably compact space is strongly starcompact, and every Hausdorff strongly starcompact space is countably compact (see [6, 9]).

It is natural in this context to introduce the following definitions:

Definition 1.1. ([4]) A space X is said to be *weakly starcompact* if for every open cover $\mathcal{U}$ of X there exists a finite subset A of X such that for every open $O \supseteq A$, $X = St(O, \mathcal{U})$.

Definition 1.2. ([5]) A space X is said to be *neighborhood star-Lindelöf* if for every open cover $\mathcal{U}$ of X there exists a countable subset A of X such that for every open $O \supseteq A$, $X = St(O, \mathcal{U})$.

From the definitions, it is clear that every weakly starcompact space is neighborhood star-Lindelöf, every strongly star-Lindelöf space is neighborhood star-Lindelöf and every neighborhood star-Lindelöf space is star-Lindelöf.

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The theory becomes more interesting when star-covering properties are considered in conjunction with other properties. In [12], the author studied the relationship between neighborhood star-Lindelöf spaces and related spaces, and investigated topological properties of neighborhood star-Lindelöf spaces. Pseudocompactness is particularly interesting in this case as it may be treated as a star-covering property (see [6, 9]). In this note, we continue to study the relationship between neighborhood star-Lindelöf spaces and related spaces, and investigate topological properties of neighborhood star-Lindelöf spaces in the classes of normal and pseudocompact spaces.

Throughout this paper, the cardinality of a set A is denoted by $|A|$. Let ω denote the first infinite cardinal, ω_1 the first uncountable cardinal, $\mathfrak{c}$ the cardinality of the set of all real numbers. For a cardinal κ, let κ^+ be the smallest cardinal greater than κ. For each pair of ordinals α, β with $\alpha < \beta$, we write $[\alpha, \beta) = \{\gamma : \alpha \leq \gamma < \beta\}$, $(\alpha, \beta] = \{\gamma : \alpha < \gamma \leq \beta\}$, $(\alpha, \beta) = \{\gamma : \alpha < \gamma < \beta\}$ and $[\alpha, \beta] = \{\gamma : \alpha \leq \gamma \leq \beta\}$. As usual, a cardinal is an initial ordinal and an ordinal is the set of smaller ordinals. Every cardinal is often viewed as a space with the usual order topology. Other terms and symbols that we do not define follow [7].

2. Main results

In [12], the author showed that there exists a Tychonoff neighborhood star-Lindelöf space X that is not weakly starcompact and there exists a Tychonoff star-Lindelöf space that is not neighborhood star-Lindelöf. But these spaces are neither normal nor pseudocompact. In the following, we construct pseudocompact and normal examples. Recall that a space is called *Urysohn* if every two distinct points have neighborhoods with disjoint closures. Clearly, the property is between the Hausdorff condition and regularity. Bonanzinga et al. in [4] showed that the three properties, countable compactness, strongly starcompactness, and weak starcompactness, are equivalent for Urysonn spaces.

Example 2.1. *There exists a pseudocompact, neighborhood star-Lindelöf Tychonoff space X that is not weakly star-compact.*

Proof. Let $X = \omega \cup \mathcal{R}$ be the Isbell-Mrówka space (see [11]), where $\mathcal{R}$ is a maximal almost disjoint family of infinite subsets of ω with $|\mathcal{R}| = \mathfrak{c}$. Then X is Tychonoff pseudocompact. Since ω is a countable dense subset of X. Then X is strongly star-Lindelöf. Thus X is neighborhood star-Lindelöf. But X is not countably compact, since $\mathcal{R}$ is an uncountable discrete closed subset of X. Thus X is not weakly starcompact, since countable compactness is equivalent to weakly starcompactness for Tychonoff spaces. □

In [12], the author gave an example showing that there exists a Tychonoff neighborhood star-Lindelöf space X that is not weakly starcompact. In fact, the space is normal.

Example 2.2. *There exists a normal neighborhood star-Lindelöf space X that is not weakly starcompact.*

For the next example, we need the following lemma.

Lemma 2.3. ([5]) *A space X is neighborhood star-Lindelöf if and only if for every open cover $\mathcal{U}$ of X there exists a countable subset A of X such that $\overline{St(x, \mathcal{U})} \cap A \neq \emptyset$ for each $x \in X$.*

Example 2.4. *There exists a pseudocompact star-Lindelöf Tychonoff space that is not neighborhood star-Lindelöf.*

Proof. Let $D = \{d_\alpha : \alpha < \mathfrak{c}\}$ be a discrete space of cardinality $\mathfrak{c}$ and let $D^* = D \cup \{d^*\}$ be the one-point compactification of D, where $d^* \notin D$.

Let
$$X = (D^* \times [0, \mathfrak{c}^+]) \setminus \{\langle d^*, \mathfrak{c}^+ \rangle\}$$
be the subspace of $D^* \times [0, \mathfrak{c}^+]$. Then X is pseudocompact Tychonoff. In fact, it has a countably compact, dense subspace $D^* \times [0, \mathfrak{c}^+)$.

First we show that X is star-Lindelöf. For this end, let $\mathcal{U}$ be an open cover of X. For each $\alpha < \mathfrak{c}$, there exists $U_\alpha \in \mathcal{U}$ such that $\langle d_\alpha, \mathfrak{c}^+ \rangle \in U_\alpha$, we can find $\beta_\alpha < \mathfrak{c}^+$ such that $\{d_\alpha\} \times (\beta_\alpha, \mathfrak{c}^+] \subseteq U_\alpha$. Let $\beta = \sup\{\beta_\alpha : \alpha < \mathfrak{c}\}$.

Then $\beta < \mathfrak{c}^+$. Let $K = D^* \times \{\beta\}$. Then K is compact and $U_\alpha \cap K \neq \emptyset$ for each $\alpha < \mathfrak{c}$. Since $\mathcal{U}$ covers K, there exists a finite subset $\mathcal{U}'$ of $\mathcal{U}$ such that $K \subseteq \bigcup \mathcal{U}'$. Then

$$D \times \{\mathfrak{c}^+\} \subseteq St(\bigcup \mathcal{U}', \mathcal{U}).$$

On the other hand, since $D^* \times [0, \mathfrak{c}^+)$ is countably compact, we can find a finite subset $\mathcal{U}''$ of $\mathcal{U}$ such that

$$D^* \times [0, \mathfrak{c}^+) \subseteq St(\bigcup \mathcal{U}'', \mathcal{U}).$$

If we put $\mathcal{V} = \mathcal{U}' \cup \mathcal{U}''$, then $\mathcal{V}$ is a finite subset of $\mathcal{U}$ such that $X = St(\bigcup \mathcal{V}, \mathcal{U})$, which shows that X is star-Lindelöf.

Next we show that X is not neighborhood star-Lindelöf. For each $\alpha < \mathfrak{c}$, let

$$U_\alpha = \{d_\alpha\} \times [0, \mathfrak{c}^+].$$

Let us consider the open cover

$$\mathcal{U} = \{U_\alpha : \alpha < \mathfrak{c}\} \cup \{D^* \times [0, \mathfrak{c}^+)\}$$

of X. It suffices to show that for any countable subset F of X, there exists a point $x \in X$ such that $\overline{St(x, \mathcal{U})} \cap F = \emptyset$ by Lemma 2.3. Let F be any countable subset of X. Then there exists an $\alpha_0 < \mathfrak{c}$ such that $F \cap U_{\alpha_0} = \emptyset$. Since U_{α_0} is the only element of $\mathcal{U}$ containing $\langle d_{\alpha_0}, \mathfrak{c}^+\rangle$, then $St(\langle d_{\alpha_0}, \mathfrak{c}^+\rangle, \mathcal{U}) = U_{\alpha_0}$. By the constructions of the topology of X and the open cover $\mathcal{U}$, we have $\overline{St(\langle d_{\alpha_0}, \mathfrak{c}^+\rangle, \mathcal{U})} = U_{\alpha_0}$. Thus we complete the proof. □

For normal spaces, we have the following example.

Example 2.5. *Assuming $2^{\aleph_0} = 2^{\aleph_1}$, there exists a star-Lindelöf normal space X that is not neighborhood star-Lindelöf.*

Proof. Let $Y = L \cup \omega$ be a separable normal T_1 space where L is a closed and discrete subset of Y with $|L| = \aleph_1$ and each element of ω is isolated. See Example E [14] for the construction of such a space. Let

$$X = L \cup ([0, \omega_1) \times [0, \omega))$$

and topologize X as follows: A basic neighborhood of $l \in L$ in X is a set of the form

$$G_{U,\alpha}(l) = (U \cap L) \cup ((\alpha, \omega_1) \times (U \cap [0, \omega)))$$

for a neighborhood U of l in Y and $\alpha < \omega_1$, and a basic neighborhood of $\langle \alpha, n\rangle \in \omega_1 \times \omega$ in X is a set of the form

$$G_V(\langle \alpha, n\rangle) = V \times \{n\},$$

where V is a neighborhood of α in ω_1. The author showed that X is normal (see [13]).

First we show that X is star-Lindelöf. To this end, let $\mathcal{U}$ be an open cover of X. Let

$$M = \{n \in \omega : (\exists U \in \mathcal{U})(\exists \beta < \omega_1)((\beta, \omega_1) \times \{n\} \subseteq U)\}.$$

For each $n \in M$, there exist $U_n \in \mathcal{U}$ and $\beta_n < \omega_1$ such that $(\beta_n, \omega_1) \times \{n\} \subseteq U_n$. If we put $\mathcal{V}' = \{U_n : n \in M\}$, then

$$L \subseteq St(\bigcup \mathcal{V}', \mathcal{U}).$$

On the other hand, for each $n \in \omega$, since $[0, \omega_1) \times \{n\}$ is countably compact, we can find a finite subfamily $\mathcal{V}_n$ of $\mathcal{U}$ such that

$$[0, \omega_1) \times \{n\} \subseteq St(\bigcup \mathcal{V}_n, \mathcal{U}).$$

Consequently, if we put $\mathcal{V} = \mathcal{V}' \cup \bigcup\{\mathcal{V}_n : n \in \omega\}$, then $\mathcal{V}$ is a countable subset of $\mathcal{U}$ and $X = St(\bigcup \mathcal{V}, \mathcal{U})$. Hence X is star-Lindelöf.

Next we show that X is not neighborhood star-Lindelöf. Since $|L| = \aleph_1$, we can enumerate L as $\{l_\alpha : \alpha < \omega_1\}$. Since $\{l_\alpha : \alpha < \omega_1\}$ is discrete and closed in Y, for each $\alpha < \omega_1$, there exists an open neighborhood V_α of l_α in Y such that

$$V_\alpha \cap L = \{l_\alpha\}.$$

Let us consider the open cover

$$\mathcal{U} = \{G_{V_{\alpha,\alpha}}(l_\alpha) : \alpha < \omega_1\} \cup \{\omega_1 \times \omega\}$$

of X. It suffices to show that for any countable subset F of X, there exists a point $x \in X$ such that $\overline{St(x, \mathcal{U})} \cap F = \emptyset$ by Lemma 2.3. To show this, let F be a countable subset of X. Since $F \cap L$ is countable, there exists $\beta' < \omega_1$ such that

$$F \cap \{l_\alpha : \alpha > \beta'\} = \emptyset.$$

On the other hand, for each $n \in \omega$, there exists an $\alpha_n < \omega_1$ such that

$$F \cap ((\alpha_n, \omega_1) \times \{n\}) = \emptyset,$$

since F is countable. Let $\beta'' = \sup\{\alpha_n : n \in \omega\}$, then $\beta'' < \omega_1$. If we pick $\beta_0 > \max\{\beta', \beta''\}$, then $F \cap G_{V_{\beta_0,\beta_0}}(l_{\beta_0}) = \emptyset$. Since $G_{V_{\beta_0,\beta_0}}(l_{\beta_0})$ is the only element of $\mathcal{U}$ containing l_{β_0}, then $St(l_{\beta_0}, \mathcal{U}) = G_{V_{\beta_0,\beta_0}}(l_{\beta_0})$. By the constructions of the topology of X and the open cover $\mathcal{U}$, we have $\overline{St(l_{\beta_0}, \mathcal{U})} = G_{V_{\beta_0,\beta_0}}(l_{\beta_0})$. Thus we complete the proof. □

Remark 2.6. The definition of the space X in the proof of Example 2.5 is more complicated than it is necessary. In fact, X is the subspace $(Y \times (\omega_1 + 1)) \setminus ((\omega \times \{\omega_1\}) \cup (L \times \omega_1))$ of the product space $Y \times (\omega_1 + 1)$. But, for the convenience of the proof of Example 2.5, we use the definition from [13].

In the following, we show an example from [1] showing that there exists a first countable, star-Lindelöf Tychonoff space that is not neighborhood star-Lindelöf. The example uses the the Alexandorff duplicate $A(X)$ of a space X. The underlying set $A(X)$ is $X \times \{0, 1\}$; each point of $X \times \{1\}$ is isolated and a basic neighborhood of $\langle x, 0\rangle \in X \times \{0\}$ is a set of the form $(U \times \{0\}) \cup ((U \times \{1\}) \setminus \{\langle x, 0\rangle\})$, where U is a neighborhood of x in X. For the next example, we need the following Lemma.

Lemma 2.7. ([12]) *A space X having a dense Lindelöf subspace is star-Lindelöf.*

Example 2.8. *There exists a first countable, star-Lindelöf Tychonoff space that is not neighborhood star-Lindelöf.*

Proof. Let $X = (A(I) \times [0, \omega]) \setminus ((I \times \{0\}) \times \{\omega\})$ where I denotes the closed unit interval. Clearly, X is first countable and Tychonoff. Since $A(I) \times [0, \omega)$ is a dense σ-compact subset of X, then X is star-Lindelöf by Lemma 2.7, since every σ-compact subset is Lindelöf.

We show that X is not neighborhood star-Lindelöf. For each $a \in I$, let

$$U_a = \{\langle a, 1\rangle\} \times [0, \omega].$$

Then

$$U_a \text{ is s clopen subset of } X$$

and

$$U_a \cap U_{a'} = \emptyset \text{ for } a \neq a'.$$

Let us consider the open cover

$$\mathcal{U} = \{U_a : a \in I\} \cup \{A(I) \times [0, \omega)\}$$

of X. It suffices to show that for any countable subset F of X, there exists a point $x \in X$ such that $\overline{St(x, \mathcal{U})} \cap F = \emptyset$ by Lemma 2.3. Let F be any countable subset of X. Then there exists $a \in I$ such that $F \cap U_a = \emptyset$. Since U_a is the only element of $\mathcal{U}$ containing $\langle\langle a, 1\rangle, \omega\rangle$, then $St(\langle\langle a, 1\rangle, \omega\rangle, \mathcal{U}) = U_a$. By the constructions of the topology of X and the open cover $\mathcal{U}$, we have $\overline{St(\langle\langle a, 1\rangle, \omega\rangle, \mathcal{U})} = U_a$. Thus we complete the proof. □

In [12], the author showed that a regular-closed subset of a Tychonoff neighborhood star-Lindelöf space X need not be neighborhood star-Lindelöf. But the space is not pseudocompact. Now we give a pseudocompact example. Here a subset A of a space X is said to be *regular-closed* in X if $cl_X int_X A = A$.

Example 2.9. *There exists a pseudocompact, neighborhood star-Lindelöf Tychonoff space having a regular-closed G_δ-subspace which is not neighborhood star-Lindelöf.*

Proof. Let S_1 be the same space X as in the proof of Example 2.4. Then S_1 is Tychonoff pseudocompact, not neighborhood star-Lindelöf.

Let $S_2 = \omega \cup \mathcal{R}$ be the Isbell-Mrówka space (see [11]), where $\mathcal{R}$ is a maximal almost disjoint family of infinite subsets of ω with $|\mathcal{R}| = \mathfrak{c}$. Then S_2 is Tychonoff pseudocompact neighborhood star-Lindelöf.

We assume $S_1 \cap S_2 = \emptyset$. Let $\pi : D \times \{\mathfrak{c}^+\} \to \mathcal{R}$ be a bijection. Let X be the quotient image of the disjoint sum $S_1 \oplus S_2$ obtained by identifying $\langle d_\alpha, \mathfrak{c}^+\rangle$ of S_1 with $\pi(\langle d_\alpha, \mathfrak{c}^+\rangle)$ of S_2 for every $\alpha < \mathfrak{c}$. Let $\varphi : S_1 \oplus S_2 \to X$ be the quotient map. Then X is pseudocompact, since S_1 and S_2 are pseudocompact. It is clear that $\varphi(S_1)$ is a regular-closed subspace of X. Let

$$U_n = \varphi(S_1 \cup (\mathcal{R} \cup \{m \in \omega : m > n\})) \text{ for each } n \in \omega.$$

Then U_n is open in X and $\varphi(S_1) = \bigcap_{n\in\omega} U_n$. Thus $\varphi(S_1)$ is a regular-closed G_δ-subspace of X. However $\varphi(S_1)$ is not neighborhood star-Lindelöf, since it is homeomorphic to S_1.

Finally we show that X is neighborhood star-Lindelöf. We need only show that X is strongly star-Lindelöf, since every strongly star-Lindelöf space is neighborhood star-Lindelöf. To this end, let $\mathcal{U}$ be an open covers of X. Since $\varphi(S_2)$ is homeomorphic to S_2, then

$$\varphi(S_2) \subseteq St(\varphi(\omega), \mathcal{U}),$$

since $\varphi(\omega)$ is a dense subset of $\varphi(S_2)$. On the other hand, since $\varphi(D^* \times [0, \mathfrak{c}^+))$ is homeomorphic to $D^* \times [0, \mathfrak{c}^+)$, then $\varphi(D^* \times [0, \mathfrak{c}^+))$ is countably compact, so there exists a finite subset F' of $\varphi(D^* \times [0, \mathfrak{c}^+))$ such that

$$\varphi(D^* \times [0, \mathfrak{c}^+)) \subseteq St(F', \mathcal{U}).$$

If we put $F = \varphi(\omega) \cup F'$. Then F is a countable subset of X such that $X = St(F, \mathcal{U})$, which completes the proof. □

For normal spaces, we have the following example.

Example 2.10. *Assuming $2^{\aleph_0} = 2^{\aleph_1}$, there exists a normal neighborhood star-Lindelöf space having a regular-closed subspace which is not neighborhood star-Lindelöf.*

Proof. Let

$$S_1 = L \cup ([0, \omega_1) \times [0, \omega))$$

be the same space X as in the proof of Example 2.5. Then S_1 is normal, not neighborhood star-Lindelöf (see the above Example 2.5).

Let $S_2 = L \cup \omega$ be the same space Y as in the proof of Example 2.5. Then S_2 is strongly star-Lindelöf, since ω is a countable dense subset of S_2. Thus S_2 is normal, neighborhood star-Lindelöf.

Let X be the quotient image of the disjoint sum $S_1 \oplus S_2$ obtained by identifying l of S_1 with l of S_2 for any $l \in L$. Let $\varphi : S_1 \oplus S_2 \to X$ be the quotient map. Then X is normal, since S_1 and S_2 are normal, and L is closed in S_1 and S_2. It is clear that $\varphi(S_1)$ is a regular-close subspace of X by the construction of the topology of X. However $\varphi(S_1)$ is not neighborhood star-Lindelöf, since $\varphi(S_1)$ is homomorphic to S_1.

Finally we show that X is neighborhood star-Lindelöf. We need only show that X is strongly star-Lindelöf, To this end, let $\mathcal{U}$ be an open cover of X. Since ω is a countable dense subset of S_2 and $\varphi(\omega)$ is homeomorphic to $\varphi(S_2)$, then $\varphi(\omega)$ is a countable dense subset of $\varphi(S_2)$, thus

$$\varphi(S_2) \subseteq St(\varphi(\omega), \mathcal{U}).$$

On the other hand, since $\varphi([0,\omega_1)\times\{n\})$ is homomorphic to $[0,\omega_1)\times\{n\}$, then $\varphi([0,\omega_1)\times\{n\})$ is countably compact for each $n\in\omega$, thus there exists a finite subset F_n of $\varphi(\omega_1\times\{n\})$ such that

$$\varphi([0,\omega_1)\times\{n\})\subseteq St(F_n,\mathcal{U}).$$

If we put $F=\varphi(\omega)\cup\bigcup\{F_n : n\in\omega\}$, then F is a countable subset of X and $X=St(F,\mathcal{U})$, which shows that X is strongly star-Lindelöf. $\square$

Remark 2.11. It is well-known that $2^{\aleph_0}=2^{\aleph_1}$ implies negation of CH. Examples 2.5 and 2.10 give consistent examples. The author does not know if there are ZFC counterexamples.

Next we give a positive result.

Theorem 2.12. *An open F_σ-subset of a neighborhood star-Lindelöf space is neighborhood star-Lindelöf.*

Proof. Let X be a neighborhood star-Lindelöf space and let $Y=\bigcup\{H_n : n\in\omega\}$ be an open F_σ-subset of X, where the set H_n is closed in X for each $n\in\omega$. To show that X is neighborhood star-Lindelöf. Let $\mathcal{U}$ be an open cover of Y. We have to find a countable subset F of Y such that for each open $O\supseteq F$, $St(O,\mathcal{U})=Y$. For each $n\in\omega$, consider the open cover

$$\mathcal{U}_n=\mathcal{U}\cup\{X\setminus H_n\}$$

of X. Since X is neighborhood star-Lindelöf, there exists a countable subset F_n of X such that for each open $O'\supseteq F_n$, such that $St(O',\mathcal{U})=X$. For each $n\in\omega$, let $M_n=F_n\cap Y$. Then M_n is a countable sunset of Y such that for each open $O\supseteq M_n$, such that $H_n\subseteq St(O,\mathcal{U})$. If we put $F=\bigcup\{M_n : n\in\omega\}$, then F is a countable subset of Y such that for each open $O\supseteq F$, $St(O,\mathcal{U})=Y$, which shows that X is neighborhood star-Lindelöf. $\square$

A *cozero-set* in a space X is a set of the form $f^{-1}(R\setminus\{0\})$ for some real-valued continuous function f on X. Since a cozero-set is an open F_σ-set, we have the following corollary of Theorem 2.12.

Corollary 2.13. *A cozero-set of a neighborhood star-Lindelöf space is neighborhood star-Lindelöf.*

Remark 2.14. Bonanzinga et al. in [5] showed that there exists a Urysohn neighborhood star-Lindelöf space that is not strongly star-Lindelöf. But the author does not know if there exists a Tychonoff (or normal) example.

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