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Asymptotic Normality of a Simple Linear EV Regression Model with Martingale Difference Errors

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Abstract. This paper considers the estimation of a linear EV (errors-in-variables) regression model under martingale difference errors. The usual least squares estimations lead to biased estimators of the unknown parametric when measurement errors are ignored. By correcting the attenuation we propose a modified least squares estimator for a parametric component and construct the estimators of another parameter component and error variance. The asymptotic normalities are also obtained for these estimators. The simulation study indicates that the modified least squares method performs better than the usual least squares method.

1. Introduction

Consider the following simple linear EV regression model

$$y_i = x_i\beta + \theta + e_i, \quad \xi_i = x_i + u_i, \quad i = 1, \cdots, n, \tag{1}$$

where y_i is the response, β and θ are unknown parameters, x_i is nonrandom design point. Due to the measuring mechanism or the nature of the environment, the variable x_i is measured with error and is not directly observable. Instead, x_i is observed through $\xi_i = x_i + u_i$, where e_i, u_i are random errors.

Model (1) belongs to a kind of model called the EV model or measurement error model which was proposed by Deaton [6] to correct for the effects of sampling error and is somewhat more practical than the ordinary regression model. There are three monographs for measurement errors models, many of the early results are summarized in Fuller [13], and updated results of linear and nonlinear measurement error models can be found in Cheng and Van Ness [3] and Carroll et al. [1]. The last two decades there have been an increasing number of applications of the linear EV model due to its simple form and wide applicability. Lai et al. [16] established strong consistency of the least squares (LS) estimators for the unknown parameter in the multiple EV regression model; Gleser (1981) obtained some large sample results of estimation in a multivariate EV regression model; Amemiya and Fuller (1984) also discussed

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the estimation for the multivariate EV model; Cui [4] proved the asymptotic normality of M-estimators in the EV model; Cui and Chen [5] constructed empirical likelihood confidence regions for the unknown parameters in model (1); Liu and Chen [20] gave consistency of the LS estimators for model (1) and also proved that the sufficient and necessary condition for $\hat{\beta}_n$ being strong and weak consistent estimate of β is $\lim_{n\to\infty} n^{-1}A_n = \infty$, where $A_n = \sum_{i=1}^{n}\left(x_i - n^{-1}\sum_{j=1}^{n} x_j\right)^2$; Miao and Yang [25] gave the loglog law for LS estimator in simple linear EV regression model (1). Miao et al. [24] obtained consistency and asymptotic normality for usual LS estimators in simple linear EV model (1). Other works of EV model can be found in Liang and Wang [19], Huang et al. [15], Liang et al. [18], Huang [14], Ma and Li [22], Lv et al. [21], Delaigle and Meister [7] and so on. However, lots of these results are based on the usual LS estimation, and it shall produces bias. At the same time, all these articles related to the simple linear EV model (1) worked under independent framework.

It is well known that the independence assumption for the errors is not always valid in applications, especially for sequentially collected economic data, which often exhibit evident dependence in the errors. Miao et al. [26] obtained the asymptotic normality and strong consistency for unknown parameters in the EV model under the assumptions that the errors are stationary negatively associated sequences. Fan et al. [11, 12] studied various statical properties for partially time-varying coefficient EV models with dependent data. Martingale difference as a more realistic error has been assumed by many authors. For example, Fan et al. [10] employed the empirical likelihood method to obtain the confidence regions for a heteroscedastic partial linear model with martingale difference errors. Among others, see Li and Liu [17], Chen and Cui [2] and Fan and Liang [8], for example.

In this paper, we assume that $\{(e_i, u_i), \mathscr{F}_i, i \geq 1\}$ is a sequence of martingale differences with

$$E\left(e_i^2|\mathscr{F}_{i-1}\right) = \sigma_1^2 \quad \text{and} \quad E\left(u_i^2|\mathscr{F}_{i-1}\right) = \sigma_2^2 \quad a.s.$$

Usual profile least squares parameter estimation method is the most basic method. It is widely used because the advantage of simple algorithm and easy to implement. However, the use of LS estimation may obtained deviation parameter estimators. In order to overcome the bias of the usual profile least squares estimation when measurement errors are ignored, we propose the modified LS estimator for the parameter β and construct the estimators of θ and error variance and establish the asymptotic normality for these estimators. Modified LS estimation is an improved method for LS estimation that can reduce influence of errors, thus improving the estimation accuracy.

The rest of this paper is organized as follows. The modified LS estimation of the parametric β and the estimators of θ and the error variance σ_1^2 are constructed in Section 2. Assumption conditions, the asymptotic normalities of the estimators are also established in Section 2. Some simulation studies are conducted in Section 3. The proofs of the main results are postponed in Section 4.

2. The Methodology and the Results

From model (1), it follow that

$$y_i = \xi_i\beta + \theta + v_i, \;\; v_i = e_i - u_i\beta, \;\; 1 \leq i \leq n.$$

Thus, we get the usual LS estimators of β and θ:

$$\hat{\beta}_{n1} = \frac{\sum_{i=1}^{n}(\xi_i - \bar{\xi}_n)(y_i - \bar{y}_n)}{\sum_{i=1}^{n}(\xi_i - \bar{\xi}_n)^2}, \quad \hat{\theta}_{n1} = \bar{y}_n - \bar{\xi}_n\hat{\beta}_{n1},$$

where $\bar{\xi}_n = n^{-1}\sum_{i=1}^{n}\xi_i$, and other similar notations, such as $\bar{y}_n, \bar{u}_n, \bar{e}_n, \bar{e}_n$ are defined in the same way.

In order to overcome the bias of the usual profile least squares estimation when measurement errors are ignored, similar to You and Chen [27] and Fan et al. [12], we propose the modified LS estimators as follows,

$$\hat{\beta}_n = \frac{\sum_{i=1}^{n} (\xi_i - \bar{\xi}_n)(y_i - \bar{y}_n)}{\sum_{i=1}^{n} (\xi_i - \bar{\xi}_n)^2 - n\sigma_2^2}, \quad \hat{\theta}_n = \bar{y}_n - \bar{\xi}_n \hat{\beta}_n.$$

Sometimes it is also necessary to estimate the error variance $\sigma_1^2 = E\left(e_i^2\right)$ for such tasks as the construction of confidence regions, model-based tests, model selection procedures, single-to-noise ratio determination and so on. From $E(y_i - x_i\beta - \theta)^2 = \sigma_1^2$ and $E(y_i - \xi_i\beta - \theta)^2 = \sigma_1^2 + \beta^2\sigma_2^2$, we define an estimator of the error variance σ_1^2 as

$$\hat{\sigma}_1^2 = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{\theta}_n - \xi_i\hat{\beta}_n)^2 - \hat{\beta}_n^2\sigma_2^2.$$

Now we list some assumptions, which are also assumed in Miao et al. [25] and Fan and Liang [8].

(A1) $\lim_{n\to\infty} n^{-1}A_n = \infty$, where $A_n = \sum_{i=1}^{n} (x_i - \bar{x}_n)^2$;

(A2) $E|e_i|^{2+\delta} < \infty$ and $E|u_i|^{2+\delta} < \infty$ for some $\delta > 0$.

Theorem 2.1. *Suppose that* $(A1)$ *and* $(A2)$ *hold. Then*

$$\frac{\sqrt{A_n}(\hat{\beta}_n - \beta)}{\sigma} \xrightarrow{d} N(0,1),$$

where $\sigma^2 = \sigma_1^2 + \sigma_2^2\beta^2$.

Theorem 2.2. *Suppose that* $(A1)$ *and* $(A2)$ *hold. Furthermore if* $A_n/(n\bar{x}_n^2) \to \infty$, *then*

$$\frac{\sqrt{n}(\hat{\theta}_n - \theta)}{\sigma} \xrightarrow{d} N(0,1).$$

Theorem 2.3. *Under the conditions of* $(A1)$ *and* $(A2)$, *we have*

$$\sqrt{n}(\hat{\sigma}_1^2 - \sigma_1^2) \xrightarrow{d} N(0,k),$$

where $k = E[(e_1 - u_1\beta)^2 - (\sigma_1^2 + \beta^2\sigma_2^2)]$.

Remark 2.4. (A1) implies that our estimators have good large-sample properties when $\{x_n, n \geq 1\}$ have greater dispersion. For example, if $\{x_n, n \geq 1\}$ is a sequence of independent and identical distributed random variables with common distribution $N(0,1)$, the above estimators have no asymptotic normality. Condition (A2) is a common moment condition, which has been used by many authors, see (A2) in Fan et al. [9] for example. Condition $A_n/\left(n\bar{x}_n^2\right) \to \infty$ in Theorem 2.2 has been used by Miao et al. [23] and Fan et al. [9].

3. Monte Carlo Simulation Study

In this section, we carry out some simulations to study the unknown parameters and error variance in linear EV regression model with martingale difference errors. The data are generated from the following regression model:

$$y_i = x_i\beta + \theta + e_i, \;\; \xi_i = x_i + u_i, \;\; i = 1, \cdots, n,$$

where $\beta = 1.5$, $\theta = 0.5$ and $x_i = i$ for $i = 1, \cdots, n$. Since $\{(e_i, u_i), \mathscr{F}_i, 1 \le i \le n\}$ is a martingale difference sequence, we first generate martingale sequence $\{\zeta_i, \sigma(\zeta_i), i \ge 1\}$, then let $\eta_i = \zeta_{i+1} - \zeta_i$. In our simulation process, we generate the first random number $\zeta_1 \sim N(0, 1)$, and according to the conditional distribution $\zeta_{i+1}|\zeta_i \sim N(\zeta_i, 0.5^2)$ for $i = 1, \cdots, n$, we can generate $\eta_2, \cdots, \eta_{n+1}$. u_i is taken by using the similar process.

The sample sizes n are chosen to be 20, 30, 50, 100, 200 and 300, respectively. In each case the number of simulated realizations is 1000. That is, for each n, we calculate the values of each estimators. Then for each n, we repeat this process 1000 times, and we get 1000 values of each estimators. The sample means and mean square errors (MSE) of the proposed estimators are then obtained. The simulation results are shown in Table 1. We also plot the histograms and QQ-normality plots in Figures 1-3 to show the asymptotic normality of the estimators.

Table 1: Sample means and MSE of various estimators for $\beta = 1.5$ and $\theta = 0.5$

n	20	30	50	100	200	300
Mean($\hat{\beta}_{n1}$)	1.4919	1.4954	1.4987	1.4996	1.4999	1.4999
MSE($\hat{\beta}_{n1}$)	0.0013	4.0338e-4	8.3240e-5	1.0239e-5	1.1737e-6	3.8257e-7
Mean($\hat{\beta}_{n}$)	1.5031	1.5004	1.5005	1.5000	1.5000	1.5000
MSE($\hat{\beta}_{n}$)	0.0013	3.8878e-4	8.2436e-5	1.0054e-5	1.1666e-6	3.7682e-7
Mean($\hat{\theta}_{n1}$)	0.5852	0.5705	0.5448	0.5245	0.5066	0.5120
MSE($\hat{\theta}_{n1}$)	0.1981	0.1320	0.0675	0.0358	0.0159	0.0116
Mean($\hat{\theta}_{n}$)	0.4670	0.4930	0.4989	0.5017	0.4953	0.5045
MSE($\hat{\theta}_{n}$)	0.1976	0.1288	0.0658	0.0352	0.0158	0.0115
Mean($\hat{\sigma}_1^2$)	0.1659	0.1922	0.2154	0.2360	0.2465	0.2479
MSE($\hat{\sigma}_1^2$)	0.1672	0.1366	0.1047	0.0817	0.0704	0.0680

Note: 4.0338e-4 means 4.0338×10^{-4} and others are the same meaning.

From these simulation results, we draw the following conclusions. In Table 1, the estimators of β, θ and σ_1^2 are very close to their real values. The mean square errors are also very small. With the sample size increases, the estimated values are more closer to the real values, and the mean square errors also getting smaller. Besides, we can seen from Table 1 that the modified LS estimators $\hat{\beta}_n$ and $\hat{\theta}_n$ are fitted better than $\hat{\beta}_{n1}$ and $\hat{\theta}_{n1}$. In addition, seen from Figures 1-3, the distribution of estimators $\hat{\beta}_{n1}$, $\hat{\beta}_n$, $\hat{\theta}_{n1}$, $\hat{\theta}_n$ and $\hat{\sigma}_1^2$ have good fit with the normal distribution and $\hat{\beta}_n$ and $\hat{\theta}_n$ performs slightly better than $\hat{\beta}_{n1}$ and $\hat{\theta}_{n1}$.

4. Proofs of Main Results

For the convenience and simplicity, let c denote positive constant whose value may vary at each occurrence. Before proving the main theorems, we give a series of lemmas.

Lemma 4.1 (Fan et al. [10], Lemma 4.3). *Let $\{\xi_i, \mathscr{F}_i\}$ be a sequence of martingale differences, suppose that $E\left(\xi_i^2 \mid \mathscr{F}_{i-1}\right) \le C$ a.s., and $\sup_i E|\xi_i|^p < \infty$ for some $p \ge 2$. Assume that $\{a_{ni}(t), 1 \le i \le n\}$ is an array of real numbers defined on $[0, 1]$ satisfying $\max_{1 \le i,k \le n} |a_{ni}(t_k)| = O(n^{-s})$ for some $s > 0$, and that $L(x) > 0$ is a positive function slowly varying at infinity. If $\max\limits_{1 \le k \le n} \sum\limits_{i=1}^{n} a_{ni}^2(Z_k) = O(n-\gamma)$, then $\max\limits_{1 \le k \le n} \left|\sum\limits_{i=1}^{n} a_{ni}(Z_k)\xi_i\right| = o[n^{-\alpha}L(n)]$ a.s. for $\alpha < \min(\gamma/2, s - 1/p)$.*

Lemma 4.2 (Fan et al. [10], Lemma 4.1). *Let $\{\xi_i, \mathscr{F}_i, 1 \le i < \infty\}$ be a sequence of martingale differences. $E\left(\xi_i^2 \mid F_{i-1}\right) = \sigma_i^2$ a.s. and $\lim_{c \to \infty} \sup_{i \ge 1} E\left(\xi_i^2 I(|\xi_i| > c) \mid \mathscr{F}_{i-1}\right) = 0$ a.s. Assume that $\{c_{nk} : 1 \le k \le n, n \ge 1\}$ is an array of real numbers satisfying*

$$\lim_{n \to \infty} \sum_{k=1}^{n} c_{nk}^2 \sigma_k^2 = 1, \quad \sup_{n \ge 1} \sum_{k=1}^{n} c_{nk}^2 < \infty, \quad \lim_{n \to \infty} \max_{1 \le k \le n} |c_{nk}| = 0.$$

Then, $\sum_{k=1}^{n} c_{nk}\xi_k \xrightarrow{d} N(0, 1)$ as $n \to \infty$.

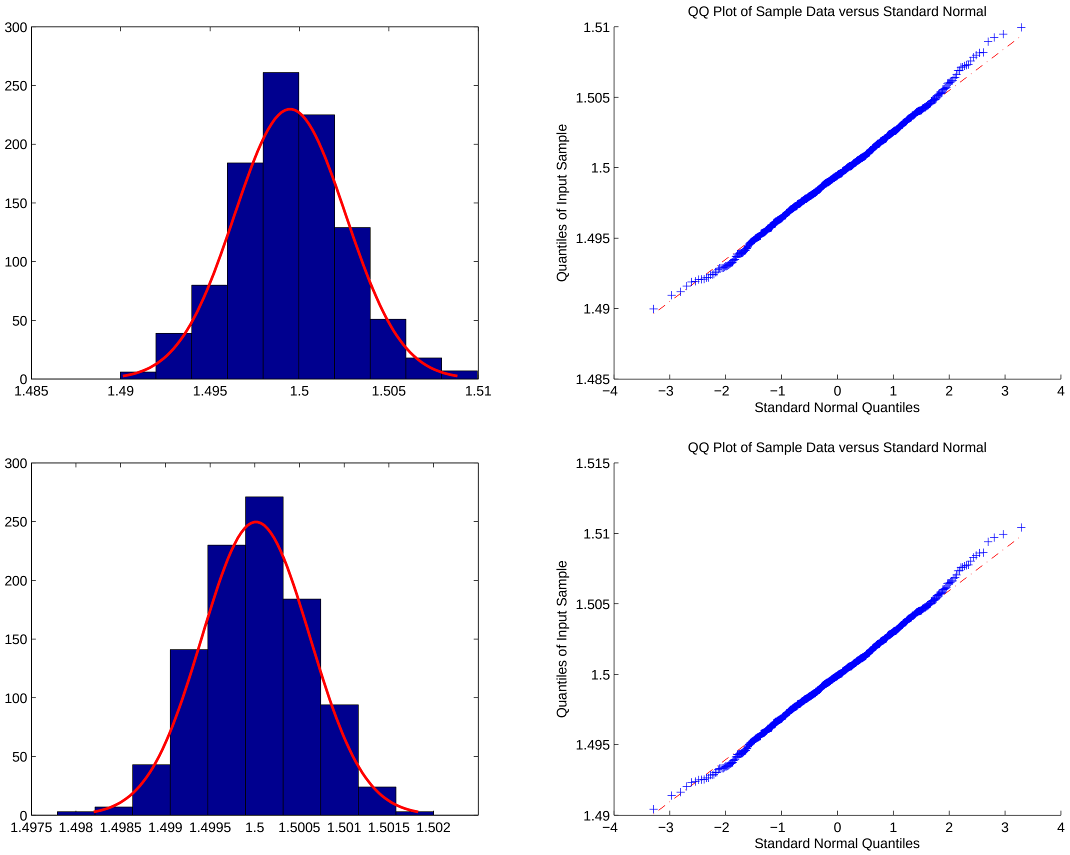

Figure 1: Histograms and QQ-normality plots for $\hat{\beta}_{n1}$(top) and $\hat{\beta}_n$(bottom) with $n = 300$

Lemma 4.3. *Suppose that conditions (A1) and (A2) hold. Then we have* $A_n^{-1} \sum_{i=1}^{n} (\xi_i - \bar{\xi}_n)^2 - 1 = o(1)$ *a.s.*

Proof. Observe that

$$\sum_{i=1}^{n} (\xi_i - \bar{\xi}_n)^2 = A_n + \sum_{i=1}^{n} (u_i - \bar{u}_n)^2 + 2 \sum_{i=1}^{n} (x_i - \bar{x}_n) u_i.$$

First, we shall establish that

$$\frac{1}{A_n} \sum_{i=1}^{n} (x_i - \bar{x}_n) u_i = o(1) \quad a.s. \tag{2}$$

(A2) and Lemma 4.1 imply that $n^{-1} \sum_{i=1}^{n} \left(u_i^2 - \sigma_2^2\right) = o(1)$ a.s. Further we have

$$\left| A_n^{-1} \sum_{i=1}^{n} (x_i - \bar{x}_n) u_i \right| \leq \sqrt{A_n^{-2} \sum_{i=1}^{n} \left(x_i - \bar{x}_n^2\right) \sum_{i=1}^{n} u_i^2} = \sqrt{A_n^{-1} n \left[n^{-1} \sum_{i=1}^{n} \left(u_i^2 - \sigma_2^2\right) + \sigma_2^2 \right]} = o(1) \quad a.s.$$

Thus, (2) holds. Hence, in order to prove Lemma 4.3, we need only to show that

$$\sum_{i=1}^{n} (u_i - \bar{u}_n)^2 = o(A_n) \quad a.s. \tag{3}$$

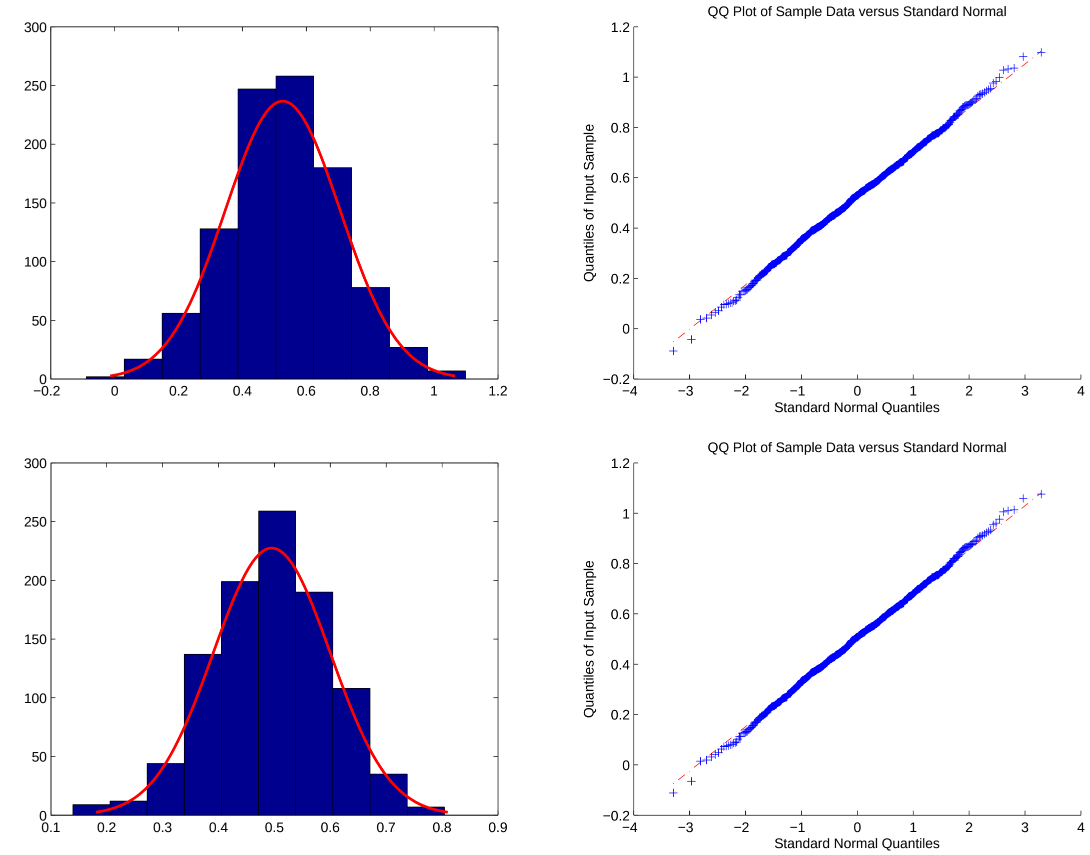

Figure 2: Histograms and QQ-normality plots for $\hat{\theta}_{n1}$(top) and $\hat{\theta}_n$(bottom) with $n = 300$

Note that

$$\sum_{i=1}^{n} (u_i - \bar{u}_n)^2 \le \sum_{i=1}^{n} u_i^2 = \sum_{i=1}^{n} \left(u_i^2 - \sigma_2^2\right) + n\sigma_2^2.$$

In view of Lemma 4.1, from (A1) and (A2), it follows that $A_n^{-1} \sum_{i=1}^{n} \left(u_i^2 - \sigma_2^2\right) = o(1)$ a.s., which together with $n\sigma_2^2 = o(A_n)$, yields (3). This completes the proof of Lemma 4.3. $\square$

Lemma 4.4. *Suppose that conditions* (A1) *and* (A2) *hold, then*

$$A_n^{-1} \sum_{i=1}^{n} (\omega_i - \bar{\omega}_n)^2 - 1 = o(1) \quad and \quad A_n^{-1/2} \sum_{i=1}^{n} (u_i - \bar{u}_n) e_i = o(1) \quad a.s.$$

where $w_i = e_i$ *and* u_i.

Proof. First, let us deal with the first equation. In view of Lemma 4.1, from (A1) and (A2) we have

$$\frac{1}{\sqrt{A_n}} \sum_{i=1}^{n} (e_i - \bar{e}_n)^2 \le \frac{1}{\sqrt{A_n}} \sum_{i=1}^{n} e_i^2 = \frac{1}{\sqrt{A_n}} \sum_{i=1}^{n} \left(e_i^2 - \sigma_1^2\right) - \frac{n\sigma_1^2}{\sqrt{A_n}} = o(1) \ a.s.$$

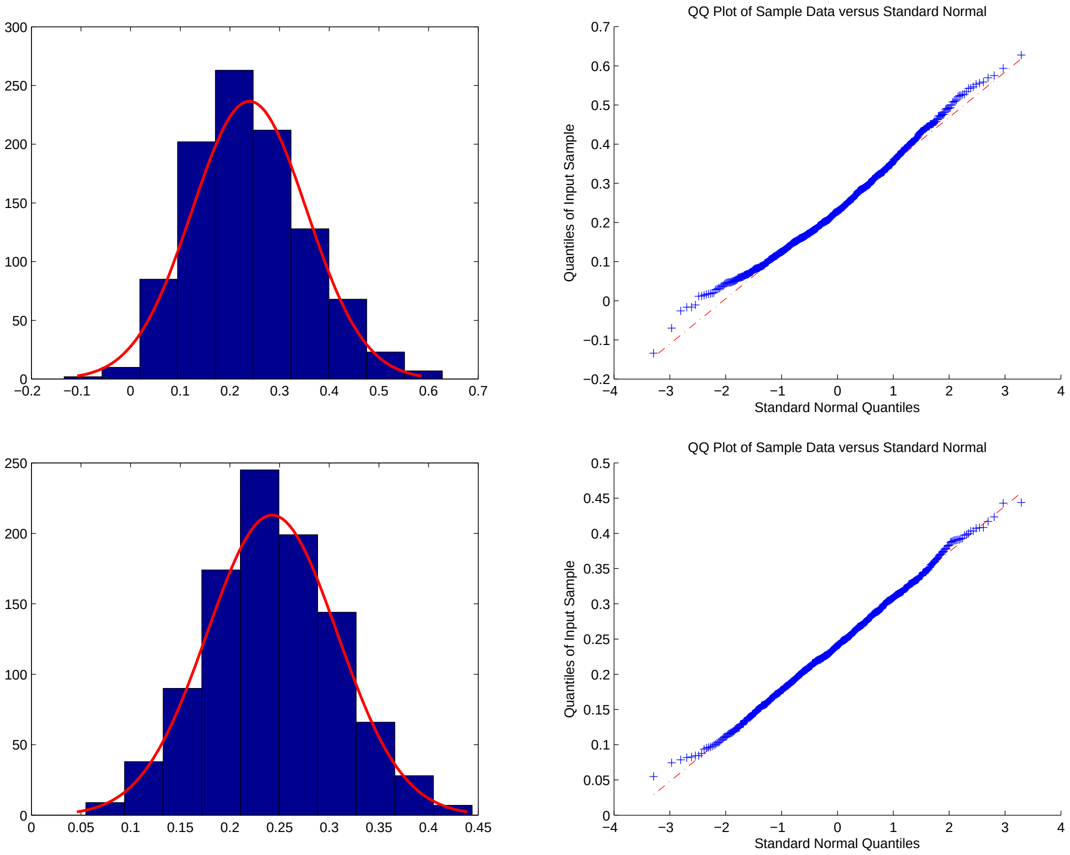

Figure 3: Histograms and QQ-normality plots for $\hat{\sigma}_1^2$ with $n = 100$ (top) and $n = 300$ (bottom)

which implies that the first equation in Lemma 4.4 holds. As to the second equation, note that

$$\frac{\sum\limits_{i=1}^{n} (u_i - \bar{u}_n) e_i}{\sqrt{A_n}} \leq \left[\frac{\sum\limits_{i=1}^{n} (u_i - \bar{u}_n)^2 + \sum\limits_{i=1}^{n} \left(e_i^2 - \sigma_1^2\right) + n\sigma_1^2}{A_n} \right]^{1/2}.$$

Then by the first equation and Lemma 4.1 give the the second equation. $\square$

Proof of Theorem 2.1. We write

$$\sqrt{A_n}(\hat{\beta}_n - \beta) = \sqrt{A_n} \frac{\sum\limits_{i=1}^{n} (u_i - \bar{u}_n) e_i + \sum\limits_{i=1}^{n} (x_i - \bar{x}_n)(e_i - u_i\beta) - \beta \sum\limits_{i=1}^{n} (u_i - \bar{u}_n)^2 + \beta n \sigma_2^2}{\sum\limits_{i=1}^{n} (\xi_i - \bar{\xi}_n)^2 - n\sigma_2^2}.$$

From (2) and Lemmas 4.3–4.4, we find

$$\sum_{i=1}^{n} (\xi_i - \bar{\xi}_n)^2 - n\sigma_2^2 = \sum_{i=1}^{n} (x_i - \bar{x}_n)^2 + \sum_{i=1}^{n} (u_i - \bar{u}_n)^2 + 2 \sum_{i=1}^{n} (x_i - \bar{x}_n)(u_i - \bar{u}_n) - n\sigma_2^2 \xrightarrow{p} A_n.$$

In view of Lemma 4.4, it follows that

$$\frac{A_n}{\sum\limits_{i=1}^{n}\left(\xi_i-\bar{\xi}_n\right)^2-n\sigma_2^2}\frac{\sum\limits_{i=1}^{n}\left(u_i-\bar{u}_n\right)e_i}{\sqrt{A_n}}=o_p\left(1\right).$$

Set $c_{nk}=\frac{A_n(x_i-\bar{x}_n)}{\left[\sum\limits_{i=1}^{n}\left(\xi_k-\bar{\xi}_n\right)^2-n\sigma_2^2\right]\sqrt{A_n}\sigma}$, then we can easily derive that

$$\lim_{n\to\infty}\sum_{k=1}^{n}c_{nk}^2\sigma^2=1,\quad \sup_{n\geq 1}\sum_{k=1}^{n}c_{nk}^2<\infty,\quad \lim_{n\to\infty}\max_{1\leq k\leq n}|c_{nk}|=0.$$

According to Lemma 4.2, we obtain that

$$\frac{\sum\limits_{i=1}^{n}A_n\left(x_i-\bar{x}_n\right)\left(e_i-u_i\beta\right)}{\left[\sum\limits_{k=1}^{n}\left(\xi_k-\bar{\xi}_n\right)^2-n\sigma_2^2\right]\sqrt{A_n}\sigma}\xrightarrow{d}N\left(0,1\right),$$

which completes the proof of Theorem 2.1. □

Proof of Theorem 2.2. We write

$$\hat{\theta}_n-\theta=\bar{x}_n(\beta-\hat{\beta}_n)+\bar{u}_n(\beta-\hat{\beta}_n)-\bar{u}_n\beta+\bar{e}_n.$$

By Lemma 4.2, we can easily get

$$\frac{1}{\sqrt{n}\sigma}\sum_{i=1}^{n}\left(e_i-u_i\beta\right)\xrightarrow{d}N\left(0,1\right),$$

i.e., $\sqrt{n}\left(\bar{e}_n-\bar{u}_n\beta\right)/\sigma\xrightarrow{d}N\left(0,1\right)$. Therefore, by Theorem 2.1, we need only to show that

$$\frac{\sqrt{n}}{\sqrt{A_n}}\left(\bar{x}_n+\bar{u}_n\right)=o\left(1\right).$$

(A1) and Lemma 4.1 imply that $\frac{\sqrt{n}}{\sqrt{A_n}}\bar{u}_n=o_p\left(1\right)$ a.s. The condition $A_n/\left(n\bar{x}_n^2\right)\to\infty$ yields that $\frac{\sqrt{n}}{\sqrt{A_n}}\bar{x}_n\to 0$. Hence, Theorem 2.2 is proved. □

Proof of Theorem 2.3. Observe that

$$\begin{aligned}\hat{\sigma}_1^2&=\frac{1}{n}\sum_{i=1}^{n}(y_i-\hat{\theta}_n-\xi_i\hat{\beta}_n)^2-\beta^2\sigma_2^2\\&=\frac{1}{n}\sum_{i=1}^{n}\left[y_i-(\hat{\theta}_n-\theta)-\theta-\xi_i(\hat{\beta}_n-\beta)-\beta\xi_i\right]^2-\beta^2\sigma_2^2.\end{aligned}$$

From Theorems 2.1-2.2, we can easily get

$$\begin{aligned}\sqrt{n}(\hat{\sigma}_1^2-\sigma_1^2)&=\frac{1}{\sqrt{n}}\sum_{i=1}^{n}\left[(y_i-\theta-\xi_i\beta)^2-\beta^2\sigma_2^2-\sigma_1^2\right]+o_p(1)\\&=\frac{1}{\sqrt{n}}\sum_{i=1}^{n}\left[(e_i-u_i\beta)^2-(\sigma_1^2+\beta^2\sigma_2^2)\right]+o_p(1).\end{aligned}$$

Then by applying central limit theorem and the law of large numbers, we can obtain the asymptotic normality for $\sqrt{n}(\hat{\sigma}_1^2-\sigma_1^2)$. □

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