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On Soft Regular Generalized Closed Sets with Respect to a Soft Ideal in Soft Topological Spaces

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Abstract. The notion of soft ideal in soft topological spaces was studied by Mustafa and Sleim [9]. They studied the concept of soft generalized closed sets with respect to a soft ideal, which is the extension of the concept of soft generalized closed sets. In this paper, we define soft regular generalized closed and open sets with respect to a soft ideal in soft topological spaces and study some of their properties. We introduce these concepts in soft topological spaces which are defined over an initial universe with a fixed set of parameters.

1. Introduction

In 1999, Molodtsov [1] proposed the concept of a soft set, which can be seen as a new mathematical approach to vagueness. He [1] applied the theory successfully in directions such as, smoothness of functions, operations research, Riemann integration, game theory, theory of probability and so on. Maji et al. [2] carried out Molodtsov's idea by introducing several operations in soft set theory. The absence of any restrictions on the approximate description in soft set theory makes this theory very convenient and easily applicable in practice. Therefore, research works on soft sets are very active and progressing rapidly in these years.

Topological structure of soft sets was studied by many authors: Shabir and Naz [3] defined the soft topological space and studied the concepts of soft open set, soft interior point, soft neighbourhood of a point, soft separation axioms and subspace of a soft topological space. They introduced these concepts which are defined over an initial universe with a fixed set of parameters. Zorlutuna et al. [4] also investigated soft interior point, soft neighbourhood and soft continuity. They introduced the relationships between soft topology and fuzzy topology. Hussain and Ahmad [5] studied and discussed the properties of soft interior, closure and boundary on soft topology. Kannan [6] defined soft generalized closed and open sets in soft topological spaces. After then Yüksel et al. [7] studied behaviour relative to soft subspaces of soft generalized closed sets and continued investigating the properties of soft generalized closed and open sets. They established their several properties in a soft compact (soft Lindelöf, soft countably compact, soft regular, soft normal) space. Also Yüksel et al. [8] defined soft regular generalized closed and open sets in soft topological spaces. They studied some basic properties of these concepts and investigated their relationships with different types of subsets of soft topological spaces with the help of counterexamples.

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Mustafa and Sleim [9] introduced the notion of a soft ideal in soft topological spaces. They introduced the concept of soft generalized closed sets with respect to a soft ideal and studied their properties in detail, which is extension of the concept of soft generalized closed sets. In this paper, we define soft regular generalized closed and open sets with respect to a soft ideal in soft topological spaces. We investigate the behaviour of soft regular generalized closed sets with respect to a soft ideal relative to union, intersection and soft subspaces. We also study the relationship between soft generalized closed set and soft regular generalized closed set with respect to a soft ideal.

2. Preliminaries

Let X be an initial universe set and E be the set of all possible parameters with respect to X. Parameters are often attributes, characteristics or properties of the objects in X. Let $P(X)$ denote the power set of X. Then a soft set over X is defined as follows.

Definition 2.1. *[1] A pair (F, A) is called a soft set over X where $A \subseteq E$ and $F : A \longrightarrow P(X)$ is a set valued mapping. In other words, a soft set over X is a parameterized family of subsets of the universe X. For $\forall \varepsilon \in A$, $F(\varepsilon)$ may be considered as the set of ε-approximate elements of the soft set (F, A). It is worth noting that $F(\varepsilon)$ may be arbitrary. Some of them may be empty, and some may have nonempty intersection.*

Definition 2.2. *[2] A soft set (F, A) over X is said to be a null soft set denoted by Φ if for all $e \in A$, $F(e) = \emptyset$. A soft set (F, A) over X is said to be an absolute soft set denoted by $\widetilde{A}$ if for all $e \in A$, $F(e) = X$.*

Definition 2.3. *[3] Let Y be a nonempty subset of X, then $\widetilde{Y}$ denotes the soft set (Y, E) over X for which $Y(e) = Y$, for all $e \in E$. In particular, (X, E) will be denoted by $\widetilde{X}$.*

Definition 2.4. *[2] For two soft sets (F, A) and (G, B) over X, we say that (F, A) is a soft subset of (G, B) if $A \subseteq B$ and for all $e \in A$, $F(e)$ and $G(e)$ are identical approximations. We write $(F, A) \sqsubseteq (G, B)$. (F, A) is said to be a soft super set of (G, B), if (G, B) is a soft subset of (F, A). We denote it by $(G, B) \sqsubseteq (F, A)$. Then (F, A) and (G, B) are said to be soft equal if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A).*

Definition 2.5. *[2] The union of two soft sets (F, A) and (G, B) over X is the soft set (H, C), where $C = A \cup B$ and for all $e \in C$, $H(e) = F(e)$ if $e \in A \backslash B$, $H(e) = G(e)$ if $e \in B \backslash A$, $H(e) = F(e) \cup G(e)$ if $e \in A \cap B$. We write $(F, A) \sqcup (G, B) = (H, C)$. [10] The intersection (H, C) of (F, A) and (G, B) over X, denoted $(F, A) \sqcap (G, B)$, is defined as $C = A \cap B$, and $H(e) = F(e) \cap G(e)$ for all $e \in C$.*

Definition 2.6. *[3] The difference (H, E) of two soft sets (F, E) and (G, E) over X, denoted by $(F, E) \backslash (G, E)$, is defined as $H(e) = F(e) \backslash G(e)$ for all $e \in E$.*

Definition 2.7. *[3] The relative complement of a soft set (F, E) over X is denoted by $(F, E)^c$ and is defined by $(F, E)^c = (F^c, E)$ where $F^c : E \longrightarrow P(X)$ is a mapping given by $F^c(e) = X \backslash F(e)$ for all $e \in E$.*

Proposition 2.8. *[3] Let (F, E) and (G, E) be the soft sets over X. Then the followings hold:*

(1) $((F, E) \sqcup (G, E))^c = (F, E)^c \sqcap (G, E)^c$
(2) $((F, E) \sqcap (G, E))^c = (F, E)^c \sqcup (G, E)^c$.

Definition 2.9. *[4] Let I be an arbitrary index set and $\{(F_i, E)\}_{i \in I}$ be a subfamily of soft sets over X. The union of these soft sets is the soft set (G, E), where $G(e) = \cup_{i \in I} F_i(e)$ for each $e \in E$. We write $\sqcup_{i \in I}(F_i, E) = (G, E)$. The intersection of these soft sets is the soft set (H, E), where $H(e) = \cap_{i \in I} F_i(e)$ for all $e \in E$. We write $\sqcap_{i \in I}(F_i, E) = (H, E)$.*

Proposition 2.10. *[2, 4] Let (F, E), (G, E), (H, E) and (K, E) be the soft sets over X. Then the followings hold:*

(1) $(F,E) \sqcap (F,E) = (F,E)$, $(F,E) \sqcap \Phi = \Phi$, $(F,E) \sqcap \widetilde{X} = (F,E)$.
(2) $(F,E) \sqcup (F,E) = (F,E)$, $(F,E) \sqcup \Phi = (F,E)$, $(F,E) \sqcup \widetilde{X} = \widetilde{X}$.
(3) $(F,E) \sqcap (G,E) = (G,E) \sqcap (F,E)$, $(F,E) \sqcup (G,E) = (G,E) \sqcup (F,E)$.
(4) $(F,E) \sqcup ((G,E) \sqcup (H,E)) = ((F,E) \sqcup (G,E)) \sqcup (H,E)$, $(F,E) \sqcap ((G,E) \sqcap (H,E)) = ((F,E) \sqcap (G,E)) \sqcap (H,E)$.
(5) $(F,E) \sqcup ((G,E) \sqcap (H,E)) = ((F,E) \sqcup (G,E)) \sqcap ((F,E) \sqcup (H,E))$, $(F,E) \sqcap ((G,E) \sqcup (H,E)) = ((F,E) \sqcap (G,E)) \sqcup ((F,E) \sqcap (H,E))$.
(6) $(F,E) \sqsubseteq (G,E)$ if and only if $(F,E) \sqcap (G,E) = (F,E)$.
(7) $(F,E) \sqsubseteq (G,E)$ if and only if $(F,E) \sqcup (G,E) = (G,E)$.
(8) If $(F,E) \sqcap (G,E) = \Phi$, then $(F,E) \sqsubseteq (G,E)^c$.
(9) If $(F,E) \sqsubseteq (G,E)$ and $(G,E) \sqsubseteq (H,E)$, then $(F,E) \sqsubseteq (H,E)$.
(10) If $(F,E) \sqsubseteq (G,E)$ and $(H,E) \sqsubseteq (K,E)$, then $(F,E) \sqcap (H,E) \sqsubseteq (G,E) \sqcap (K,E)$.
(11) $(F,E) \sqcup (F,E)^c = \widetilde{X}$.
(12) $(F,E) \sqsubseteq (G,E)$ if and only if $(G,E)^c \sqsubseteq (F,E)^c$.

Definition 2.11. *[3] Let (F,E) be a soft set over X and $x \in X$. $x \in (F,E)$ read as x belongs to the soft set (F,E) whenever $x \in F(e)$ for all $e \in E$. For any $x \in X$, $x \notin (F,E)$, if $x \notin F(e)$ for some $e \in E$.*

Definition 2.12. *[3] Let $\widetilde{\tau}$ be the collection of soft sets over X, then $\widetilde{\tau}$ is said to be a soft topology on X if*

(1) $\Phi, \widetilde{X} \in \widetilde{\tau}$
(2) If (F,E), $(G,E) \in \widetilde{\tau}$, then $(F,E) \sqcap (G,E) \in \widetilde{\tau}$
(3) If $\{(F_i,E)\}_{i\in I} \in \widetilde{\tau}$, $\forall i \in I$, then $\sqcup_{i\in I}(F_i,E) \in \widetilde{\tau}$.

The triplet $(X, \widetilde{\tau}, E)$ is called a soft topological space. Every member of $\widetilde{\tau}$ is called a soft open set. A soft set (F,E) is called soft closed in X if $(F,E)^c \in \widetilde{\tau}$.

Proposition 2.13. *[3] Let $(X, \widetilde{\tau}, E)$ be a soft topological space over X and $\widetilde{\tau}'$ denotes the collection of all soft closed sets. Then*

(1) $\Phi, \widetilde{X} \in \widetilde{\tau}'$
(2) If (F,E), $(G,E) \in \widetilde{\tau}'$, then $(F,E) \sqcup (G,E) \in \widetilde{\tau}'$
(3) If $\{(F_i,E)\}_{i\in I} \in \widetilde{\tau}'$, $\forall i \in I$, then $\sqcap_{i\in I}(F_i,E) \in \widetilde{\tau}'$.

Definition 2.14. *[3] Let $(X, \widetilde{\tau}, E)$ be a soft topological space over X and (F,E) be a soft set over X. Then the soft closure of (F,E), denoted by $(F,E)^-$, is the intersection of all soft closed super sets of (F,E).*

Clearly, $(F,E)^-$ is the smallest soft closed set over X which contains (F,E).

Theorem 2.15. *[3] Let $(X, \widetilde{\tau}, E)$ be a soft topological space over X, (F,E) and (G,E) soft sets over X. Then*

(1) $\Phi^- = \Phi$ and $(\widetilde{X})^- = \widetilde{X}$
(2) $(F,E) \sqsubseteq (F,E)^-$
(3) (F,E) is a soft closed set if and only if $(F,E) = (F,E)^-$
(4) $((F,E)^-)^- = (F,E)^-$
(5) $(F,E) \sqsubseteq (G,E)$ implies $(F,E)^- \sqsubseteq (G,E)^-$
(6) $((F,E) \sqcup (G,E))^- = (F,E)^- \sqcup (G,E)^-$
(7) $((F,E) \sqcap (G,E))^- \sqsubseteq (F,E)^- \sqcap (G,E)^-$.

Definition 2.16. *[3] Let $(X, \widetilde{\tau}, E)$ be a soft topological space over X, (G,E) be a soft set over X and $x \in X$. Then x is said to be a soft interior point of (G,E) and (G,E) is said to be a soft neighbourhood of x if there exists a soft open set (F,E) such that $x \in (F,E) \sqsubseteq (G,E)$.*

Definition 2.17. *[5] Let $(X, \tilde{\tau}, E)$ be a soft topological space over X and (F, E) be a soft set over X. Then the soft interior of (F, E), denoted by $(F, E)^{\circ}$, is the union of all soft open sets contained in (F, E).*

Clearly, $(F, E)^{\circ}$ is the largest soft open set contained in (F, E).

Theorem 2.18. *[5] Let $(X, \tilde{\tau}, E)$ be a soft topological space over X, (F, E) and (G, E) soft sets over X. Then*

(1) $\Phi^{\circ} = \Phi$ and $(\tilde{X})^{\circ} = \tilde{X}$
(2) $(F, E)^{\circ} \sqsubseteq (F, E)$
(3) (F, E) is a soft open set if and only if $(F, E) = (F, E)^{\circ}$
(4) $((F, E)^{\circ})^{\circ} = (F, E)^{\circ}$
(5) $(F, E) \sqsubseteq (G, E)$ implies $(F, E)^{\circ} \sqsubseteq (G, E)^{\circ}$
(6) $((F, E) \sqcap (G, E))^{\circ} = (F, E)^{\circ} \sqcap (G, E)^{\circ}$
(7) $((F, E) \sqcup (G, E))^{\circ} \sqsupseteq (F, E)^{\circ} \sqcup (G, E)^{\circ}$.

Theorem 2.19. *[5] Let $(X, \tilde{\tau}, E)$ be a soft topological space over X and (F, E) be a soft set over X. Then*

(1) $((F, E)^{c})^{\circ} = ((F, E)^{-})^{c}$
(2) $((F, E)^{c})^{-} = ((F, E)^{\circ})^{c}$.

Definition 2.20. *[3] Let (F, E) be a soft set over X and Y be a nonempty subset of X. Then the soft subset of (F, E) over Y denoted by $(^{Y}F, E)$, is defined as $^{Y}F(e) = Y \cap F(e)$, for all $e \in E$. In other words $(^{Y}F, E) = \tilde{Y} \sqcap (F, E)$.*

Definition 2.21. *[3] Let $(X, \tilde{\tau}, E)$ be a soft topological space over X and Y be a nonempty subset of X. Then $\tilde{\tau}_Y = \{(^{Y}F, E) : (F, E) \in \tilde{\tau}\}$ is said to be the soft relative topology on Y and $(Y, \tilde{\tau}_Y, E)$ is called a soft subspace of $(X, \tilde{\tau}, E)$. We can easily verify that $\tilde{\tau}_Y$ is, in fact, a soft topology over Y.*

Theorem 2.22. *[3] Let $(Y, \tilde{\tau}_Y, E)$ be a soft subspace of a soft topological space $(X, \tilde{\tau}, E)$ and (F, E) be a soft set over X, then*

(1) (F, E) is soft open in Y if and only if $(F, E) = \tilde{Y} \sqcap (G, E)$ for some $(G, E) \in \tilde{\tau}$.

(2) (F, E) is soft closed in Y if and only if $(F, E) = \tilde{Y} \sqcap (G, E)$ for some soft closed set (G, E) in X.

Definition 2.23. *[9, 11] Let $(X, \tilde{\tau}, E)$ be a soft topological space over X, (F, E) and (G, E) soft sets over X. Two soft sets (F, E) and (G, E) are said to be soft disconnected sets if $(F, E)^{-} \sqcap (G, E) = \Phi$ and $(G, E)^{-} \sqcap (F, E) = \Phi$.*

Definition 2.24. *[6] A soft set (F, E) is called soft generalized closed (soft g-closed) in a soft topological space $(X, \tilde{\tau}, E)$ if $(F, E)^{-} \sqsubseteq (G, E)$ whenever $(F, E) \sqsubseteq (G, E)$ and $(G, E) \in \tilde{\tau}$.*

Definition 2.25. *[6] A soft set (F, E) is called soft generalized open (soft g-open) in a soft topological space $(X, \tilde{\tau}, E)$ if the relative complement $(F, E)^{c}$ is soft g-closed in X.*

Definition 2.26. *[8] A soft set (F, E) is called soft regular open (soft regular closed) in a soft topological space $(X, \tilde{\tau}, E)$ if $(F, E) = ((F, E)^{-})^{\circ}$ $((F, E) = ((F, E)^{\circ})^{-})$.*

Remark 2.27. *[8] Every soft regular open set in a soft topological space $(X, \tilde{\tau}, E)$ is soft open.*

Definition 2.28. *[8] A soft set (F, E) is called soft regular generalized closed (soft rg-closed) in a soft topological space $(X, \tilde{\tau}, E)$ if and only if $(F, E)^{-} \sqsubseteq (G, E)$ whenever $(F, E) \sqsubseteq (G, E)$ and (G, E) is soft regular open in $(X, \tilde{\tau}, E)$.*

Definition 2.29. *[9] A nonempty collection I of soft subsets over X is called a soft ideal on X if the following holds*

(1) If $(F, E) \in I$ and $(G, E) \sqsubseteq (F, E)$ then $(G, E) \in I$ (heredity)
(2) If (F, E) and $(G, E) \in I$, then $(F, E) \sqcup (G, E) \in I$ (additivity).

Definition 2.30. *[9] A soft set (F, E) is called soft generalized closed with respect to a soft ideal I (soft I-g-closed) in a soft topological space $(X, \tilde{\tau}, E)$ if and only if $(F, E)^{-} \setminus (G, E) \in I$ whenever $(F, E) \sqsubseteq (G, E)$ and (G, E) is soft open.*

3. Soft Regular Generalized Closed Sets with Respect to a Soft Ideal

Definition 3.1. *A soft set (F,E) is called soft regular generalized closed with respect to a soft ideal I (briefly soft I-rg-closed) in a soft topological space $(X,\tilde{\tau},E)$ if and only if $(F,E)^{-}\setminus(G,E) \in I$ whenever $(F,E) \sqsubseteq (G,E)$ and (G,E) is soft regular open in $(X,\tilde{\tau},E)$.*

Example 3.2. *Let $X = \{x_1,x_2,x_3\}$, $E = \{e_1,e_2\}$ and $\tilde{\tau} = \{\Phi,\tilde{X},(F_1,E),(F_2,E),(F_3,E)\}$ where*
$F_1(e_1) = \{x_1\},\ F_1(e_2) = \{x_1\}$,
$F_2(e_1) = \{x_2\},\ F_2(e_2) = \emptyset$,
$F_3(e_1) = \{x_1,x_2\}, F_3(e_2) = \{x_1\}$.
Then $(X,\tilde{\tau},E)$ is a soft topological space over X. Let $I = \{\Phi,(G_1,E),(G_2,E),(G_3,E),...,(G_7,E)\}$ be a soft ideal on X, where
$G_1(e_1) = \{x_3\},\ G_1(e_2) = \{x_2,x_3\}$,
$G_2(e_1) = \{x_3\},\ G_2(e_2) = \{x_3\}$,
$G_3(e_1) = \emptyset, G_3(e_2) = \{x_3\}$,
$G_4(e_1) = \{x_3\},\ G_4(e_2) = \{x_2\}$,
$G_5(e_1) = \emptyset,\ G_5(e_2) = \{x_2\}$,
$G_6(e_1) = \emptyset, G_6(e_2) = \{x_2,x_3\}$,
$G_7(e_1) = \{x_3\},\ G_7(e_2) = \emptyset$.
Clearly, (F_2,E) is soft I-rg-closed. In fact, $(F_2,E) \sqsubseteq (F_2,E)$ and (F_2,E) is soft regular open. Hence we obtain $(F_2,E)^{-}\setminus(F_2,E) \in I$.

Proposition 3.3. *Every soft rg-closed set is soft I-rg-closed.*

Proof. Let (F,E) be a soft rg-closed set in a soft topological space $(X,\tilde{\tau},E)$. Let $(F,E) \sqsubseteq (G,E)$ and (G,E) is soft regular open in $(X,\tilde{\tau},E)$. Since (F,E) is soft rg-closed, then $(F,E)^{-} \sqsubseteq (G,E)$ and hence $(F,E)^{-}\setminus(G,E) = \Phi \in I$. Therefore (F,E) is soft I-rg-closed. □

The converse of this proposition is not true in general as can be seen from the following example.

Example 3.4. *Let us take the soft topology $\tilde{\tau}$ on X in Example 3.2. Let (H,E) be a soft set over X such that $H(e_1) = \{x_1\},\ H(e_2) = \emptyset$. Clearly, (H,E) is soft I-rg-closed but it is not soft rg-closed.*

Theorem 3.5. *Every soft I-g-closed set is soft I-rg-closed.*

Proof. Let (F,E) be a soft I-g-closed set in a soft topological space $(X,\tilde{\tau},E)$. We show that (F,E) is soft I-rg-closed. Suppose that $(F,E) \sqsubseteq (G,E)$, where (G,E) is soft regular open. If (G,E) is soft regular open, then (G,E) is soft open. Thus $(F,E) \sqsubseteq (G,E)$ and (G,E) is soft open. Since (F,E) is soft I-g-closed, then $(F,E)^{-}\setminus(G,E) \in I$. Therefore (F,E) is a soft I-rg-closed set. □

Example 3.6. *Let $X = \{x_1,x_2,x_3,x_4\}$, $E = \{e_1,e_2\}$ and $\tilde{\tau} = \{\Phi,\tilde{X},(F_1,E),(F_2,E),(F_3,E)\}$ where*
$F_1(e_1) = \{x_1\},\ F_1(e_2) = X$,
$F_2(e_1) = \{x_1,x_2\},\ F_2(e_2) = X$,
$F_3(e_1) = \{x_1,x_2,x_3\}, F_3(e_2) = X$.
Then $(X,\tilde{\tau},E)$ is a soft topological space over X. Let $I = \{\Phi,(G_1,E),(G_2,E),(G_3,E)\}$ be a soft ideal on X, where
$G_1(e_1) = \{x_2\},\ G_1(e_2) = \emptyset$,
$G_2(e_1) = \emptyset,\ G_2(e_2) = \{x_3\}$,
$G_3(e_1) = \{x_2\}, G_3(e_2) = \{x_3\}$.
Let (H,E) be a soft set over X such that $H(e_1) = \{x_1\},\ H(e_2) = \emptyset$. Clearly, (H,E) is soft I-rg-closed in $(X,\tilde{\tau},E)$, but it is not soft I-g-closed.

Theorem 3.7. *Let $(X, \tilde{\tau}, E)$ be a soft topological space, (F, E) and (G, E) soft sets over X. If (F, E) and (G, E) are soft I-rg-closed sets in $(X, \tilde{\tau}, E)$, then $(F, E) \sqcup (G, E)$ is soft I-rg-closed in $(X, \tilde{\tau}, E)$.*

Proof. Let (F, E) and (G, E) be soft I-rg-closed sets in $(X, \tilde{\tau}, E)$. Suppose that $(F, E) \sqcup (G, E) \sqsubseteq (H, E)$ and (H, E) is soft regular open. Then $(F, E) \sqsubseteq (H, E)$ and $(G, E) \sqsubseteq (H, E)$. Since (F, E) and (G, E) are soft I-rg-closed sets, then $(F, E)^- \backslash (H, E) \in I$ and $(G, E)^- \backslash (H, E) \in I$. Therefore $[(F, E) \sqcup (G, E)]^- \backslash (H, E) = [(F, E)^- \backslash (H, E)] \sqcup [(G, E)^- \backslash (H, E)] \in I$. Hence we obtain that $(F, E) \sqcup (G, E)$ is soft I-rg-closed. □

Remark 3.8. *The intersection of two soft I-rg-closed sets is generally not a soft I-rg-closed set.*

Example 3.9. *Let $X = \{x_1, x_2, x_3, x_4\}$, $E = \{e_1, e_2\}$ and $\tilde{\tau} = \{\Phi, \tilde{X}, (F_1, E), (F_2, E), (F_3, E), (F_4, E)\}$ where*
$F_1(e_1) = \{x_1\}$, $F_1(e_2) = \{x_1\}$,
$F_2(e_1) = \{x_2\}$, $F_2(e_2) = \{x_2\}$,
$F_3(e_1) = \{x_1, x_2\}$, $F_3(e_2) = \{x_1, x_2\}$,
$F_4(e_1) = \{x_1, x_2, x_3\}$, $F_4(e_2) = X$.
Then $(X, \tilde{\tau}, E)$ is a soft topological space over X. Let $I = \{\Phi, (G_1, E), (G_2, E), (G_3, E)\}$ be a soft ideal on X, where
$G_1(e_1) = \{x_4\}$, $G_1(e_2) = \emptyset$,
$G_2(e_1) = \emptyset$, $G_2(e_2) = \{x_2\}$,
$G_3(e_1) = \{x_4\}$, $G_3(e_2) = \{x_2\}$.
Let (H, E) and (K, E) be two soft sets over X such that $H(e_1) = \{x_1, x_4\}$, $H(e_2) = \{x_1\}$ and $K(e_1) = \{x_1, x_2, x_3\}$, $K(e_2) = \{x_1, x_3\}$. Clearly, (H, E) and (K, E) are soft I-rg-closed sets in $(X, \tilde{\tau}, E)$ but $(H, E) \sqcap (K, E)$ is not a soft I-rg-closed set.

Theorem 3.10. *Let $(X, \tilde{\tau}, E)$ be a soft topological space, (F, E) and (H, E) two soft sets over X such that $(H, E) \sqsubseteq (F, E)^- \backslash (F, E)$ where (H, E) is soft regular closed. If (F, E) is soft I-rg-closed, then $(H, E) \in I$.*

Proof. Suppose that (F, E) is soft I-rg-closed. Let (H, E) be a soft regular closed subset of $(F, E)^- \backslash (F, E)$. Then $(H, E) \sqsubseteq (F, E)^- \sqcap (F, E)^c$ and so $(F, E) \sqsubseteq (H, E)^c$. Since (F, E) is a soft I-rg-closed set, then $(F, E)^- \backslash (H, E)^c \in I$. Since $(H, E) \sqsubseteq (F, E)^- \backslash (H, E)^c$, by the properties of soft ideal $(H, E) \in I$. □

We note that the converse of this theorem is not true in general as shown in the following example.

Example 3.11. *Let us take the soft topology $\tilde{\tau}$ on X in Example 3.9., we have $(F_2, E)^- \backslash (F_2, E)$ contains soft set (G, E) such that $(G, E) = \Phi$ is only soft regular closed set implies $(G, E) = \Phi \in I$. But (F_2, E) is not a soft I-rg-closed set in $(X, \tilde{\tau}, E)$.*

Theorem 3.12. *Let $(X, \tilde{\tau}, E)$ be a soft topological space, (F, E) and (G, E) soft sets over X. If (F, E) is soft I-rg-closed and $(F, E) \sqsubseteq (G, E) \sqsubseteq (F, E)^-$ such that $(H, E) \sqsubseteq (G, E)^- \backslash (G, E)$ where (H, E) is soft regular closed, then $(H, E) \in I$.*

Proof. Let (F, E) be a soft I-rg-closed set and $(F, E) \sqsubseteq (G, E) \sqsubseteq (F, E)^-$. Suppose that $(H, E) \sqsubseteq (G, E)^- \backslash (G, E)$ and (H, E) is soft regular closed. If $(F, E) \sqsubseteq (G, E)$, then $(G, E)^c \sqsubseteq (F, E)^c$ ...(1). If $(G, E) \sqsubseteq (F, E)^-$, then $(G, E)^- \sqsubseteq ((F, E)^-)^- = (F, E)^-$ ...(2). That is $(G, E)^- \sqsubseteq (F, E)^-$. From (1) and (2), we obtain $[(G, E)^- \sqcap (G, E)^c] \sqsubseteq [(F, E)^- \sqcap (F, E)^c]$ and $[(G, E)^- \backslash (G, E)] \sqsubseteq [(F, E)^- \backslash (F, E)]$. By assumption $(H, E) \sqsubseteq (G, E)^- \backslash (G, E)$ and $[(G, E)^- \backslash (G, E)] \sqsubseteq [(F, E)^- \backslash (F, E)]$, we obtain $(H, E) \sqsubseteq [(G, E)^- \backslash (G, E)] \sqsubseteq [(F, E)^- \backslash (F, E)]$. Since (F, E) is soft I-rg-closed and by Theorem 3.10., then $(H, E) \in I$. □

If $(X, \tilde{\tau}, E)$ is a soft topological space over X and Y is a nonempty subset of X, then we shall denote the soft closure of $(F, E) \sqsubseteq \tilde{Y}$ with respect to relative soft topology on $(Y, \tilde{\tau}_Y, E)$ by $((F, E))^-_Y = (F, E)^- \sqcap \tilde{Y}$.

Theorem 3.13. *Let $(X, \tilde{\tau}, E)$ be a soft topological space, $Y \subset X$ be a nonempty subset of X and (F, E) be a soft set over Y. If $\tilde{Y}$ is soft open in $(X, \tilde{\tau}, E)$ and (F, E) is soft I-rg-closed in $(X, \tilde{\tau}, E)$, then (F, E) is soft I-rg-closed relative to the soft topological subspace $(Y, \tilde{\tau}_Y, E)$ and with respect to the soft ideal $I_Y = \{(H, E) \sqsubseteq \tilde{Y} : (H, E) \in I\}$.*

Proof. Let $(F,E) \sqsubseteq \widetilde{Y} \sqcap (G,E)$ and suppose that (G,E) is soft regular open in $(X,\widetilde{\tau},E)$. So, $\widetilde{Y} \sqcap (G,E)$ is soft regular open in $(Y,\widetilde{\tau}_Y,E)$ and $(F,E) \sqsubseteq (G,E)$. Since (F,E) is soft I-rg-closed in $(X,\widetilde{\tau},E)$, then $(F,E)^-\backslash(G,E) \in I$. Now $((F,E)^-_Y)\backslash(\widetilde{Y} \sqcap (G,E)) = (\widetilde{Y} \sqcap (F,E)^-)\backslash(\widetilde{Y} \sqcap (G,E)) = (\widetilde{Y} \sqcap ((F,E)^-\backslash(G,E))) \in I_Y$ whenever (F,E) is soft I-rg-closed relative to subspace $(Y,\widetilde{\tau}_Y,E)$. □

4. Soft Regular Generalized Open Sets with Respect to a Soft Ideal

Definition 4.1. *A soft set (F,E) is called soft regular generalized open with respect to a soft ideal I (briefly soft I-rg-open) in a soft topological space $(X,\widetilde{\tau},E)$ if and only if $(F,E)^c$ is soft I-rg-closed in $(X,\widetilde{\tau},E)$.*

Theorem 4.2. *A soft set (F,E) is soft I-rg-open in a soft topological space $(X,\widetilde{\tau},E)$ if and only if $(G,E)\backslash(H,E) \sqsubseteq (F,E)^\circ$ for some $(H,E) \in I$, whenever $(G,E) \sqsubseteq (F,E)$ and (G,E) is soft regular closed in $(X,\widetilde{\tau},E)$.*

Proof. Suppose that (F,E) is soft I-rg-open, $(G,E) \sqsubseteq (F,E)$ and (G,E) is soft regular closed. Then $(G,E)^c$ is soft regular open. Then $(F,E)^c \sqsubseteq (G,E)^c$. Since $(F,E)^c$ is soft I-rg-closed, $((F,E)^c)^-\backslash(G,E)^c \in I$. This implies that $((F,E)^c)^-\backslash(G,E)^c = ((F,E)^c)^- \sqcap (G,E) = (H,E) \in I$, so $[((F,E)^c)^- \sqcap (G,E)] \sqcup (G,E)^c = (H,E) \sqcup (G,E)^c$. Hence $((F,E)^c)^- \sqsubseteq (G,E)^c \sqcup (H,E)$ for some $(H,E) \in I$. So $[(G,E)^c \sqcup (H,E)]^c \sqsubseteq (((F,E)^c)^-)^c = (F,E)^\circ$ and therefore $(G,E)\backslash(H,E) = (G,E) \sqcap (H,E)^c \sqsubseteq (F,E)^\circ$.

Conversely, assume that $(G,E) \sqsubseteq (F,E)$ and (G,E) is soft regular closed in $(X,\widetilde{\tau},E)$. These imply that $(G,E)\backslash(H,E) \sqsubseteq (F,E)^\circ$ for some $(H,E) \in I$. Now we show that $(F,E)^c$ is soft I-rg-closed. Let $(F,E)^c \sqsubseteq (K,E)$ and (K,E) is soft regular open. Then $(K,E)^c \sqsubseteq (F,E)$. By assumption $(K,E)^c\backslash(H,E) \sqsubseteq (F,E)^\circ = (((F,E)^c)^-)^c$ for some $(H,E) \in I$. This gives that $[(K,E) \sqcup (H,E)]^c \sqsubseteq (((F,E)^c)^-)^c$. Then $((F,E)^c)^- \sqsubseteq (K,E) \sqcup (H,E)$ for some $(H,E) \in I$. Thus $((F,E)^c)^-\backslash(K,E) \sqsubseteq ((K,E) \sqcup (H,E))\backslash(K,E) = ((K,E) \sqcup (H,E)) \sqcap (K,E)^c = (H,E) \sqcap (K,E)^c \sqsubseteq (H,E) \in I$. This shows that $((F,E)^c)^-\backslash(K,E) \in I$ (from the properties of soft ideal). Hence $(F,E)^c$ is soft I-rg-closed and (F,E) is soft I-rg-open. □

Remark 4.3. *The union of two soft I-rg-open sets is generally not soft I-rg-open. (See Example 3.9.)*

Theorem 4.4. *Let $(X,\widetilde{\tau},E)$ be a soft topological space. If (F,E) and (G,E) are soft disconnected I-rg-open sets in $(X,\widetilde{\tau},E)$, then $(F,E) \sqcup (G,E)$ is soft I-rg-open in $(X,\widetilde{\tau},E)$.*

Proof. Let (H,E) be a soft regular closed subset of $(F,E) \sqcup (G,E)$. Then $(H,E) \sqcap (F,E)^- \sqsubseteq (F,E)$ and hence by Theorem 4.2., $[(H,E) \sqcap (F,E)^-]\backslash(K,E) \sqsubseteq (F,E)^\circ$ for some $(K,E) \in I$. Similarly $(H,E) \sqcap (G,E)^- \sqsubseteq (G,E)$ and $[(H,E) \sqcap (G,E)^-]\backslash(L,E) \sqsubseteq (G,E)^\circ$ for some $(L,E) \in I$. This means that $[(H,E) \sqcap (F,E)^-]\backslash(F,E)^\circ \in I$ and $[(H,E) \sqcap (G,E)^-]\backslash(G,E)^\circ \in I$. Then $[((H,E) \sqcap (F,E)^-)\backslash(F,E)^\circ] \sqcup [((H,E) \sqcap (G,E)^-)\backslash(G,E)^\circ] \in I$. Hence $[(H,E) \sqcap ((F,E)^- \sqcup (G,E)^-)]\backslash[(F,E)^\circ \sqcup (G,E)^\circ] \in I$. But $(H,E) = (H,E) \sqcap ((F,E) \sqcup (G,E)) \sqsubseteq (H,E) \sqcap ((F,E) \sqcup (G,E))^-$, and we have

$$\begin{aligned}(H,E)\backslash[(F,E) \sqcup (G,E)]^\circ &\sqsubseteq [(H,E) \sqcap ((F,E) \sqcup (G,E))^-]\backslash[(F,E) \sqcup (G,E)]^\circ \\ &\sqsubseteq [(H,E) \sqcap ((F,E) \sqcup (G,E))^-]\backslash[(F,E)^\circ \sqcup (G,E)^\circ] \in I.\end{aligned}$$

From the properties of soft ideal, we obtain $(H,E)\backslash[(F,E) \sqcup (G,E)]^\circ \in I$. Now we take $(D,E) = (H,E)\backslash[(F,E) \sqcup (G,E)]^\circ \in I$. Then $(H,E)\backslash(D,E) = (H,E)\backslash[(H,E)\backslash[(F,E) \sqcup (G,E)]^\circ] \sqsubseteq ((F,E) \sqcup (G,E))^\circ$. Hence $(H,E)\backslash(D,E) \sqsubseteq ((F,E) \sqcup (G,E))^\circ$ for some $(D,E) \in I$. Hence we obtain that $(F,E) \sqcup (G,E)$ is soft I-rg-open. □

Corollary 4.5. *If (F,E) and (G,E) are soft I-rg-open sets in a soft topological space $(X,\widetilde{\tau},E)$, then $(F,E) \sqcap (G,E)$ is soft I-rg-open in $(X,\widetilde{\tau},E)$.*

Proof. If (F,E) and (G,E) are soft I-rg-open sets, then $(F,E)^c$ and $(G,E)^c$ are soft I-rg-closed. By Theorem 3.7., $[(F,E) \sqcap (G,E)]^c = (F,E)^c \sqcup (G,E)^c$ is soft I-rg-closed. This shows that $(F,E) \sqcap (G,E)$ is soft I-rg-open. □

If $(X, \tilde{\tau}, E)$ is a soft topological space over X and $Z \subset X$ is a nonempty subset of X, then we shall denote the soft interior of $(F, E) \sqsubseteq \tilde{Z}$ with respect to relative soft topology on $(Z, \tilde{\tau}_Z, E)$ by $((F, E))^\circ_Z = (F, E)^\circ \sqcap \tilde{Z}$.

Theorem 4.6. *Let $(X, \tilde{\tau}, E)$ be a soft topological space and $Z \subset X$ be a nonempty subset of X. If $(F, E) \sqsubseteq \tilde{Z}$ is soft I-rg-open relative to $(Z, \tilde{\tau}_Z, E)$ and $\tilde{Z}$ is soft open relative to $(X, \tilde{\tau}, E)$, then (F, E) is soft I-rg-open relative to $(X, \tilde{\tau}, E)$.*

Proof. Suppose that $(F, E) \sqsubseteq \tilde{Z}$, $(G, E) \sqsubseteq (F, E)$ and (G, E) is a soft regular closed set relative to $(X, \tilde{\tau}, E)$. Then (G, E) is soft regular closed relative to $(Z, \tilde{\tau}_Z, E)$. Since $(F, E) \sqsubseteq \tilde{Z}$ is soft I-rg-open relative to $(Z, \tilde{\tau}_Z, E)$, by Theorem 4.2., $(G, E) \backslash (H, E) \sqsubseteq (F, E)^\circ_Z = (F, E)^\circ \sqcap \tilde{Z}$ for some $(H, E) \in I_Z$. Since the soft topological subspace $(Z, \tilde{\tau}_Z, E)$ and with respect to the soft ideal $I_Z = \{(H, E) \sqsubseteq \tilde{Z} : (H, E) \in I\}$, we obtain $(H, E) \in I$. It follows then that $(G, E) \backslash (H, E) \sqsubseteq (F, E)^\circ$ for some $(H, E) \in I$. Using Theorem 4.2., we get that (F, E) is soft I-rg-open in $(X, \tilde{\tau}, E)$. $\square$

Theorem 4.7. *Let $(X, \tilde{\tau}, E)$ be a soft topological space and (F, E) be a soft set over X. If a soft set (F, E) is soft I-rg-closed, then $(F, E)^- \backslash (F, E)$ is soft I-rg-open.*

Proof. Suppose that (F, E) is soft I-rg-closed and $(H, E) \sqsubseteq (F, E)^- \backslash (F, E)$, where (H, E) is soft regular closed. Then $(H, E) \in I$ and hence $(H, E) \backslash (G, E) = \Phi$ for some $(G, E) \in I$. Clearly $(H, E) \backslash (G, E) \sqsubseteq ((F, E)^- \backslash (F, E))^\circ$. By Theorem 4.2., $(F, E)^- \backslash (F, E)$ is soft I-rg-open. $\square$

The converse of this theorem is not true in general as can be seen from the following example.

Example 4.8. *From Example 3.9., we have $(F_1, E)^- \backslash (F_1, E)$ is a soft I-rg-open set. But (F_1, E) is not soft I-rg-closed in $(X, \tilde{\tau}, E)$.*

5. Conclusions

We have studied some new concepts in soft topological spaces which are defined over an initial universe with a fixed set of parameters such as soft regular generalized closed and open sets with respect to a soft ideal. Also, we have investigated many basic properties of these concepts. In future more general types of soft regular generalized closed sets may be defined and using of them characterizations related with soft separation axioms and soft continuity may be studied. Hence we expect that some research teams will be actively working on soft regular generalized closed (open) sets and different types of subsets of soft topological spaces with respect to a soft ideal.

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