



Pricing Strategy of Closed-Loop Supply Chain Under Disruptions

Xiaofeng Xu^a, Jiaguo Liu^{b,c}, Yongli Li^d, Xuelong Zhou^e

^aCollege of Economics and Management, China University of Petroleum, Qingdao 266580, China;

^bTransportation Management College, Dalian Maritime University, Dalian 116026, China;

^cCollaborative Innovation Center for Transport Studies, Dalian Maritime University, Dalian 116026, China;

^dSchool of Management Administration, Northeastern University, Shenyang, 110819, China;

^eSchool of Economics and Management, Harbin Engineering University, Harbin, 150001, China

Abstract.

The aim of this paper is to understand how disruptions effect on closed-loop supply chain, and how to achieve the coordination of closed-loop supply chain under disruptions situation for the whole supply chain system.

Design/methodology/approach- This paper analyzed the effect caused by the disruptions of the closed-loop supply chain from both supply and demand, and applied the established margin profits sharing contract to achieve coordination under the disruptions.

Findings- From both supply side and demand side, it drew how the disruptions affect the closed-loop supply chain. When the disruptions have less effect on the forward and reverse cost of closed-loop supply chain, the impact of the closed-loop supply chain is controllable; when the disruptions have lager effect on the forward and reverse cost of closed-loop supply chain, the impact of the closed-loop supply chain is uncontrollable.

Research limitations/implications- In this paper, the contract costs are ignored for zero. While in actual situation, to maintain the efficient implementation of this contract must spend considerable cost. Sometimes the cost may be too high to become hinder between manufacturers and retailers. This paper also does not make in-depth study on this issue, so it will be further study in the future.

Originality/value- This paper also designed a set of differential profit sharing contract based on revenue sharing contract mechanism. The contract is easy to be managed, easy to be operated, making the closed-loop supply chain system can achieve coordination whether in steady state or under disruptions.

1. Introduction

In recent years, frequent disruptions, such as the Philips factory fire incident in March 2000, the New York World Trade Center "9.11" terrorist attacks in 2001, "Tonyred" event at the beginning of 2005, Japan earthquake incidents in March 2011, Ya'an earthquake and the Yuyao flood incidents in 2013, not only have caused the loss of property, but also have a big shock on the closed-loop supply chain, which may make the cooperation between the various enterprises of closed-loop supply chain change their ways of

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Email address: liujiaguogmail.com (Jiaguo Liu)

cooperation and the decision-making behavior, break the coordination of closed-loop supply chain under normal conditions. Therefore, the establishment of pricing strategy and coordination mechanism of closed loop the emergency supply chain is imperative.

Emergency management of supply chain disruptions is proposed by Causen et al^[1]. It was first well used ^[2,3] to solve the airline to respond to disruptions. Qi ^[4] studied deterministic demand changes caused by disruptions and proposed similar quantity discount contract to coordinate supply chain under the emergency. Xu^[5] did lots of research on the problem of coordination of supply chain with nonlinear demand function. Gurfer and Parla^[6] studied the emergency coordination of two suppliers in the two level supply chain, and discussed the strategy of emergency coordination for the supplier in two cases. Abboud^[7] considered the coordination of supply chain under the condition of the failure of the machine in the production and inventory system. Axsater^[8] analyzed the emergency replenishment problem in the critical period of single stage, and found a heuristic decision rules, using to trigger emergency order, to reduce the expected cost. S.Sodhi and S.Tang^[9] extended the deterministic linear programming model, using stochastic programming, to study the problem of supply chain planning with uncertain demand over inventory. Erhan Kutanoglu^[10] established joint inventory model based on Markov chain, aiming at the failure of supply chain risk, and analyzed the influence of transverse joint on the inventory system service level and inventory cost. Chen and Xiao^[11] proposed two mechanisms to coordinate the supply chain of one manufacturer and many retailers. The classic quantity discount contract to coordinate the supply chain in the event of disruptions was studied by domestic scholars such as Yu ^[12]. Hu ^[13] considered the maximum size of the demand in the event of an emergency interference, and used the price discount contract achieving the coordination of three level supply chain. Cao ^[14] considered the demand uncertainty, through improving the contract parameters in the original contract, then established the revenue sharing contract with the resistance to the unexpected event. Gao^[15] interested in how to use the revenue sharing contract to coordinate the supply chain when factor distribution function changes. Xu et al^[16] have established a robust optimization model for the dynamic operation of the closed-loop supply chain under uncertain environment. And Wang Yuyan^[17, 18] respectively adjusted income sharing contract and buy back contract.

On the basis of previous research, this paper considers that the closed-loop supply chain under disruptions, and disruptions will bring changes to the product supply and demand in the market. From the two aspects, it analyzed influence brought by the emergency to the closed-loop supply chain, and used the established different profits sharing contract to achieve the closed-loop supply chain coordination under disruptions.

2. Methodology

The impact of disruptions on the closed-loop supply chain is divided into three categories: supply, internal enterprise and demand. The following will be based on the basic closed-loop supply chain model proposed by scholars Liu ^[19] and analyzed the three above effects respectively, to study how each parameter in the system changes when disruptions (such incidents may be disruptions in the external environment or disruptions occur within the supply chain) happen based on the steady state in the closed-loop supply chain. So we can analyze how disruptions affect closed loop supply chain and how it changes and response to the disruptions, making the closed loop supply chain operate better and achieve the coordination under the disruptions, to reduce the loss of profits (or to increase corporate profits).

The effect of disruptions on the closed-loop supply chain internal enterprise shows as damage to the internal equipment, personnel and systems. Whether it is production equipment damage or core business personnel casualties, the essence is that it affects the production cost of new or remanufacturing products in the closed-loop supply chain system. The reason why it happens is just because disruptions affect internal supply chain, which is different from product supply. Still, its external performance is familiar with disruptions effect on product supply, so in order to simplify the analysis, this aspect is combined to the product supply. As consequence, the paper only analyzes the influence of disruptions from product supply and the demand influence two aspects in detail in the following the parameter change analysis part.

A simple closed-loop supply chain system is considered as a composition of a manufacturer and a retailer. The operation mode of the system in the market is: manufacturers produce new products with

materials and distribute new products through the retailer. While the retailer entrusts the manufacturer with the responsibility of recycling waste products on the market, and then recycles the waste products at a certain price from the retailer. Retailers recycle the waste products at a certain price from consumers, as to form the reverse recovery process. Thus, manufacturers can be produced using of new materials and waste products that is recycled after waste products recycling. The products that produced by raw materials are defined as new products. The product that is produced by waste product after the reproduction and the remanufacturing process is called remanufacturing product. Manufacturers sell two kinds of product to the junior retailers with two different prices, and the retailer is responsible for putting the two products on the market. Obviously, above closed-loop supply chain includes two processes of supplying new products, remanufacturing product and reversing recycling of waste products, as shown in figure 1.

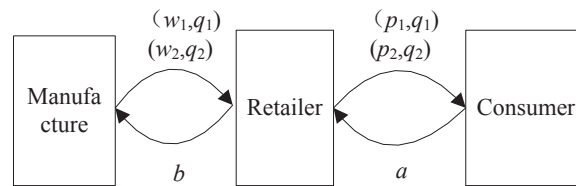


Fig. 1 Closed-loop supply chain system for different product quality

According to the above description of the closed-loop supply chain problems, we can make the following assumptions and notations:

Suppose that the consumer preference for a product is: when the consumer product quality is q and the product price is p , the utility of the consumer $U = \theta q - p$, otherwise the utility of the consumer U is 0. Note that q represents for product quality characteristics number, $\theta (\theta > 0)$ means the utility consumers get when quality $q = 1$, and the number of each product quality characteristics were less than a mass limit q_0 , namely the quality of each product are not unlimited high and its quality characteristics number could not less than 0. And θ is known as the consumer's quality preference coefficient, while p is the price of the product and $\theta q - p$ stands for the consumer surplus. Set θ is a random variable, the greater value, the greater the utility of consumer. With $F(\theta)$ representing distribution function, $f(\theta)$ is the density function.

Suppose that the cost and quality of product manufacturer produced through two material are different: the quality characteristics number of product made from the new material is q_1 , with production cost c_1 ; the quality characteristics number of product made from the recycling material is q_2 , with production cost c_2 ; the manufacturer spend no additional expense on the two kinds of products when delivered to the retailers; besides, $q_1 > q_2, c_1 > c_2$.

Manufacturers sell two kinds of products with different quality to retailers at price of w_1 and w_2 , then retailers will sell at price of p_1 and p_2 in the market, which takes that in the actual situation retailers will ensure they are profitable and generally there is a positive relationship between every kind of product quality and price in the market into account. We can draw the following assumptions: $w_1 > w_2, p_1 > p_2, p_1 > w_1, p_2 > w_2$.

Suppose that retailers do not need to spend additional expense when selling two different quality products on the market. The cost of products in the sales process of transportation, advertising and others are 0.

If new products and remanufactured products have no different in the recovery, i.e., whether it is by new products or remanufactured products, the value is the same when they enter the reproduction program after the use of consumer, so the recycling price is also the same as a , manufacturers recycle the waste products on price of b , and manufacturers can turn all the waste products into remanufactured products, i.e., there is no loss. In order to ensure the enterprise making profits by recycling products, the inequality $a < b < c_1 - c_2$ and $c_2 + b < w_2$ should be established.

Use π_m, π_r, π stand for the profits of manufacturers, retailers and the whole system in the closed-loop supply chain system; use D_1, D_2 represent two different kinds quality of products in the market, and D_1 represents for the demand for new products, D_2 for remanufacturing products.

3. The impact of disruptions on the supply side

The impact of disruptions on the supply of closed-loop supply chain is the raw materials or parts of the products in the process of supplying to the corresponding manufactures, products are shipped to retailers from manufactures and delivered to the consumer from the retailers and the retailers return the waste products to the manufacturers. This kind of influence will usually cause great negative impact on the closed-loop supply chain, and even the collapse of the whole system.

Concrete analysis, the impact of disruptions on product supply can be divided into two categories: the first one is the increase in the cost of new products caused by disruptions increase in the price of raw materials, the second one is the increase of the cost of recycling waste products or the increase of the production cost of remanufacturing products caused by the increase of the value of the waste products. Of course, it is also possible that the cost of both new products and remanufacturing products changes. The reason of the increase of production cost may be the shortage of raw material and waste products or the increase of transportation cost caused by road transportation. Such as the 2008 snowstorm caused the transport capacity of the south declined, led the cost of goods transportation in the system increasing. Of course, another reason that increases the cost of two kinds of products is the internal of node enterprise suffered heavy losses. No matter how it influences, the reaction in the model is the impact of production cost (c_1, c_2) or the price a of waste products recycling.

First, when disruptions only affect the production cost of the new product, we can assume that the cost of production has changed from c_1 to $c_1 + \Delta c_1$ ($\Delta c_1 > 0$). There is two possibilities:

1) When $c_1 + \Delta c_1$ is still less than the wholesale price w_1 of the manufacturer, the impact of the unexpected event on the closed-loop supply chain is controllable. Under the centralized decision-making model, we can simply launch the price of the two kinds of product in closed-loop supply chain will become

$$p_1^* = \frac{q_1 + c_1 + \Delta c_1}{2}, p_2^* = \frac{q_2 + c_2 + a}{2} \tag{1}$$

That means the new product pricing will increase by , and the pricing of the remanufacturing product has not changed. The overall profit of the closed-loop supply chain will be changed to

$$\pi^*_0 = \frac{(q_1q_2 - c_1q_2 - \Delta c_1q_2)(q_1 - q_2 - c_1 - \Delta c_1 + c_2 + a)}{4q_2(q_1 - q_2)} + \frac{(q_2 - a - c_2)(c_1q_2 + \Delta c_1q_2 - aq_1 - c_2q_1)}{4q_2(q_1 - q_2)} \tag{2}$$

Overall profit margins will be reduced, and the changed value of its profit is

$$\Delta\pi^* = \frac{\Delta c_1^2 + 2(c_1 + q_2 - c_2 - a - q_1)\Delta c_1}{4(q_1 - q_2)} \tag{3}$$

It can be inference that the new product pricing and wholesale prices are increased respectively by $0.25\Delta c_1, 0.5\Delta c_1$. Then the profits of the retailer, manufacturer, the overall system under the decentralized decision-making model of closed-loop supply chain in of profit decrease by $0.25\Delta\pi^*, 0.5\Delta\pi^*, 0.75\Delta\pi^*$. At this time, the profit gap of the difference between two kinds of decision-making mode is $0.25(\pi^* - \Delta\pi^*)$. According to the profit sharing contract, the profits of the manufacturer and the retailer can be obtained:

$$\begin{cases} \pi_m = 0.5(\pi^* - \Delta\pi^*) + 0.25\lambda(\pi^* - \Delta\pi^*) \\ \pi_r = 0.25(\pi^* - \Delta\pi^*) + 0.25(1 - \lambda)(\pi^* - \Delta\pi^*) \end{cases} \tag{4}$$

2) When $c_1 + \Delta c_1$ is greater than the wholesale price w_1 of the manufacturer, the impact of the disruptions on the closed-loop supply chain is not controllable. In this case, manufacturers use new materials to produce new products that will cost more than the wholesale price for the retailers. So the previous closed-loop supply chain becomes a one-way reverse supply chain, such a demand D for products is: $D = D_1 + D_2 = 1 - \frac{p_2}{q_2}$

At this point, the seller’s profit function is: $\pi_r = (p_2 - w_2)D + (b - a)D$

Manufacturer’s profit function is: $\pi_m = (w_2 - c_2 - b)D$

The profit of the whole closed loop supply chain is: $\pi = (p_2 - c_2 - a)D$

According to the above situation, this part can also be divided into two kinds: the centralized decision-making model and the decentralized decision-making model. The methods are the same, so we give the result directly.

The pricing strategy and the overall profit of the system under the centralized decision mode are respectively as:

$$p_2^* = \frac{q_2 + c_2 + a}{2} \quad \pi_1^* = \frac{(q_2 - a - c_2)^2}{4q_2} \tag{5}$$

The pricing strategy based on the decentralized decision model is:

$$w_2 = \frac{q_2 + c_2 + 2b - a}{2}, p_2 = \frac{3q_2 + c_2 + a}{4} \tag{6}$$

The total profit of the retailers, manufacturers and the system are:

$$\pi_r = \frac{(q_2 - a - c_2)^2}{16q_2} = 0.25\pi_1^* \tag{7}$$

$$\pi_m = \frac{(q_2 - a - c_2)^2}{8q_2} = 0.5\pi_1^* \tag{8}$$

$$\pi = \pi_m + \pi_r = \frac{3(q_2 - a - c_2)^2}{16q_2} = 0.75\pi_1^* \tag{9}$$

Thus the profit of decentralized decision model of closed-loop supply chain still equals 75% of centralized decision mode, and the loss of profit rate is 25%. Using the differential profit sharing contract can also enable the system to achieve the effect of the centralized decision-making model, the contract of each corporate profit:

$$\begin{cases} \pi_m = 0.5\pi_1^* + 0.25\lambda\pi_1^* \\ \pi_r = 0.25\pi_1^* + 0.25(1 - \lambda)\pi_1^* \end{cases} \tag{10}$$

Second, when disruptions only affect the production cost of the re manufacturing product, we can assume that the cost of production has changed from c_2 to $c_2 + \Delta c_2$ ($\Delta c_2 > 0$). According to the previous hypothesis $a < b < c_1 - c_2$ and $c_2 + b < w_2$, there are two possibilities:

1) When $c_2 + \Delta c_2$ is still less than $c_1 - b$ and $w_2 - b$, the impact of the disruptions on the closed-loop supply chain is controllable. We can simply launch the price of the two kinds of product under centralized decision-making model in closed-loop supply chain will become

$$p_1^* = \frac{q_1 + c_1}{2}, p_2^* = \frac{q_2 + c_2 + \Delta c_2 + a}{2} \tag{11}$$

That means the remanufacturing product pricing will increase by , and the pricing of the new product has not changed. The overall profit of the closed-loop supply chain will be changed to

$$\pi_2^* = \frac{(q_1q_2 - c_1q_2)(q_1 - q_2 - c_1 + c_2 + \Delta c_2 + a)}{4q_2(q_1 - q_2)} + \frac{(q_2 - a - c_2 - \Delta c_2)(c_1q_2 - aq_1 - c_2q_1 - \Delta c_2q_1)}{4q_2(q_1 - q_2)} \tag{12}$$

Overall profit margins will be reduced, and the changed value of its profit is

$$\Delta\pi^* = \frac{\Delta c_2(2c_1q_2 - \Delta c_2q_1 - 2q_1a - 2c_2q_1)}{4q_2(q_1 - q_2)} \tag{13}$$

At the same time, it can be concluded that the price and the wholesale price of two kinds of products of the closed-loop supply chain in decentralized decision mode are:

$$p_1 = \frac{3q_1 + c_1}{4}, p_2 = \frac{3q_2 + c_2 + \Delta c_2 + a}{4} \quad w_1 = \frac{q_1 + c_1}{2}, w_2 = \frac{q_2 + c_2 + \Delta c_2 + 2b - a}{2} \tag{14}$$

According to the profit sharing contract, the profits of the manufacturer and the retailer can be obtained:

$$\begin{cases} \pi_m = 0.5(\pi^* - \Delta\pi^*) + 0.25\lambda(\pi^* - \Delta\pi^*) \\ \pi_r = 0.25(\pi^* - \Delta\pi^*) + 0.25(1 - \lambda)(\pi^* - \Delta\pi^*) \end{cases} \tag{15}$$

2) When $c_2 + \Delta c_2$ is greater than $c_1 - b$ or $w_2 - b$, the impact of the disruptions on the closed-loop supply chain is not controllable.

In this case, manufacturers use waste products to produce remanufacturing products that will cost more than the wholesale price for the retailers. They will end the wholesale production of remanufacturing products, only for the wholesale production of new products, so that there are only new products on the market. Those who are in the market to make product preferences can only choose a new product to meet their own utility. So, a closed-loop supply chain becomes a traditional one-way forward supply chain. At this time by the consumer's utility function $U = \theta q - p$ can draw when the utility is greater than 0 consumers will buy the products, demand D for products becomes:

$$D = P \left\{ \theta \geq \frac{p_1}{q_1} \right\} = 1 - \frac{p_1}{q_1} \tag{16}$$

At this point, the seller's profit function is: $\pi_r = (p_1 - w_1)D$

Manufacturer's profit function is: $\pi_m = (w_1 - c_1)D$

The profit of the whole closed loop supply chain is: $\pi = (p_1 - c_1)D$

According to the above situation, this part can also be divided into two kinds: the centralized decision-making model and the decentralized decision-making model. The methods are the same, so we give the result directly.

The pricing strategy and the overall profit of the system under the centralized decision mode are respectively as:

$$p_1^* = \frac{q_1 + c_1}{2} \quad \pi_3^* = \frac{(q_1 - c_1)^2}{4q_1} \tag{17}$$

The pricing strategy based on the decentralized decision model is:

$$w_1 = \frac{q_1 + c_1}{2}, p_1 = \frac{3q_1 + c_1}{4} \tag{18}$$

The total profit of the retailers, manufacturers and the system are:

$$\pi_r = \frac{(q_1 - c_1)^2}{16q_1} = 0.25\pi_3^* \tag{19}$$

$$\pi_m = \frac{(q_1 - c_1)^2}{8q_1} = 0.5\pi_3^* \tag{20}$$

$$\pi = \pi_m + \pi_r = \frac{3(q_1 - c_1)^2}{16q_1} = 0.75\pi_3^* \quad (21)$$

Thus the profit of decentralized decision model of closed-loop supply chain still equals 75% of centralized decision mode, and the loss of profit rate is 25%. Using the differential profit sharing contract can also enable the system to achieve the effect of the centralized decision-making model, the contract of each corporate profit:

$$\begin{cases} \pi_m = 0.5\pi_3^* + 0.25\lambda\pi_3^* \\ \pi_r = 0.25\pi_3^* + 0.25(1 - \lambda)\pi_3^* \end{cases} \quad (22)$$

Third, when disruptions have effect on the production cost of new products and remanufacturing products impact at the same time, we can on the assumption that the new product production cost from c_1 to $c_1 + \Delta c_1$ ($\Delta c_1 > 0$); The remanufacturing production costs from c_2 to $c_2 + \Delta c_2$ ($\Delta c_2 > 0$). When $c_1 + \Delta c_1$ is less than the manufacturer's wholesale price w_1 , $c_2 + \Delta c_2$ is still greater than $c_1 - b$ or $w_2 - b$, as well as $c_1 + \Delta c_1$ is greater than the manufacturer's wholesale price w_1 , $c_2 + \Delta c_2$ is less than $c_1 - b$ and $w_2 - b$, the change of closed-loop supply chain will be similar to the described situation above. For problems of one-way forward cost change of supply chain and reverse supply chain is relatively simple, no more repeat there. So we only consider the following two cases.

1) When $c_1 + \Delta c_1$ is still less than the wholesale price w_1 of the manufacturer, and $c_2 + \Delta c_2$ is still less than $c_1 - b$ and $w_2 - b$, the impact of the disruptions on the closed-loop supply chain is controllable. Under the centralized decision-making model, we can simply launch the price of the two kinds of product under centralized decision-making model in closed-loop supply chain. The impact of disruptions on the supply cost of the closed-loop supply chain can express in Table 1.

Fourth, when disruptions have effect on the price a of scrap products in the closed-loop supply chain, we can assume that the price of waste products change from a to $a + \Delta a$ ($\Delta a > 0$). According to the hypothesis, we know that the recycling of waste products prices will only affect the reverse recovery of the closed-loop supply chain, which is similar to the situation second incident only for remanufacturing products influence, namely when waste materials price is more than the price p_2 of remanufactured products due to disruptions or more than the price of the remanufacturing cost difference $c_1 - c_2$. manufacturers can withstand, the manufacturer and retailer will terminate the recycling of waste products, and there is only new products in the market. This situation is totally same with situation Second and is also a kind of uncontrollable influence. While the influence of disruptions on the recycling price is controlled, except the rise of production cost change to the rise of recycling price, the rest is similar to the First situation.

Synthesizing the impact of four kinds of unexpected things above on the production cost of the closed-loop supply chain forward new products process and reverse remanufacturing products process. We can know that when the disruptions have less effect on the forward and reverse cost of closed-loop supply chain, the impact of the closed-loop supply chain is controllable; when the disruptions have lager effect on the forward and reverse cost of closed-loop supply chain, the impact of the closed-loop supply chain is uncontrollable.

Table 1. The impact of disruptions on cost

Influence way	Pricing of new and remanufacturing products	Changes of System profit	Influence type	Whether using share the contract or not
$c_1 + \Delta c_1 < w_1$	Increase in new products price; no change of remanufacturing products	Reduce	Controllable	Yes
$c_1 + \Delta c_1 > w_1$	New products out of the market; no change of remanufacturing products	Reduce	Uncontrollable	Yes
$c_2 + \Delta c_2 < c_1 - b$ and $c_2 + \Delta c_2 < w_2 - b$	No change in the new products price; increase in remanufacturing products	Reduce	Controllable	Yes
$c_2 + \Delta c_2 > c_1 - b$ or $c_2 + \Delta c_2 > w_2 - b$	No change in the new product price; remanufacturing products out of the market	Reduce	Uncontrollable	Yes
$c_1 + \Delta c_1 < w_1$ and $c_2 + \Delta c_2 < c_1 - b$ and $c_2 + \Delta c_2 < w_2 - b$	Increase in new products price; increase in remanufacturing products	Reduce	Controllable	Yes
$c_1 + \Delta c_1 > w_1$ and $c_2 + \Delta c_2 < c_1 - b$ or $c_2 + \Delta c_2 < w_2 - b$	New products out of the market; remanufacturing products out of the market	0	corrupted	-

4. The impact of disruptions on the demand side

The impact of disruptions on the demand of the closed-loop supply chain is the impact on demand for new products and remanufacturing products on the closed-loop supply chain. Specifically, the unexpected event may affect the demand for new products or the demand for the remanufacturing products. This change may be a single new product demand changes, it may also be a single remanufacturing product demand changes, and there may also be a change in the demand for the two kinds of products (specific to the model established in this paper, that impact is the two kinds of product demand change). This kind of impact of disruptions always has the potential to be a negative effect. sales plummeted, and it may be a positive effect, such as Beijing’s haze makes the demand for masks unexpectedly increased, stimulating consumers consumption. So, in terms of demand, the impact of the unexpected event on the closed-loop supply chain is likely to be an opportunity or a disaster.

Combined with the previous hypothesis and the explanation, when the product quality is q and the product price is p , the consumer’s utility is $U = \theta q - p$, otherwise the consumer’s utility is 0. And q represents for the number of product quality characteristics, and $\theta (\theta > 0)$ represents for the utility consumer got when $q = 1$, which is called the consumer’s quality preference coefficient. Suppose θ is a random variable. The bigger value, the more utility consumers received from the unit mass consumption. With $F(\theta)$ standing for distribution function, $f(\theta)$ is the density function.

According to the analysis of the mathematical model mentioned above, we can get the demand function of the two kinds of products which are obtained by the game between the new product and the

remanufacturing product:

$$\begin{cases} D_1(p_1, p_2) = 1 - F(\frac{\partial p}{\partial q}) \\ D_2(p_1, p_2) = F(\frac{\partial p}{\partial q}) - F(\frac{p_2}{q_2}) \end{cases} \quad (23)$$

From the demand function of the new product and remanufacturing product D_1, D_2 , we can see that the demand function of the two kind of products is related to the distribution function θ of the consumer's quality preference coefficient $F(\theta)$. Therefore, the demand change analysis of the two kinds of products in this model can be transformed into the change of consumer's quality preference coefficient θ . θ changes can be understood as the occurrence of disruptions that makes the consumer to change the utility value that obtains from the products unit quality. From the previous hypothesis, the consumer's quality preference coefficient is a random variable. In the event of disruptions, the change of θ expression can be represented by the change of $F(\theta)$, the expression of the specific function of the A is likely to become big and the small, so we can make the following assumptions: After the occurrence of disruptions, the distribution function of the consumer's quality preference coefficient θ will be changed from stationary state $F(\theta)$ to $G(\theta)$, as the same, $g(\theta)$ can be used to express the density function of θ . The distribution function $G(\theta)$ of the quality preference coefficient θ under the influence of disruptions will produce the same effect as the $F(\theta)$ in the stationary state. It constitutes a new demand function for new products and remanufacturing products under the influence of disruptions:

$$\begin{cases} D_1(p_1, p_2) = 1 - G(\frac{\partial p}{\partial q}) \\ D_2(p_1, p_2) = G(\frac{\partial p}{\partial q}) - G(\frac{p_2}{q_2}) \end{cases} \quad (24)$$

In order to make a comparison with the basic model which was made in the stationary state in the previous section of the paper. Still assume that θ is a random variable, and its distribution function $G(\theta)$ is uniform distribution. In order to make the calculation simple, the uniform distribution of $G(\theta)$ is assumed to satisfy the (1,2), we can get the demand function under the disruptions:

$$\begin{cases} D_1(p_1, p_2) = 2 - \frac{p_1 - p_2}{q_1 - q_2} \\ D_2(p_1, p_2) = \frac{p_1 - p_2}{q_1 - q_2} - \frac{p_2}{q_2} \end{cases} \quad (25)$$

In centralized decision mode, we can get:

$$p_1^* = \frac{2q_1 + c_1}{2}, p_2^* = \frac{2q_2 + c_2 + a}{2} \quad (26)$$

The overall profit of the whole closed loop supply chain under the centralized decision mode is:

$$\pi^{**} = \frac{(2q_1q_2 - c_1q_2)(2q_1 - 2q_2 + c_2 - c_1 + a)}{4q_2(q_1 - q_2)} + \frac{(2q_2 - a - c_2)(c_1q_2 - aq_1 - c_2q_1)}{4q_2(q_1 - q_2)} \quad (27)$$

Thus, the optimal pricing strategy of new product and remanufacturing product in centralized decision mode is increased by $0.5q_1, 0.5q_2$. This makes the total profit of the system changed $0.25(3q_1 - 2c_1)$, the overall profit of the system increases in this case.

On the other hand, based on the optimal pricing we can get the demand function of the market under the condition of emergency:

$$\begin{cases} D_1(p_1, p_2) = 2 - \frac{p_1^* - p_2^*}{q_1 - q_2} = \frac{2q_1 - 2q_2 + c_2 - c_1 + a}{2(q_1 - q_2)} \\ D_2(p_1, p_2) = \frac{p_1^* - p_2^*}{q_1 - q_2} - \frac{p_2^*}{q_2} = \frac{c_1q_2 - c_2q_1 - aq_1}{2q_2(q_1 - q_2)} \end{cases} \quad (28)$$

So we can see the situation variety of demand that new products and remanufactured products compares with the demand in the stationary state is: new product demand added 0.5, the demand for the

remanufacturing product has not changed. We can deduce the total demand for the product added 0.5, because the occurrence of disruptions.

When the closed-loop supply chain system is decentralized decision mode, it can be obtained: Manufacturer’s wholesale price strategy£

$$(w_1, w_2): w_1 = \frac{2q_1+c_1}{2}, w_2 = \frac{2q_2+c_2+2b-a}{2} \tag{29}$$

Considering the wholesale price (w_1, w_2) , the final sell price (p_1, p_2) is:

$$p_1 = \frac{6q_1 + c_1}{4}, p_2 = \frac{6q_2 + c_2 + a}{4} \tag{30}$$

Put these equation into the corresponding retailers, manufacturers and system profit function, we can obtain:

Retailers profit:

$$\pi_r = \frac{(2q_1q_2 - c_1q_2)(2q_1 - 2q_2 - c_1 + c_2 + a)}{16q_2(q_1 - q_2)} + \frac{+(2q_2 - c_2 - a)(c_1q_2 - c_2q_1 - aq_1)}{16q_2(q_1 - q_2)} \tag{31}$$

Manufacturers profit:

$$\pi_m = \frac{(2q_1q_2 - c_1q_2)(q_1 - q_2 - c_1 + c_2 + a)}{8q_2(q_1 - q_2)} + \frac{+(2q_2 - c_2 - a)(c_1q_2 - c_2q_1 - aq_1)}{8q_2(q_1 - q_2)} \tag{32}$$

The overall profit of the closed-loop supply chain is:

$$\pi = \pi_r + \pi_m = 3 \left[\frac{(2q_1q_2 - c_1q_2)(q_1 - q_2 - c_1 + c_2 + a)}{16q_2(q_1 - q_2)} + \frac{+(2q_2 - c_2 - a)(c_1q_2 - c_2q_1 - aq_1)}{16q_2(q_1 - q_2)} \right] \tag{33}$$

It can be noticed when the disruptions only affect consumers’ preference coefficient θ , and it was from the original $F(\theta)$ to $G(\theta)$. Profits of retailers and manufacturers in the decentralized decision model have increased by $0.0625(3q_1 - 2c_1), 0.125(3q_1 - 2c_1)$. At the same time, the total profit of the system in the decentralized decision model is still 75% of the centralized decision mode, which means that the system will lose 25% of the total profit. So we can continue to use the difference between the profit sharing contracts to coordinate the closed-loop supply chain. We can get the profits of the enterprises in the closed-loop supply chain after the contract coordination with using the same processing method.

$$\begin{cases} \pi_m = 0.5\pi^{**} + 0.25\lambda\pi^{**} \\ \pi_r = 0.25\pi^{**} + 0.25(1 - \lambda)\pi^{**} \end{cases} \tag{34}$$

Retailers and manufacturers in the closed-loop supply chain have increased their profits. The overall profit of the system has reached the state of the centralized decision mode.

From the above analysis we can see that in the incident that makes consumer’s quality preference coefficient change and then influence to demand, this effect of the closed-loop supply chain is controlled. It can also coordinate the closed-loop supply chain with using the profit sharing contract and make each enterprise on the node to achieve the maximum profit, reduce the unnecessary loss of profit, and improve the efficiency of closed-loop supply chain.

5. Case studies

5.1. Case background

There is a manufacturer M located in Beijing for the production of mouse in the market. M has another retailer R located in Jinan. In Jinan, there are two kinds of mouse products, which are produced by M enterprise, and R enterprise is responsible for selling, that is, the new mouse and the remanufacturing

mouse. Consumers in the market buy the mouse according to their own preferences. Under the operation of the normal steady state in the market, M enterprises and R enterprises according to their own interests of the game to make itself to obtain the maximum profit, but in the steady state, a total of following two incidents occurred in the closed-loop supply chain:

1) Due to the market prices of raw materials which is required by the manufacturer M enterprises manufacturing new mouse rises sharply because of the impact of the disruptions, leads the cost of making a new mouse in the production of M is unexpectedly increased.

2) The performance of the notebook computer has developed rapidly because of the unexpected development of the computer technology, this makes the consumer pursuit the high quality of the relevant supporting products, such as the mouse, keyboard and other quality requirements will be increased. Under the impact of the unexpected event, more and more consumers demand for high quality of the mouse, the consumer’s quality preference coefficient rose.

5.2. Case analysis and conclusions

According to the above case background, we can get the relevant parameter values through the access to the enterprise and the market survey, specific values are as follows: the number of the new mouse and remanufacturing mouse, the production costs of the new mouse and remanufacturing mouse is $q_1 = 100, q_2 = 60, c_1 = 20, c_2 = 5$. On the market, the consumer’s quality preference coefficient θ obeys uniform distribution $(0, 1)$, we can use the model established by the previous chapter to determine the value of a by different recycling prices. From the hypothesis and the first conclusion of part 3, we can know the relationship between the recovery price a to meet the $2aq_1 + 2c_2q_1 - 2c_1q_2 \leq 0$, and then we can get the value range of the recovery price a is $0 < a \leq 7$, according to the model, the optimal solution is shown in Table 2.

Table 2. Pricing, demand and profit of two kinds of decision making under different numerical a

a	Centralized decision					Decentralized decision					Margin profit
	p_1	p_2	D_1	D_2	π	p_1	p_2	D_1	D_2	π	$\Delta\pi$
1	60	33	0.325	0.125	16.375	80	46.5	0.1625	0.0625	12.2812	4.0938
3	60	34	0.350	0.083	16.167	80	47	0.175	0.042	12.125	4.042
5	60	35	0.375	0.042	16.042	80	47.5	0.188	0.021	12.0315	4.0105

From the numerical that shows in table 2 we can verify the following conclusion: that is the new mouse market pricing p_1 has no relationship with the recovery price a , the demand of new mouse D_1 increases with the increase of the recycling price a ; the remanufacturing mouse market pricing p_2 increases with the increase of the recovery price a , the demand of remanufacturing mouse D_2 descends with the increase of the recovery price; system profit under two situations of centralized and decentralized decision descends with the increase of the recovery price a , margin profit $\Delta \pi$ descends with the increase of the recovery price a , this is consistent with the actual situation of the closed-loop supply chain of the mouse production and sales.

In addition, when R enterprise and M enterprise that in the closed-loop supply chain in a certain proportion to share the margin profits which loses due to the dispersion decision, two enterprises will realized when the distribution λ changes in the proportion of manufacturers M and retailers R at the margin of profit sharing contract under the self profit will also change accordingly. Thus, according to the enterprises own influence and pay effort and other factors to determine the value λ . Thus the profit function of the manufacturer M and the retailer R is a function of the recovery price a and the difference between the profit distribution ratios λ , it is:

$$\begin{cases} \pi_m = \frac{(0.5+0.25\lambda)(a^2-14a+1585)}{96} \\ \pi_r = \frac{(0.5-0.25\lambda)(a^2-14a+1585)}{96} \end{cases} \tag{35}$$

If the recycling price a determined assuming $a = 3$, manufacturer M and retailer's R profit function can be expressed as function of the profit distribution coefficient λ , then we can get the situation that system and two enterprise's profit changes, as shown in Figure 2.

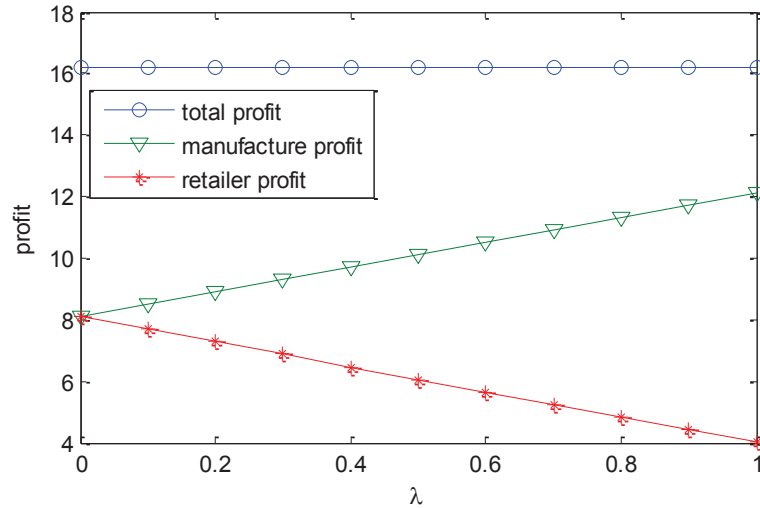


Fig. 2 Manufacturer and retailer profit changing trend

In this case, we can deduce every node of the closed-loop supply chain enterprise's overall profits is decreased with the rise of recycling of used products in the market price, so in the actual operation, in order to achieve the maximization profit, the retailers reduces the recovery price, this will make the total profit of manufacturers and system has also been a corresponding increase.

When the second kinds of disruptions occurred (the new mouse production costs rising), analysis of the model from the 2.4 chapter, when $c_1 + \Delta c_1 < w_1$, that is $0 < \Delta c_1 < 40$, it is controlled impact of closed-loop supply chain, the overall profit of the closed-loop supply chain is (assume $a = 5$):

$$\pi^*_0 = \frac{3\Delta c_1^2 - 180\Delta c_1 + 7700}{480} \tag{36}$$

Compared with the stationary state, the change value is:

$$\Delta\pi^* = \frac{\Delta c_1^2 + 2(c_1 + q_2 - c_2 - a - q_1)\Delta c_1}{4(q_1 - q_2)} = \frac{\Delta c_1^2 - 60\Delta c_1}{160} \tag{37}$$

It can be seen that because of $0 < \Delta c_1 < 40$, so $\Delta\pi^* < 0$, the overall profit of the system will be reduced. It is uncontrolled impact of closed-loop supply chain when $\Delta c_1 > 40$, the overall profit of the closed-loop supply chain is changed to:

$$\pi^* = \frac{(q_2 - a - c_2)^2}{4q_2} = \frac{125}{12} \tag{38}$$

From the above analysis, the total profit of the system can be influenced by the disruptions, as shown in Figure 3:

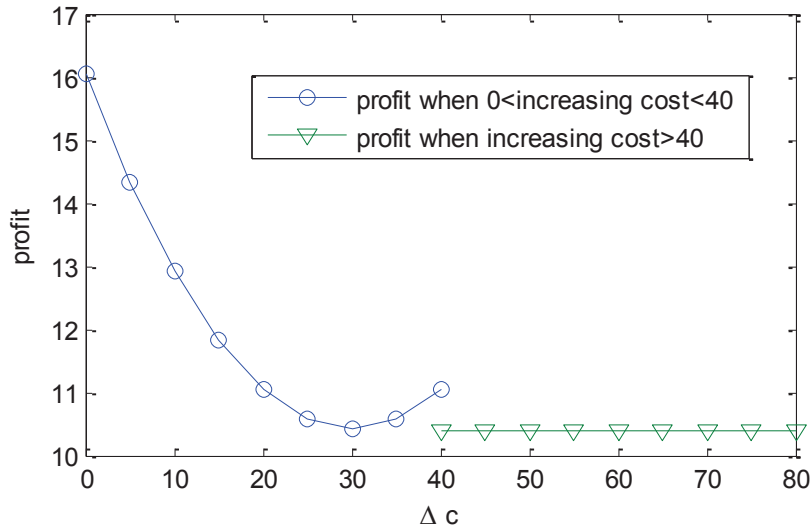


Fig. 3 The changing trend figure of the total profit of the system increases with the increasing of the production costs of new mouse

When the second kinds of disruptions occurred (i.e., the improvement of the performance of the notebook makes the consumer’s quality preference for the mouse). The consumer’s quality preference coefficient is increased from the θ obeys uniform distribution on $(0, 1)$ to the uniform distribution on $(1, 2)$. At this point, it can be drawn about the impact of disruptions on the demand for the mouse to control the impact of the conditions, thus can be drawn (assume $a = 5$): the market price of new mouse and remanufacturing mouse and the total profit of the closed-loop supply chain in the centralized decision is: $p_1 = 110, p_2 = 65, \pi^{**} = 81.041$, the demand of new mouse and remanufacturing mouse changed to:

$$\begin{cases} D_1(p_1, p_2) = 2 - \frac{p_1^* - p_2^*}{q_1 - q_2} = \frac{2q_1 - 2q_2 + c_2 - c_1 + a}{2(q_1 - q_2)} = 0.875 \\ D_2(p_1, p_2) = \frac{p_1^* - p_2^*}{q_1 - q_2} - \frac{p_2^*}{q_2} = \frac{c_1 q_2 - c_2 q_1 - a q_1}{2q_2(q_1 - q_2)} = 0.042 \end{cases} \quad (39)$$

So we can see the situation that demand of new products and remanufactured products compared with the demand in the stationary state is: the demand for new products increased by 0.5, remanufacturing products demand has not changed. This can be deduced in the product of the total demand of the product increased by 0.5 because of the occurrence of disruptions.

At the same time, it can derive under the decentralized decision-making model, after the profit sharing contract coordinated; the profit function of the two enterprises is changed to:

$$\begin{cases} \pi_m = 0.5\pi^{**} + 0.25\lambda\pi^{**} = 40.521 + 20.260\lambda \\ \pi_r = 0.25\pi^{**} + 0.25(1 - \lambda)\pi^{**} = 40.521 - 20.260\lambda \end{cases} \quad (40)$$

Manufacturers and retailers have increased their profits respectively $20.260\lambda, 20.260(1 - \lambda)$ by the coordination contract, so that the closed-loop supply chain can be achieved in the event of a win-win situation.

From the above analysis of disruptions, we can see that the improvement of the performance of the notebook makes the consumer to improve the quality of the mouse, the demand for new mouse will be improved, and then the demand for the remanufacturing mouse will keep unchanged. In this situation, the profit of the manufacturer M and retailer R, the overall profit of the closed-loop supply chain are mentioned, it is a controllable closed-loop supply chain impact, that event is an event that can increase the profit of the system. We should actively face the occurrence of this kind of disruptions, improve the production of new products, adapt to the market.

6. Conclusions

On the basis of steady supply chain product quality game model, this paper studied the impact of disruptions specific to the corresponding model parameters. From supply and demand two aspects, it emphasized how the disruptions affect the closed-loop supply chain. This paper also designed a set of differential profit sharing contract based on revenue sharing contract mechanism. The contract is easy to be managed, and easy to be operated, which makes the closed-loop supply chain system achieve coordination whether in steady state or under disruptions.

In this paper, the contract costs are ignored for zero. While in actual situation, contract costs must be considered to maintain the efficient implementation of this contract. In some cases, the cost may be too high to become a hinder between manufacturers and retailers. This paper also does not make an intensive study of this issue, so it will be further study in the future.

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