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Local Government Debt Risk Assessment and Future Borrowing Planning in the Process of Urbanization

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Abstract. With the deepening of new urbanization, local government debt risk in China is accumulating and amplifying. This paper first constructs local government debt risk index system and utilizes factor analysis to calculate the overall debt risk interval. Secondly, this paper builds a micro cash flow model based on genetic algorithm to estimate the reasonable scale of future borrowing under the cash flow constraint and risk index constraints. The objective is to achieve the optimal disposable revenue of local government. The model not only estimates the total borrowing scale, but also forms the different strategies of issuing bonds at different times and with different maturities. At the same time, we can change specific parameters (such as land transfer revenue) to analyse the variation of the future debt scale. Finally, this paper carries on an empirical study of city X to testify the methods and models, which provides scientific management of local government debt in the process of new urbanization.

1. Introduction

The issuing of "National new urbanization planning" (2014–2020) means the formal launching of the new urbanization construction in China. While new urbanization construction promotes the infrastructure of the city, it also brings risks. The local government bears most of the financing and investment task of infrastructure construction. From 2007 to 2014, local government debt increased more than 3 times. In 2014, the debt balance was as high as 2.4 trillion yuan, the pressure is heavy. Due to the unreasonable systems and mechanisms (Such as: imbalance of financial resources and power of local government, Chinese unique land management system, assessment of government officials following GDP evaluation mechanism, "soft budget constraints", "no constraints outside budget"), fiscal opportunism behavior of local government will emerge (Qiang Gong, 2011). Local governments mainly fulfill the huge financing needs with the aid of

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land financing (Chenggang Xu, 2010; Chunjie Wu, 2013). With the accumulation of debts and weakening of land transfer revenue, debt default risk of local government is increasing rapidly (Canming Yang, Yuanping Lu, 2013). Improper local government debt management is bound to transfer debt pressure to higher levels of government, threatening the central financial security (Yuqing Guo, 2011; Xiaolan Deng, 2013).

The State Council promulgated the "Opinion on strengthening local government debt management" (2014) (hereinafter referred to as the "Opinion"), which suggests local governments take positive roles in and defusing financial risks, and promote the healthy and sustained development of the national economy. The introduction of "Opinion" prompted local governments to establish a unity of debt management mechanism including borrowing, using and repayment. "Opinion" enhances the power of government regulation and the efficiency of local government investment under the new market conditions. The core content is the establishment of standardized debt management mechanism. Therefore, the establishment of the risk warning mechanism and the standard debt financing mechanism is badly needed for local governments.

Measuring the overall risk of local government debt needs to establish a debt risk index system, which is reflected the in the practice all over the world (Kaminsky et al., 1998; Kumar et al., 2003; Gray & JOBST, 2010). At present, some foreign countries have established local government financial risk warning systems through debt risk indexes, such as the "local fiscal monitoring plan and financial crisis method" in United States, "local government borrowing limitation" in Brazil, "traffic lights system" in Columbia. These results have certain exemplary roles for debt risk warning. In addition, Andrew (1999) (International Monetary Fund) proposed the DCSD(Developing Country Studies Division) model for monetary crisis in developing countries. NAG & Mitra (1999) build the artificial neural network model and Abiad (2003) constructed Markov switching model with variable transition probability. These models, which mainly focused on the monetary crisis warning, need further improvement for debt risk assessment.

China has a late start applying these risk assessment methods in the field of local government debt risk. In recent years, Yu Pei and Huasheng Ouyang (2006) first discuss the basic principles of establishing early warning mechanism of local government debt risk. In addition, a number of people propose to establish debt indexes as the core of the local government debt risk warning system (Yi Liu et al., 2004). The overall risk of debt needs integrated evaluation of multi dimension indexes: Yi Liu (2004) weights debt risk indexes using entropy method; Weiyu Shao (2008) builds comprehensive index for debt risk indexes using neural network; Wang Zhou et al. (2015) use cluster analysis. This paper constructs the local government debt risk indexes with concise, clear and practical principles, and uses factor analysis to build the comprehensive risk index, which realizes the assessment of the overall debt risk of local government.

After qualitative assessment of local government debt risk, we need to estimate the reasonable scale of future borrowing. In the relatively limited research at present, using KMV model to calculate the default probabilities corresponding with the borrowing scale is common (Liyan Han et al., 2003; Miaoru Wang, 2012; Dan Chen, 2013; Qinggang Tan, Ping Qi, 2013). However, there are many limitations in the existing research: First is that KMV model needs to make pre assumptions that do not conform to the actual, such as negelecting the interest rate on the debt and prohibiting borrowing new debt to repay old debt; Secondly, the existing models only plans the overall debt scale other than taking into account the different duration of government debt; The third is that the existing models do not consider the cash flow constraints; The fourth is that the number of debt constraints is not enough according to the benchmark constraints on the debt scale. In the process of new urbanization, the local government needs the best borrowing strategy to ensure the maximum disposable revenue without liquidity risk. Polackova et al. (2000) and Dafflon (2009) come up with the thought of managing comprehensive assets and liabilities - that is, managing the local government debt risk under the premise of safety and liquidity. The absolute matching strategy proposed by Lu Wang and Chong Wang (2010) , which requires perfect match of cash flow from assets and liabilities, is the basic idea of local government borrowing scale planning. Yu et al. (2012) and Fernandeza et al. (2013) use genetic algorithm to allocate funds to investment project with multi-objective attribute, which provides a reference for borrowing scale planning.

Based on the thought of managing assets and liabilities and cash flow matching, this paper applies genetic algorithm to build the micro cash flow model. The model not only estimates the optimal total borrowing scale, but also forms the different strategies of issuing bonds at different times and with different maturities. Since the input and output variables and model parameters can be flexibly adjusted, the model can be applied in scenario analysis. With transportability greatly enhanced, the model is one of the effective methods of local government borrowing in new urbanization process.

The above two complementing steps effectively evaluate local government debt risk, take furute borrowing scale under control, provide borrowing strategies, and make scenario analysis. They can effectively stop risk from transferring to the central finance in the process of new urbanization.

2. Local government debt risk index system

2.1. The construction of local government debt risk index system

Specifically, the local government debt risk index system should follow the principles in Table 1:

Table 1: Principles of index selection

Systematic principle	The emergence and evolvement of the local government debt risk is a systemic problem, the indexes must reflect the comprehensive effect of various factors.
Sensitivity principle	The indexes should highly summarize the local government debt risk, and react to small changes when running the debt fund.
Functional principle	The index system is the scientific basis for the macro financial decision, improving the financial quality and the function of "accurate estimation".
Dynamic principle	The local government debt risk is in the course of social economic operation, it is a kind of fiscal operating status and changing trend.
Reality principle	The index can reflect the formation and characteristics of the local government debt risk.

Local government debt risk is closely related to the government revenue, expenditure, debt scale, macroeconomic situation and public risk. This paper constructs the following debt risk index system, which contains three dimensions: debt scale, solvency and debt potential. As is shown in Table 2:

Table 2: Risk index system

Dimensions	Name	Formula
Debt scale	Debt repayment ratio	(Interest + repayment of principal)/Revenue
	Debt burden ratio	Debt balance/GDP
	Deficit dependence ratio	Deficit/Spending
Solvency	GDP growth rate	$(GDP(t) - GDP(t-1))/GDP(t-1)$
	Land transfer burden	Debt balance/Land transfer revenue
	Revenue/GDP	Revenue/GDP
	Solvency ratio	Debt balance/Disposable revenue
Debt potential	Revenue growth rate	$(Revenue(t) - Revenue(t-1))/Revenue(t-1)$
	Revenue elasticity	Revenue growth rate/GDP growth rate

All parameters and indexes in Table 4 are benchmarked against local government.

The risk interval of each index must be established to measure the risk of debt. Scientifically defining the critical value for risk interval is essential for evaluating local government debt risk. Determining the critical value for risk interval is mainly based on the following principles: refer to the domestic and foreign standards and researches; use uniform interval if there are no references to determine the critical value. Under above principles, the risk interval of the index is divided into three levels: low risk, medium risk and high risk.

Based on domestic and foreign standards and the financial structure of local government, this paper sets up three standard intervals for the debt scale indexes (i.e. debt service ratio, debt burden rate, deficit dependence); GDP growth rate in solvency indexes is based on Keqiang Li Prime Minister's "A reasonable range of economic operation" and the recent twenty-five years' GDP growth rate of China (from 1990 to

2014). While this paper takes the lowest GDP growth rate 3.8% in 1990 during twenty-five years as the lower limit. The second lowest GDP growth rate 7.4% in 2014 during twenty-five years is a reasonable upper limit. Critical value of the land transfer burden and revenue/GDP are determined by PhD thesis of Weiyu Shao (2008), Suzhou University. Solvency ratio follows 90%–150% standard developed by International Monetary Fund (IMF); Among debt potential indexes, revenue growth rate and revenue elasticity have no reasonable basis on critical value, so take average intervals as a unified standard. The results are shown in Table 3:

Table 3: Risk index intervals

Dimensions	Indexes	Low risk	Medium risk	High risk
Debt scale	Debt repayment ratio	0–20%	20%–40%	> 40%
	Debt burden ratio	0–30%	30%–60%	> 60%
	Deficit dependence ratio	0–5%	5%–10%	> 10%
Solvency	GDP growth rate	> 7.4%	3.8–7.4%	0–3.8%
	Land transfer burden	0–1	1–3	> 3
	Revenue/GDP	> 20%	10%–20%	0–10%
	Solvency ratio	0–90%	90%–150%	> 150%
Debt potential	Revenue growth rate	> 30%	0–30%	< 0
	Revenue elasticity	> 2.57	0–2.57	< 0

2.2. *Assessment of the overall debt risk of local government*

Conflicts will occur when using the debt risk index system: while some indexes show high debt risk, some other indexes show debt risk is in reasonable range. Relying on the single index lacks accuracy, it's crucial to build the overall debt risk evaluation system. Firstly, use factor analysis to extract several principal factors that represent overall risk of the debt from all the risk indexes. Then, use the score formula to obtain the comprehensive score of the total debt risk. Higher score means higher level of risk and vice versa. So far, the overall debt risk comprehensive score model is completed. Secondly, apply the critical values of each risk interval of each index into the factor model, use the score formula to get the overall debt risk range. For clearer interpretation, this paper takes X City as an example.

2.2.1. *The construction of debt risk index matrix*

The local government debt risk indexes is shown in Table 4:

Table 4: Debt risk indexes

X1	X2	X3	X4	X5	X6	X7	X8	X9
GDP growth rate (%)	Land transfer burden	Solvency ratio	Debt repayment ratio (%)	Debt burden ratio (%)	Revenue/ GDP (%)	Deficit dependence ratio (%)	Revenue growth rate (%)	Revenue elasticity

Assuming the actual sampling data of n years, the local government debt risk indexes constitute a $n \times 9$ debt risk index matrix:

$$X = \begin{bmatrix} X_{11} & X_{12} & \cdots & X_{19} \\ X_{21} & X_{22} & \cdots & X_{29} \\ \vdots & \vdots & \vdots & \vdots \\ X_{n1} & X_{n2} & \cdots & X_{n9} \end{bmatrix} = \begin{bmatrix} X(1,:) \\ X(2,:) \\ \vdots \\ X(n,:) \end{bmatrix} \quad (1)$$

The sample covariance matrix is

$$S = \frac{1}{n} X^T (I_n - \frac{1}{n} e_n e_n^T) X = (s_{ij})_{9\times 9} \quad (2)$$

Where

$$s_{ij} = \frac{1}{n}\sum_{\alpha=1}^{n}(X_{\alpha i} - \bar{X}_i)(X_{\alpha j} - \bar{X}_j)$$
$$\bar{X}_i = \frac{1}{n}\sum_{\alpha=1}^{n} X_{\alpha i}$$
$$\bar{X}_j = \frac{1}{n}\sum_{\alpha=1}^{n} X_{\alpha j} \quad (i, j = 1, 2, \cdots, 9) \tag{3}$$

As the unit of debt risk indexes are different, the influence should be eliminated before calculation. So we use $\frac{X_{\alpha j} - \bar{X}_j}{\sqrt{s_{jj}}}$ to substitute the sample data $X_{\alpha j}$ in matrix X ($\alpha = 1, 2, \cdots, n$, $j = 1, 2, \cdots, 9$). Thus the sample covariance matrix become:

$$S = \frac{1}{n} X^T X \tag{4}$$

Note that the elements in X have been standardized.

2.2.2. *Judge the suitability of debt risk index for factor analysis*

The factor analysis is a process of reconstructing a few representative indexes from many debt risk indexes. Its potential requirements: strong correlations between the original variables. Therefore, correlation coefficient matrix of debt risk indexes needs to be computed before factor analysis.

The sample correlation coefficient matrix can be obtained from the sample covariance matrix, which takes form as follows:

$$\hat{R} = (r_{ij})_{9\times 9} \tag{5}$$

Where $r_{ij} = \frac{s_{ij}}{\sqrt{s_{ii}}\sqrt{s_{jj}}}(i, j = 1, 2, \cdots, 9)$. If most of the correlation coefficients are less than 0.3 and do not pass the statistical test, the risk indexes are not suitable for factor analysis.

2.2.3. *Principal factor extraction*

The original 9 debt risk indexes, which needs dimensional reduction, are transformed into 9 linearly independent comprehensive indexes by the factor analysis model. The process is as follows:

Firstly, calculate the characteristic root ($\lambda_1, \lambda_2, \cdots, \lambda_9; \lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_9$) of the correlation coefficient matrix $\hat{R}$, then calculate the linearly independent unit feature vector (Principal factor) corresponding with $\lambda_1, \lambda_2, \ldots, \lambda_9$:

$$\Gamma = \begin{pmatrix} F_1 \\ F_2 \\ \vdots \\ F_9 \end{pmatrix} \tag{6}$$

Easy to verify

$$\Gamma^T \hat{R} \Gamma = \begin{bmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_9 \end{bmatrix} = \Lambda \tag{7}$$

If the variance of the former m principal factor accounts for 85% of the total variance, the former m principal factor basically contains the original information of the 9 debt risk indexes.

That is, select the smallest positive integer $m (m \leq 9)$ and make

$$\frac{\lambda_1 + \lambda_2 + \cdots \lambda_m}{\lambda_1 + \lambda_2 + \cdots \lambda_9} \geq 85\% \tag{8}$$

The first main factor F_1 has the largest variance among linear combinations of debt risk index $X1$, $X2$, $\cdots$, $X9$; the second principal factor F_2 has the second largest variance among linear combinations of debt risk index $X1, X2, \cdots, X9$ and is not related to F_1; ... the principal factor F_m the m largest variance among linear combinations of debt risk index $X1$, $X2$, $\cdots$, $X9$.

Based on m unit feature vectors F_1, F_2, $\cdots$, F_m and the n years' sample $X(1,:)$, $X(2,:)$, $\cdots$, $X(n,:)$, we get former m principal factor score functions:

$$\begin{cases} X(1,:) = K_{11}F_1 + K_{12}F_2 + \ldots K_{1m}F_m + \varepsilon_1 \\ X(2,:) = K_{21}F_1 + K_{22}F_2 + \ldots K_{2m}F_m + \varepsilon_2 \\ \ldots \\ X(n,:) = K_{n1}F_1 + K_{n2}F_2 + \ldots K_{nm}F_m + \varepsilon_n \end{cases} \tag{9}$$

Where

$$K = \begin{bmatrix} K_{11} & K_{12} & \cdots & K_{19} \\ K_{21} & K_{22} & \cdots & K_{29} \\ \vdots & \vdots & \vdots & \vdots \\ K_{n1} & K_{n2} & \cdots & K_{n9} \end{bmatrix} = [K(:,1), K(:,2), \ldots, K(:,9)] \tag{10}$$

2.2.4. The naming of comprehensive index

The naming of the comprehensive indexes is another key problem of the factor analysis. Debt risk indexes are variables that have actual economic implications, and we need to get economic implications of the new comprehensive indexes. In practical application, the new comprehensive indexes are named by analyzing the load matrix. Factor rotation can make the comprehensive indexes more explanatory. Calculate the comprehensive index load, build load matrix A.

$$A = \Gamma^T \Lambda = (F_1, F_2, \ldots, F_n) \begin{bmatrix} \sqrt{\lambda_1} & & & \\ & \sqrt{\lambda_2} & & \\ & & \ddots & \\ & & & \sqrt{\lambda_m} \end{bmatrix} \tag{11}$$

$$= \begin{bmatrix} F_{11}\sqrt{\lambda_1} & F_{21}\sqrt{\lambda_2} & \vdots & F_{m1}\sqrt{\lambda_m} \\ F_{12}\sqrt{\lambda_2} & F_{22}\sqrt{\lambda_2} & \vdots & F_{m2}\sqrt{\lambda_m} \\ \vdots & \vdots & \vdots & \vdots \\ F_{19}\sqrt{\lambda_m} & F_{29}\sqrt{\lambda_2} & \vdots & F_{m9}\sqrt{\lambda_m} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{21} & \vdots & a_{m1} \\ a_{12} & a_{22} & \vdots & a_{m2} \\ \vdots & \vdots & \vdots & \vdots \\ a_{19} & a_{29} & \vdots & a_{m9} \end{bmatrix} \tag{12}$$

Where $a_{ij} = F_{ij}\sqrt{\lambda_i}$ $(i = 1, 2, \cdots, m, j = 1, 2, \cdots, 9, m < 9)$.

The i-th column in load matrix A is load coefficients of 9 debt risk indexes on the i-th principal factor F_i. The economic meaning of i-th principal factor F_i can be obtained by synthesizing the economic meanings of debt risk indexes whose corresponding load coefficients have absolute values that closed to 1.

2.2.5. Calculate the comprehensive risk score F

$$F = \frac{\lambda_1}{\lambda_1 + \lambda_2 + \cdots + \lambda_9} K(:,1) + \frac{\lambda_2}{\lambda_1 + \lambda_2 + \cdots + \lambda_9} K(:,2) + \cdots + \frac{\lambda_m}{\lambda_1 + \lambda_2 + \cdots + \lambda_9} K(:,m) \tag{13}$$

The overall risk of local government debt can be evaluated by comprehensive score F in formular (13), the greater the value, the higher the overall debt risk.

3. Micro cash flow model of local government borrowing

3.1. Model overview

In the process of new urbanization, local government needs to reasonably develop borrowing strategies over the next few years. The essence of borrowing strategies is to increase government disposable revenue as much as possible (disposable revenue = fiscal revenue + land transfer revenue + net debt borrowing) under constraints and overall risk control. This section will establish genetic algorithm (GA) based micro cash flow model, under which the future overall borrowing scale and the borrowing scale at specific time and of certain duration can be obtained under cash flow constraints and risk index constraints according to the current and future estimates of financial performance indexes.

3.2. Model structure

The micro cash flow model has three parts: the input platform, the calculation platform and the output platform, as is shown in Figure 1:

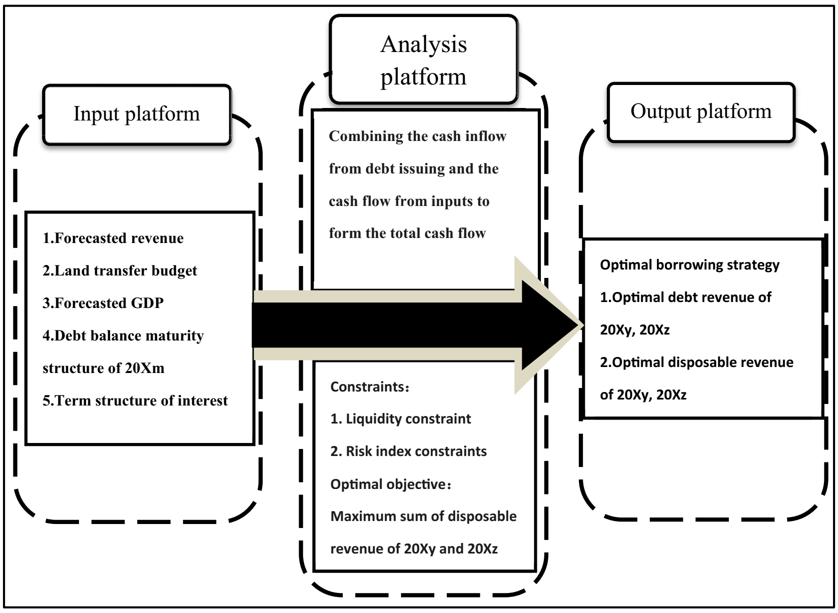

Figure 1: Platform structure

3.2.1. Input platform

Time series is the basis of micro cash flow model analysis, and it is the most basic principle generating all cash flow. Time series of the model is set to be from year $20Xm$ to year $20Xn$ (m, n for different integers from 1 to 9, $m < n$; X is unknown integer from 1 to 9). Cash flow at each time point forms cash flow sequence of $m - n + 1$, as shown in Table 5:

Table 5: Time setting

20Xm	20Xm + 1	20Xm + 2	20Xm + 3	...	20Xn
$t = 0$	$t = 1$	$t = 2$	$t = 3$	...	$t = m - n$

Input platform contains four elements: fiscal revenue forecast F_j, land transfer revenue forecast L_i, GDP forecast G_i, fixed cash flow on behalf of debt balance maturity structure D_i. A set of fixed cash flows are formed according to the above four inputs, called fixed cash flow $IN_i = F_i + L_i + G_i + D$. Where $i = 1, 2, \ldots, m + n - 1$, representing the time series index.

3.2.2. *Analysis platform*

Genetic algorithm simulation generates tens of thousands of kinds of borrowing strategies. The i-th borrowing strategy produces debt revenue RD_{ij}, and debt repayment cash flow PD_{ij}. The net cash flow from local government for the next $m-n+1$ years is I_{ij}. Where $I_{ij} = IN_{ij} + RD_{ij}$, $j = 1, 2, \ldots, m-n+1$.

Net cash flow I_{ij} contains both valid solutions and invalid solutions. Based on the objectives and constraints (liquidity constraint and debt risk index constraints), measurement platform uses genetic algorithm to filter valid solutions that satisfy the constraints, and selects the optimal solutions from the valid solutions.

The objective of the model is flexible and diverse, and the most representative objective is the maximum sum of the disposable revenue (disposable revenue = fiscal revenue + land transfer revenue + net debt borrowing) over the next one year/few years. This paper selects maximum sum of the disposable revenue of fiscal year $20Xy$ and $20Xz$ (y, z are different integers from 1 to 9) as the objective.

Liquidity constraint, which is the most basic constraint, ensures the timely repayment of interest and principal; Debt risk index constraint is formed by the specific critical values of four debt indexes (solvency ratio, debt burden ratio, land transfer burden, debt repayment ratio) that are not only significant parts of risk index system but also Chinese official criteria of local government debt risk assessment.

3.2.3. *Output platform*

The output platform includes three elements: the optimal borrowing strategy, the optimal debt revenue in $20Xz$ and $20X$y, and the optimal local government disposable revenue in $20Xz$ and $20X$y. All interest rates are simple rates referred to the current existing local government debt, all repayments of principal occurs at expiration. The term structure of the optimal borrowing strategies is shown in Table 6:

Table 6: Term structure of optimal borrowing strategies

Borrowing time	$20Xm+1$					$20Xm+2$				$20Xm+3$			…		$20Xn$
Duration	1	2	3	…	$n-m$	1	2	…	$n-m-1$	1	…	$n-m-2$	…	…	1

Due to the limited length of the local government budget data, it is of little significance to consider debt risk for too long. This paper analyses years from $20Xm+1$ to $20Xn$. So the longest life of the debt at $20Xm+1$ is $n-m$ years, which expires at $20Xn+1$ without repayment of principal at $20Xn$. Similarly, the longest borrowing at $20Xm+2$ is $n-m-1$, $20Xm+3$ for $n-m-2\ldots$. Based on the term structure of optimum debt strategies, the model uses genetic algorithm to filter valid solutions that satisfy the constraints, and selects the optimal solutions from the valid solutions.

4. Empirical Study – Taking City X as an example

4.1. *The empirical research on the overall debt risk of* X *government*

Municipal debt risk indexes and their values are listed in Table 7:

Table 7: Debt risk indexes of City X

The year	X1	X2	X3	X4	X5	X6	X7	X8	X9
	GDP growth rate (%)	Land transfer burden rate	Solvency ratio	Debt repayment ratio (%)	Debt burden ratio (%)	Revenue/ GDP (%)	Deficit dependence (%)	Revenue growth rate (%)	Revenue elasticity
2012	0.1033	3.4722	1.6890	0.3816	0.2413	0.1520	0.0605	−0.2704	−2.6178
2011	0.1815	1.2546	0.9905	0.3317	0.1834	0.2298	0.0608	0.1214	0.6691
2010	0.1792	1.5343	1.1113	0.4095	0.2485	0.2421	0.0469	0.8845	4.9356
2009	0.0329	3.4415	1.9791	0.3963	0.2629	0.1515	0.0182	0.2720	8.2724
2008	0.1886	1.8664	1.3106	0.3176	0.1395	0.1230	0.0017	0.6474	3.4327
2007	0.2103	1.7653	0.6203	0.2095	0.0811	0.0888	−0.0119	0.1577	0.7501

Data source: X Municipal Finance Bureau

After statistical verification, the correlation coefficients are suitable for factor analysis for the correlation coefficients between indexes are more than 0.3. According to Table 8, three principal factors are extracted from the nine basic risk indexes. The explanation of the first principal factor to the overall risk of debt is 49.407%, the second is 26.467%, and the third is 20.631%. Statistically, if accumulative percentage contribution of $m(m < p)$ factors exceed 80%, then m principal factors will be able to maximize the retention of the information of original 9 indexes. From Table 8, we can see that the sum of the eigenvalues of the first 3 principal factors accounts for 96.505% of total variance. So consider only the first 3 three principal factors.

Table 8: The explanatory power of the principal factors to the overall debt risk

Principal factor	Eigenvalues			Extraction of sum of squares loaded			Rotate the sum of squares loaded		
	Total	Variance%	Sum%	Total	Variance%	Sum%	Total	Variance %	Sum %
1	4.447	49.407	49.407	4.447	49.407	49.407	3.777	41.969	41.969
2	2.382	26.467	75.874	2.382	26.467	75.874	2.929	32.548	74.517
3	1.857	20.631	96.505	1.857	20.631	96.505	1.979	21.988	96.505

Maximum variance orthogonal rotation matrix illustrates which basic indexes provide risk information that reflected by each principal factor (factor rotation increases explanatory power of each basic index in principal factors), as shown in Table 9:

Table 9: Principal factors after rotation

		Principal factors		
		1	2	3
GDP growth rate	(X1)	−0.972	−0.123	−0.036
Land transfer burden	(X2)	0.953	−0.113	−0.230
Solvency ratio	(X3)	0.950	0.191	0.091
Debt repayment ratio	(X4)	0.567	0.758	0.271
Debt burden ratio	(X5)	0.665	0.716	0.167
Revenue/GDP	(X6)	−0.173	0.952	0.196
Deficit dependence ratio	(X7)	0.108	0.925	−0.361
Revenue growth rate	(X8)	−0.341	0.116	0.899
Revenue elasticity	(X9)	0.314	−0.034	0.916

As is shown in Table 9, load coefficients $X1$, $X2$, $X3$ (Namely, the GDP growth rate, the land transfer burden and the debt coverage ratio) of principal factor 1 are as high as −0.972, 0.953 and 0.95 respectively. So, give the principal factor 1 the economic meaning – solvency factor $F1$; principal factor 2 has closely relationship with three basic indexes: debt repayment ratio, debt burden ratio, revenue/GDP, given the economic meaning – the risk of debt scale factor $F2$; Similarly, principal factor 3 is defined as debt potential factor $F3$.

From the formula (9) and (13), we can get the score of the principal factor and the comprehensive risk score of each year. Higher value means higher risk level of the debt, as shown in Table 10:

Table 10: Ranking of the principal factor scores and comprehensive score

	F1	Rank	F2	Rank	F3	Rank	F	Rank
2012	0.881	2	0.408	3	−1.528	6	0.228	3
2011	−0.812	5	0.904	2	−0.563	5	−0.278	5
2010	−0.540	4	1.135	1	1.056	1	0.251	2
2009	1.583	1	−0.270	4	0.819	2	0.880	1
2008	−0.283	3	−0.713	5	0.627	3	−0.199	4
2007	−0.829	6	−1.464	6	−0.411	4	−0.882	6

Comprehensive risk scores represent the relative level of debt risk in each year. But cannot objectively measure the level of debt risk. Put the critical value into the formula (9) and (13), the critical value of risk factor score is shown in Table 11:

Table 11: The factor score of the main factor and the risk threshold

	F1	F2	F3	F
Critical value 1	−0.776	0.242	−0.067	−0.333
Critical value 2	0.665	0.472	−0.418	0.367

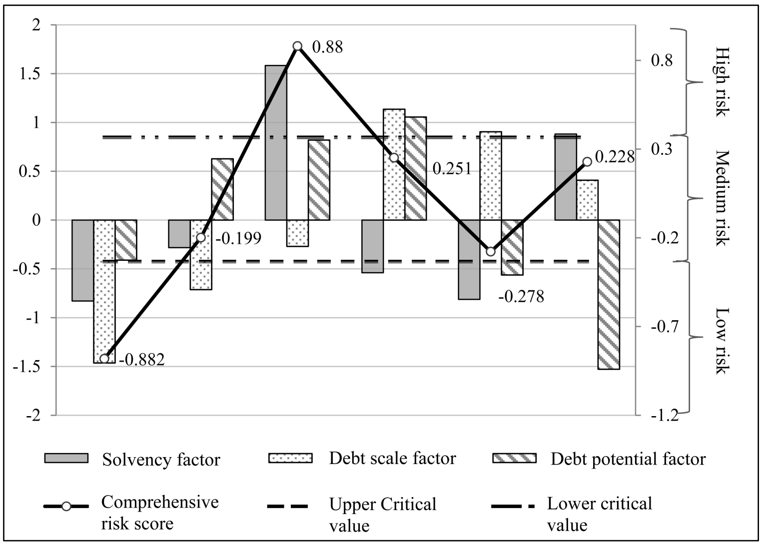

Figure 2: Three risk areas for the comprehensive evaluation of debt risk

The comprehensive scores fall below the lower critical value is at the low risk interval, and between the lower critical value and upper critical value is at the intermediate risk interval. The high risk interval is above the upper critical value. The risk interval of overall debt risk over the years is shown in Figure 2. The debt risk of X city generally lies in the medium risk level, the overall risk reached the high-risk level. After adjusting the input data (Table 7) and index risk interval (Table 3), the overall debt risk level of different local governments can be obtained by the same method.

4.2. X government borrowing strategy and the overall debt scale estimation

4.2.1. Model input

a. Forecasted data of financial revenue

Estimation for future borrowing plan is inseparable from future financial revenues. Here we use historical financial revenue growth rate of City X from 2006–2013 (Arithmetic mean is 17%) to forecast the 2014–2018 values, as is shown in Table 12:

Table 12: Forecasted financial revenue

Year	2013	2014	2015	2016	2017	2018
Forecast (million ¥)	473.379	553.853	648.009	758.170	887.059	1,037.859

b. Land transfer revenue budget

2014–2016 annual land transfer revenue budget is the given by Financial Bureau of City X. 2017, 2018 land transfer revenues are assumed the average value of 2006 to 2016, as is shown in Table 13:

Table 13: Land transfer revenue

Year	2013	2014	2015	2016	2017	2018
Forecast (million ¥)	473.379	7,000.00	5,000.00	5,000.00	4,261.53	4,261.53

c. GDP forecast data

For the future estimates of GDP, we use 8% growth rate under the current new economic situation, as is shown in Table 14:

Table 14: GDP

Year	2013	2014	2015	2016	2017	2018
Forecast (million ¥)	57,751	62,578	67,405	72,232	77,059	81,886

d. Debt repayment timetable from 2014–2018

Debt balance at the end of 2013 is 19.33 billion yuan. Debt repayment in 2014 is 6.89 billion yuan, which can be obtained according to the debt balance data of City X in 2013. Debt repayment in 2015 is 6.05 billion yuan. Due to only the sum repayment of 2016 to 2018 are given, the sum repayment is averaged and equally allocated to 3 years from 2014 to 2018. The corresponding repayment amount from 2014–2018 is shown in Table 15:

Table 15: Debt repayment timetable from 2014–2018

Year	2014	2015	2016	2017	2018
Forecast (million ¥)	6,893.80	6,050.68	2,126.95	2,126.95	2,126.95

e. Future debt maturity structure

All debts are simple with repayment of principal at expiration. Interest rates are gathered from interest rates of existing debt of City X. The future debt maturity structure is shown in Table 16:

Table 16: Future debt maturity structure

Duration(year)	1	2	3	4	5
Interest(%)	7.25	7.50	7.50	7.75	8.00

4.2.2. *Model constraints*

The liquidity constraint is the basic constraint, and the four debt risk index constraints are taken from the risk indexes. In this paper, the critical values of debt risk index constraints are set at the maximum value of corresponding indexes from 2006 to 2013 of City X, as is shown in Table 17:

Table 17: Model constriants

Constriants	Liquidity	Solvency ratio	Debt burden ratio	Land transfer burden	Debt repayment ratio
Criteria	Disposable revenue > 0	< 200%	< 34%	< 3	< 60%

4.2.3. *Next two years borrowing strategy of City* X

a. Single simulation

Figure 3 shows the optimal borrowing strategy of 2014 and 2015 from single simulation. The 2014-1 on the horizontal axis represents one year municipal bond issued at 2014; 2014-2 represents municipal bond issued at 2014 with two years' maturity, and so forth. At the same time, the simulation generates the optimal debt revenues: 801 million yuan at 2014 , 938 million yuan at 2015.

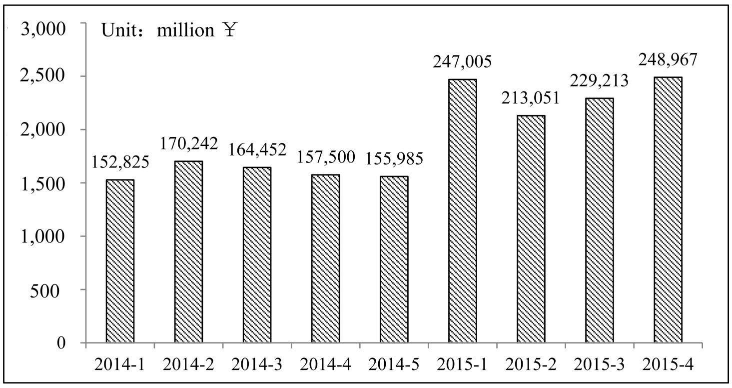

Figure 3: Optimal borrowing strategy of 2014 and 2015

b. Repeated simulation

Due to the randomness of single simulation result, it cannot be the standard of total size of debt revenue. In order to measure the critical value of the government debt revenue and debt balance of X City, 10000 simulations were carried out. Debt revenue calculated by each simulation is the optimal amount meeting the risk constraints. The "Basel agreement" requires regulators to take 3 or 4 times the VaR(measures both market risk and credit risk) to measure the overall risk. Thus, high critical value of debt revenue and debt balance is obtained by adding 4 standard deviation to the average.

The average debt revenue in 2014 is 7.23 billion yuan, with standard deviation 279.72 million yuan. The average debt revenue in 2015 is 9.10 billion yuan, with standard deviation 257.23 million yuan. Average debt balance in 2014 is 19.75 billion yuan, with standard deviation 279.72 million yuan. The average debt balance in 2015 is 22.25 billion yuan, with standard deviation 173.57 million yuan, as Table 18 shows:

Table 18: Statistical indicators of the debt income and debt balance after 10000 simulations

	Year	2014	2015
Debt revenue(million ¥)	Sample mean	7,319.19	9,103.06
	Sample standard deviation σ	279.72	257.23
	4σ upper bond	8,438.07	10,131.97
Debt balance(million ¥)	Sample mean	19,750.71	22,251.29
	Sample standard deviation σ	279.72	173.56
	4σ upper bond	20,869.60	22,945.55
High risk critical value of net debt revenue		1,544.27	2,075.96

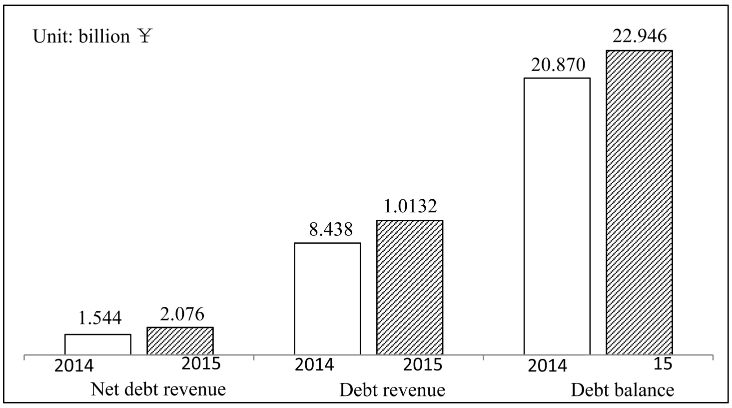

Figure 4: High critical value of debt revenue and debt balance

As is shown in Figure 4, the high risk critical values of net debt revenue in 2015 and 2014 are 1.54 and 2.08 billion yuan, the high risk critical values of debt revenue in 2015 and 2014 are 8.44 and 10.13 billion yuan, the high risk critical value of debt balance are 20.87 and 22.95 billion yuan respectively

At the same time, this paper gets the database which contains 10000 kinds of borrowing strategies. Figure 5 lists 3 different borrowing strategies:

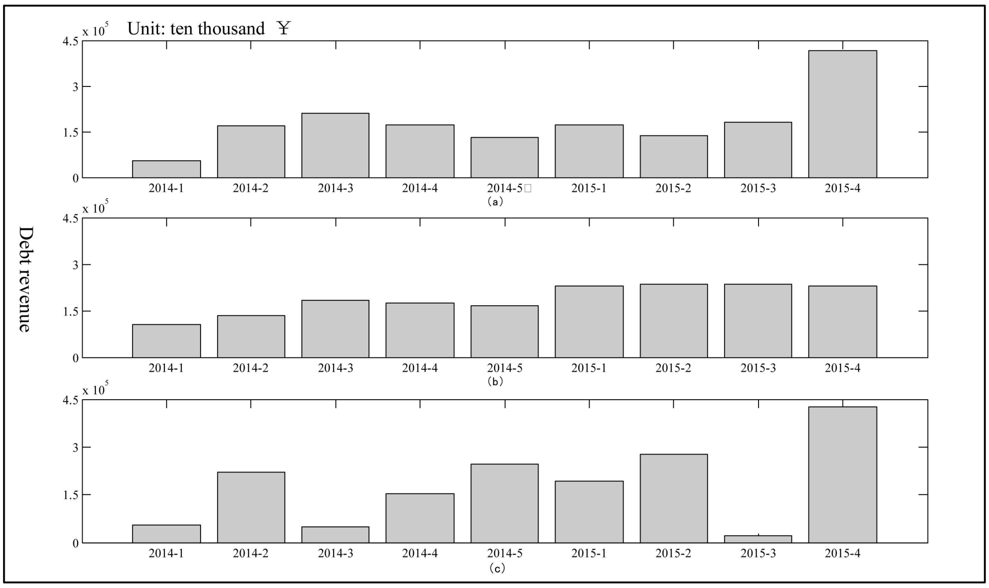

Figure 5: 3 kinds of different borrowing strategies generated by simulation

Comparing the three sub graphs, it isn't hard to find that under the condition of ensuring the debt risk under control in the next 5 years, the debt strategies of 2014 and 2015 are diverse. For example, the borrowing strategy in Figure 5(b) is more balanced, 2014 and 2015 borrowings are evenly allocated to the short, medium and long term. The borrowing strategies in Figure 5(c) appears to be more extreme, where 2014 1-year, 3-year and 2015 3-year borrowings are little comparing to other terms, especially 2015 4-year loan reached nearly 45 billion.

Although the three debt strategies are different in Figure 5, but the total debt revenue of 2014 and 2015 is relatively stable, as is shown in Figure 6:

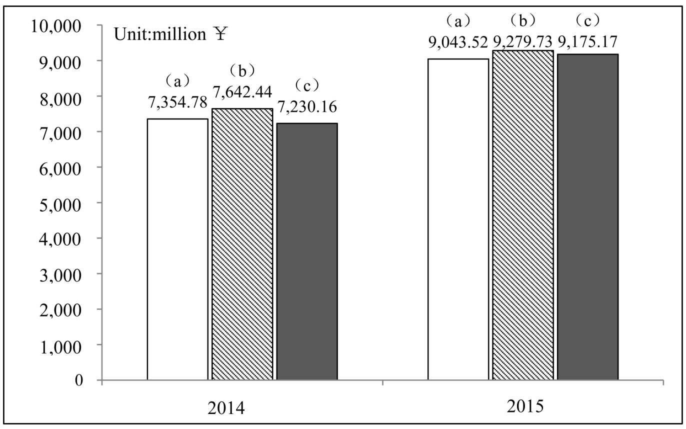

Figure 6: Comparison of debt revenues of three borrowing strategies of 2014 and 2015

Therefore, X city government can select debts of suitable maturity structures from database of optimal strategies to achieve the maximization of the use of debts.

4.2.4. The impact of the change of land transfer revenue on the debt revenue

In the process of urbanization, the risk of local government debt is strongly influenced by land transfer revenue. The land transfer revenue, which serves as hard security of government debt issuing, is bound to restrict the local government debt scale. If the government expects land transfer revenue is optimistic, the government will issue more debt according to the financial needs of the infrastructure projects. On the contrary, if land revenue will deteriorate over the next two years, the government needs to control issuing debt to prevent large scale of debt repayment in the future.

According to the land revenue forecast of 2014 and 2015, this paper sets four scenarios for the land transfer revenue changes: 2014-2015 land transfer revenue decreases by 30%, a decrease of 15%, an increase by 15%, an increase by 30%. The corresponding debt revenues, percentage changes of debt revenues, and net debt revenues are shown in Table 19 and Figure 7:

Table 19: Debt revenues, percentage changes of debt revenues, and net debt revenues

Land transfer revenue	2014			2015		
	Debt revenue (million ¥)	Percentage change	Net debt revenue (million ¥)	Debt revenue (million ¥)	Percentage change	Net debt revenue (million ¥)
−30%	5,151.60	−38.95%	−773.26	7,608.37	−24.91%	438.07
−15%	6,513.28	−22.81%	895.14	9,131.14	−9.88%	1,886.43
Unchanged	8,438.07	0.00%	1,544.26	10,131.97	0.00%	2,075.95
+15%	9,907.88	+17.42%	3,014.08	12,571.40	+24.08%	4,938.44
+30%	11,174.50	+32.42%	4,280.69	13,577.91	+34.01%	5,123.77

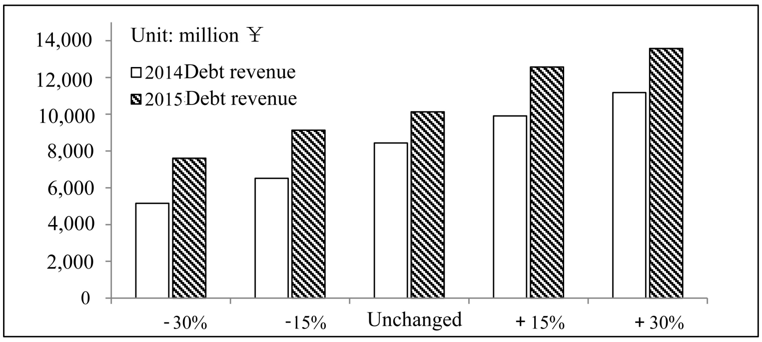

Figure 7: The impact of land transfer revenue on debt revenue

As we can see, X city government borrowing scale is positively correlates with land transfer revenue. Especially in 2014, the change of land transfer revenue leads to greater change of debt revenue. Therefore, the government needs to reasonably plan land transfer revenue for the next two years based on the condition of the local economy as well as the macro-control policy of central government.

4.2.5. Conclusion

By implementing the financial operation data of City X into the model, it is concluded that the average debt revenue in 2014 is 7.23 billion yuan, with standard deviation 279.72 million yuan. The average debt revenue in 2015 is 9.10 billion yuan, with standard deviation 257.23 million yuan. Average debt balance in 2014 is 19.75 billion yuan, with standard deviation 279.72 million yuan. The high risk critical values of net debt revenue in 2015 and 2014 are 1.54 and 2.08 billion yuan, the high risk critical values of debt revenue in 2015 and 2014 are 8.44 and 10.13 billion yuan, the high risk critical value of debt balance are 20.87 and 22.95 billion yuan respectively.

This paper also gets the database which contains 10000 equal borrowing strategies, which can be used to issue municipal bonds accordingly.

X city government borrowing scale is positively correlates with land transfer revenue. Especially in 2014, the change of land transfer revenue leads to greater change of debt revenue. So the government needs to carefully plan land transfer revenue for the next several years based on the current debt condition and future financing needs.

5. Conclusion

Based on the data from Financial Bureau of City X, this paper establishes the concrete methods and models of local government debt risk assessment and future borrowing scale estimation.

This paper first establishes the local government debt risk index system, which includs 9 indexes of three categories: debt scale, solvency and the debt potential. The low, medium and high risk interval of each index are determined according to the researches and standards domestic and abroad. Afterwards, this paper uses factor analysis to determine the low, medium and high risk interval of the overall debt risk of local government. The comprehensive score of local government debt risk, which can be used to determine where it lies in overall debt risk interval, can also be calculated. Local government debt risk assessment methods above contributes to the establishment of local government debt risk early warning mechanism suggested by Ministry of Finance, which leads to the control of outstanding debt to a reasonable degree.

Afterwards, a micro cash flow model based on genetic algorithm is constructed to estimate the future borrowing scale. The model inputs are forecasted financial revenue, land transfer revenue budget, forecasted GDP and the maturity structure of debt balance. The model constraints are liquidity constraints and debt risk index constraints. The objective is to achieve the optimal disposable revenue of local government. The model can not only estimate the total borrowing scale, but also can form the different strategies of issuing bonds at different times and with different maturities. Since the input and output variables and model parameters can be flexibly adjusted, the model can be applied into scenario analysis. With transportability greatly enhanced, the model is one of the effective planning methods of local government borrowing in new urbanization process. The application of micro cash flow model can help local governments to form the conscious control of the debt scale, strengthen the government debt management and promote the quality of local government borrowing in new market condition.

This study conforms to the thought of "opinion" which prompts the local government to establish a unity of debt management mechanism of borrowing, using and repayment. Local government debt risk assessment method and borrowing scale estimation model can promote the internal control and establish debt financing mechanism commensurate with the risk under the new market conditions, which will improve the local government bond market and effectivly control the problem of local government debt.

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