Filomat 30:15 (2016), 3963–3974
DOI 10.2298/FIL1615963Z

Published by Faculty of Sciences and Mathematics,
University of Niš, Serbia
Available at: http://www.pmf.ni.ac.rs/filomat

# Game Model of Enterprises and Government Based on the Tax Preference Policy for Energy Conservation and Emission Reduction

**Guoxing Zhang[a], Zhenhua Zhang[a], Yongjing Cui[a], Chun Yuan[a]**

[a]*School of Management, Lanzhou University, Lanzhou University, Lanzhou 730000, People's Republic of China*

**Abstract.** In recent years, greater efforts in tax preference policy for energy conservation and emission reduction (ECER) have been implemented in our country. Based on the tax preference of enterprise income for comprehensive utilization of resources, the constraints to achieve completely successful equilibrium are studied in the single period and multiple periods. In the single period, the key to achieve separating equilibrium is analyzed carefully by constructing a signaling game model of enterprises and government on tax preference of enterprise income. In the multiple periods, with the stochastic evolutionary game model based on the stochastic differential equation (SDE) theory, the constraints of keeping the separating equilibrium stable and continuing in a long term will be further investigated. It gives the optimal number of tax preference of enterprise income, camouflage cost and expected cost of risk under the state of separating equilibrium. The optimal result of completely successful equilibrium is obtained in single period by the analysis of numerical example for enterprises and government signaling game model. The simulation experiment is successfully finished to test the effectiveness of the stochastic evolutionary game model by using mathematical software MATLAB.

## 1. Introduction

According to China's the first release Chinese Annual Report [1], China's industrial solid waste comprehensive utilization rate is nearly 60the annual utilization amount is about 2 billion tons, reducing stockpiling on the ground more than 100 thousand acres by the comprehensive utilization of a variety of solid waste in 2011. It can be said that China has made considerable progress in terms of resource comprehensive utilization for energy conservation and emission reduction (ECER). In order to further promote ECER, Chinese government has promulgated a large number of economic policies, laws, regulations, and executive orders etc. to promote the resource comprehensive utilization, especially tax preference of enterprise income in terms of institutional construction. Twelfth Five-Year Guidance [2] pointed out the need to establish and improve tax preferences measures for encouraging resource comprehensive utilization, and to implement the preferential policies for resource comprehensive utilization. Twelfth Five-Year Energy-Saving [3] clearly

2010 *Mathematics Subject Classification*. Primary 62P20; Secondary 90B50
*Keywords*. signaling game, stochastic evolutionary game, tax preference policy
Received: 15 March 2015; Accepted: 12 May 2016
Communicated by Dr. Alex Maritz and Dr. Charles Xie
Research supported by New Century Excellent Talents Support Program from Education Ministry (NCET-13-0267), Humanities and Social Sciences Planning Foundation of Ministry of Education (15YJA630097), Philosophy and Social Sciences Planning Foundation of Gansu Province (YB003), and Fundamental Research Funds for the Central Universities from Lanzhou University (15LZUJBWYJ040, 15LZUJBWZY119).
*Email address:* zhangzh_10@lzu.edu.cn (Zhenhua Zhang)

put forward the requirement of increasing support efforts on fiscal policy and strictly implementing the existing tax preference policy for resources comprehensive utilization. In order to further clarify the specific conditions of implementing tax preference policy, Jilin, Liaoning, Zhejiang, Shanghai, Shandong, Tianjin and other local governments have formulated implementation details combined with local circumstances in accordance with the relevant provisions of the central government. Economic growth needs energy consumption while it generates carbon emission[4]. China is constantly improving the related tax preference policy and implementation details to promote the development of ECER.

Tax policy is one of the usual means for the government to regulate the market in ECER. Appropriate and reasonable tax policy plays important role in the prevention and control of pollution. In order to alleviate environmental problems brought by development of economic, Pigou [5] already pointed out that the government should impose polluters Pigouvian tax, making up the gap between polluters' private cost and social costs. Despite that pollution taxes still remain unpopular, Hsu et al. [6] thought that the case of pollution taxation is gradually generally becoming stronger with the development of enforcement and monitoring technology. With respect to achieving the socially desired level of pollution, Karp [7] pointed out that charging the ambient tax will be one of the useful ways to measure individual emissions of firms and each firm will pay a unit tax on aggregate emissions. Fullerton and West [8] found that 71The purpose of the western countries that impose carbon tax is to achieve effect of reducing fossil fuel consumption and carbon dioxide emissions through taxation in proportion to their carbon content for fossil fuel products. Baranzini et al. [9] put forward that carbon taxes may be a useful policy option as a cost-effective instrument for reducing emissions, and the main negative impacts could be reduced by the use of the generated fiscal revenues and the design of the tax. Metcalf [10] gave an idea of designing a carbon tax to reduce US greenhouse gas emissions. Gerlagh and Lise [11] showed it to us that carbon taxes accelerate the speed of substitution of carbon-free energy for fossil fuels with induced technological changes. Tol [12] implied that tourist destinations which rely heavily on intercontinental flights or on short-haul flights will see a decline in international tourism numbers because of the effect of high emissions and carbon tax on aviation fuel. Sathre and Gustavsson [13] indicated that higher energy and carbon taxation rates could increase the economic competitiveness of wood construction materials due to both the increased economic value of biomass by-products used to replace fossil fuel and the lower energy cost for material manufacture.

Tax preference is one of the specific tax policies in ECER. Some scholars have studied the mechanism of tax preferences in ECER. On the basis of preferential diesel fuel tax promoting the development of the car, Minjares et al. [14] showed that if the carbon emission control technologies are in place, particulate matter emissions from diesel vehicles will be reduced, thereby maximizing climate benefits. Cansino et al. [15] provided a comprehensive overview of the main tax incentives which are used in the EU-27 member states to promote green electricity. Timilsina et al. [16] referred that government implementing tax credits for solar technology for a long time continually can promote growth of the solar industry in the whole national industries. Proper use of tax preferences can promote the development of the national industry of ECER. Singh [17] pointed out that the fiscal policy of tax preferences has accelerated the development of the cause of India's renewable energy. Markandya et al. [18] presented the results of analyzing the potential cost-effectiveness of different policies for fostering the production and consumption of energy-efficient appliances and found that tax credits on boilers seem to be a cost-effective option in Denmark and Italy. Minter [19] summarized that Chinese government adopted tax preferences and other policy measures to support electronic waste recycler for further development of the national recovery system.

In the implementation of tax preference for ECER, there are actually game behaviors happening about the application and approval of tax preference for enterprises and government. Game theory can be applied to analyzing implementation process of tax preference policy for ECER. Scholars research ECER theory by using game theory at present mainly concentrating on exploring which kind of conditions to be met for different equilibrium, getting stable strategy of equilibrium by analysis of the game, and proposing appropriate policy recommendations from the different results of the game. Liu et al. [20] analyzed the main electricity bidding mechanisms in electricity auction markets for designing a proper bidding mechanism to decrease the generators' market power based on the signaling game theory. Xie et al. [21] built up a model of system dynamics to study evolutionary games between lots of developers and government in the aspect of decision making about greening building, with the results showing that evolutionary equilibrium would not exist under the support of static government incentive. By the analysis of the manufacturing and retailing firms having uncooperative, oligopolistic game through sharing customer demand such that a firm's decisions impact the product prices, Chung et al. [22] found that an important implication is that effects of taxation depend on the oligopolistic game structure. Unfortunately, there are little scholars attempting to systematically expound specific game process between enterprises and government with respect to tax preferences.

Overall, the current research on tax is mainly focused on how to analyze tax effect in the field of ECER, how to adjust the different combinations of taxes and how to understand the role of tax preference policy as part of tax policies etc. It still needs to explain and analyze in more details about exploring application process of tax preference on ECER and researching equilibrium influencing factors by comprehensive application of game theory. Learning from analytical thought of signaling game and stochastic evolutionary game, this paper will attempt to explore the constraints achieving completely successful equilibrium by selecting "the tax preference of enterprise income for comprehensive utilization of resources" as a specific research object in different periods for ECER.

## 2. Model building and analysis

To further clarify specific implementation details on tax preference policy for resource comprehensive utilization in [3], Certified Management Approach [23] that is time-sensitive with specific implementation details, has clarified procedural steps for enterprises applying for tax preferences. Because of its strong representation, this paper will analyze based on [23]. Since tax preference policy for ECER which is settled and cleared within one year (single period) keeps on going in the years (multiple periods), this paper will try to find out the constraints to achieve completely successful equilibrium in the single period and multiple periods for ECER. For the game between enterprises and government in single period, enterprises apply for tax preferences as signal senders; government accepts the request as signal recipient, and then decides whether to grant tax preferences or not. This dynamic Bayesian game with mechanism of information transmission is collectively referred to as signaling game. For the game between enterprises and government in multiple periods, enterprises will continue to achieve game equilibrium by the approach of trial and error; random factors such as game history, institutional factors, and psychological speculation of enterprises as well as some details of equilibrium process will cause interference effect for game strategy selection of both sides. This game with many random factors in decision-making environment is called stochastic evolutionary game. Based on the above analysis, this section will try to build signaling game model in single period and stochastic evolutionary game model in multiple periods respectively between enterprises and government.

### 2.1. *Signaling game model of enterprises and government*

According to the requirement of [23], enterprises should submit an application certifying comprehensive utilization of resources to government. Then, government decides whether to agree with certification application or not depending on the actual situation. Certified enterprises can get authorization book for comprehensive utilization of resources as a qualified certificate to apply for tax preferences. In actual operation, enterprises may misrepresent or overstate products income for comprehensive utilization of resources in order to get more tax preferences after getting the authorization book. Thus, enterprises and government will start signaling game by focusing on applying for and granting tax preferences in single period: first, game side $O$ (nature) randomly selects a type for signal sender $S$ (enterprise) by a certain probability, making $S$ understand its type; then $S$ choose one behavior in its strategy space, namely sending a signal; finally, signal recipient $R$ (government) need to judge its type and choose their own behavior based on the signal sent by $S$.

#### 2.1.1. *The basic setting of model*

(1) According to the contents of Circular on tax preference [24] and Circular of related problems [25], enterprises whose part of the income is eligible to apply for tax preferences (P-enterprises) apply for part of

income tax preferences, indicating that the enterprises strictly abide the provision which is that income for non-comprehensive utilization of resources should be separately accounted with income for comprehensive utilization of resources. Enterprises which do not meet the criteria to apply for any tax preferences (N-enterprises) apply for part of the income tax preferences, indicating that the enterprises pretend in strict compliance with the above provisions. Accordingly, the type set of enterprises is $T = \{$ all the income is eligible to apply for tax preferences $g$, part of the income is eligible to apply for tax preferences $a$ , none of the income is eligible to apply for tax preferences $b$ $\}$. Action space for enterprises is $M = \{$ applying for tax preferences for all the income $h$ ; applying for tax preferences for part of the income $l$ ; applying for tax preferences for none of the income $O$ $\}$.

(2) Set $L_1$ as tax preference provided by government to enterprises for all the income; $L_2$ as tax preference for part of the income; $V_1$ as social benefit obtained by government if all the income of enterprises is eligible to apply for tax preferences; $V_2$ as social benefit obtained by government if part of the income of enterprises is eligible to apply for tax preferences; $O$ as social benefit obtained by government if none of the income of enterprises is eligible to apply for tax preferences; and $V_1 > L_1 > V_2 > L_2 > 0$ .

(3) Based on the consideration of maximizing own interests, enterprises whose all the income is eligible to apply for tax preferences (A-enterprises) will certainly apply for all the income tax preferences with application cost of 0; P-enterprises will also apply for part of the income tax preferences with application cost of 0. $C_1$ is the camouflage cost paid by enterprises if P-enterprises apply for all the income tax preferences; $C_2$ is the camouflage cost paid by enterprises if N-enterprises apply for part of the income tax preferences; $C_3$ is the camouflage cost paid by enterprises if N-enterprises apply for all the income tax preferences; and $C_1 < C_2 < C_3$. According to the provisions of Article 16 in [23], the tax preference will be cancelled and the enterprises will be punished, once discovering that enterprises adopt fraudulent means to have the related tax preferences and certificating qualification for comprehensive utilization of resources. Therefore if the camouflage behaviors of these three enterprises are discovered in the latter re-inspection checks, it will cause losses of $N_1$, $N_2$ and $N_3$ with constraint relations of $N_1 < N_2 < N_3$. The probability that will be found in the latter re-inspection checks is $k\,(0 < k < 1)$. If government refuses to give tax preferences even though the enterprises are eligible to apply for tax preferences, it will cause some damage of initiative for the enterprises adopting behavior of comprehensive utilization of resources in the future; it will also cause additional losses $J$ for government, and $J > L_1$ .

*2.1.2. Game analysis of enterprises and government*

When the government decides to grant tax preferences and the enterprise applies for all the income tax preferences, benefits of the enterprise and government are respectively $(L_1, V_1 - L_1)$ if the enterprise is an A-enterprise; benefits of the enterprise and government are respectively $(L_1 - C_1 - kN_1, V_2 - L_1)$ if the enterprise is a P-enterprise; and benefits of the enterprise and government are respectively $(L_1 - C_3 - kN_3, -L_1)$ if the enterprise is an N-enterprise. When the government decides to grant tax preferences and the enterprise applies for part of the income tax preferences, benefits of the enterprise and government are respectively $(L_2, V_2 - L_2)$ if the enterprise is a P-enterprise; benefits of enterprise and government are respectively $(L_2 - C_2 - kN_2, -L_2)$ if the enterprise is an N-enterprise.

When the government refuses to grant tax preferences and the enterprise applies for all the income tax preferences: benefits of enterprise and government are respectively $(0, V_1 - J)$ if the enterprise is an A-enterprise; benefits of enterprise and government are respectively $(-C_1, V_2 - J)$ if the enterprise is a P-enterprise; and benefits of enterprise and government are respectively $(-C_3, 0)$ if the enterprise is an N-enterprise. When the government refuses to grant tax preferences and the enterprise applies for part of the income tax preferences: benefits of the enterprise and government are respectively $(0, V - J)$ if the enterprise is a P-enterprise; benefits of the enterprise and government are respectively $(-C_2, 0)$ if the enterprise is an N-enterprise.

In addition, in order to make the analysis much closer to reality, set $V_1 - L_1 > V_2 - L_2 > 0$, namely social benefits created by A-enterprise which applies for tax preferences for all the income are better than social benefits created by P-enterprise which applies for tax preferences for part of the income, while the latter social benefits are better than social benefits created by N-enterprise which does not apply for tax preferences. The game process is represented as an extended form shown in Fig. 1.

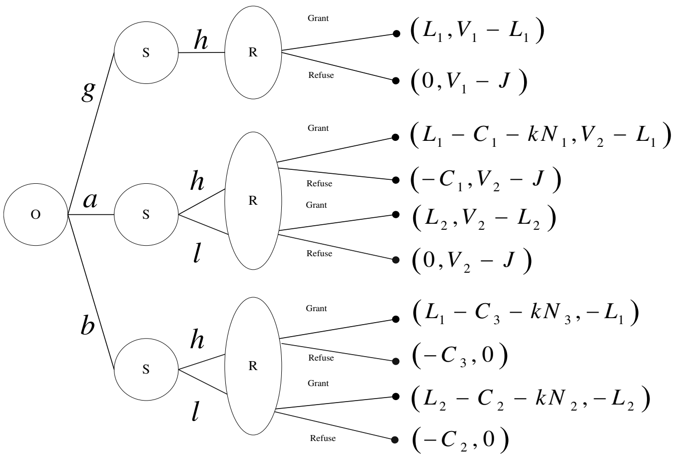


Fig. 1 Extended form for game of enterprises and government

If we set the government as risk-neutral game player, the probabilities of existing A-enterprises, P-enterprises and N-enterprises are respectively $p(g)$, $p(a)$ and $p(b)$, and $p(g) + p(a) + p(b) = 1$.

Expected benefit of government which decides to grant tax preferences for all the income is given by (Eq. 1):

$$Eh = p\,(g/h)\,(V_1 - L_1) + p\,(a/h)\,(V_2 - L_1) + p\,(b/h)\,(-L_1) \tag{1}$$

Expected benefit of government which refuses to grant tax preferences for all the income is that expressed by (Eq. 2):

$$Eh' = p\,(g/h)\,(V_1 - J) + p\,(a/h)\,(V_2 - J) \tag{2}$$

Expected benefit of government which decides to grant tax preferences for part of the income is given by (Eq. 3):

$$El = p\,(p/l)\,(V_2 - L_2) + p\,(a/l)\,(-L_2) \tag{3}$$

Expected benefit of government which refuses to grant tax preferences for part of the income is calculated by (Eq. 4):

$$El' = p\,(a/l)\,(V_2 - J) \tag{4}$$

Among the above four formulas, $p\,(g/h)$, $p(a/h)$ and $p(b/h)$ are respectively conditional probabilities of occurring A-enterprises, P-enterprises and N-enterprises when applying for tax preferences for all the income; similarly, $p(a/l)$ and $p(b/l)$ are respectively conditional probabilities of occurring P-enterprises and N-enterprises when applying for tax preferences for part of the income; and $p(g/h) + p(a/h) + p(b/h) = 1$, $p(a/l) + p(b/l) = 1$.

*2.1.3. Separating equilibrium of completely successful market*

$L_1 - C_1 - kN_1 < L_2$ means that the expected benefit of P-enterprises applying for tax preferences for all the income is not as good as the expected benefit of enterprises honestly applying for tax preferences for part of the income; $L_1 < C_3 + kN_3$ means that the sum of the camouflage cost and expected cost of risk for N-enterprises applying for tax preferences for all the income is greater than the corresponding expected benefit; $L_2 < C_2 + kN_2$ means that the sum of the camouflage cost and expected cost of risk for N-enterprises applying for tax preferences for part of the income is greater than the corresponding expected benefit. The game will achieve perfect Bayesian equilibrium of completely successful market only if the three inequalities above are all satisfied. In this case, the enterprise applying for tax preference for all or part of the income can fully reflect that the enterprise's all or part of the income is eligible for the tax preference. The corresponding judgments which show combination of strategies are as follows: A-enterprise applies for tax preference for all the income, P-enterprise applies for tax preference for part of the income, and N-enterprise does not apply for tax preference; tax preferences will be granted by government according to the actual application forms.

It can be proved by Backward Induction Procedure that the combination of both strategies and corresponding judgments constitute the perfect Bayesian equilibrium. When the enterprise applies for tax preference for all the income, the expected return of government is $Eh = 1\,(V_1 - L_1) + 0\,(V_2 - L_1) + 0\,(-L_1) = V_1 - L_1$ if the government decides to grant tax preference, otherwise, the expected return of government is $Eh' = 1(V_1 - J) + p(a/h)0 = V_1 - J$. When the enterprise applies for tax preference for part of the income, the expected return of government is $El = 1(V_2 - L_2) + 0(-L_2) = V_2 - L_2$ if the government decides to grant tax preference; otherwise, the expected return of government is $El' = V_2 - J$. We get $Eh > Eh'$ and $El > El'$ because of $J > L_1 > L_2$, which means the return is the least if the government refuses to grant tax preference. Therefore, giving is the absolutely best strategy compared with the refusal of granting tax preference for the government.

If the enterprise which applies for tax preference is an A-enterprise, this enterprise is bound to apply for tax preference for all the income with the enterprise return of $L_1$. If the enterprise which applies for tax preference is a P-enterprise, it's reasonable to apply for tax preference for part of the income because of $L_1 - C_1 - kN_1 < L_2$. If the enterprise is an N-enterprise, the best choice is not to apply for any tax preference because of $L_1 < C_3 + kN_3$ and $L_2 < C_2 + kN_2$ (namely $L_1 - C_3 - kN_3 < 0$ and $L_2 - C_2 - kN_2 < 0$).

Therefore, the above strategies of enterprises and government have met rational requirements; judgments of information set on the equilibrium path comply with equilibrium strategies and Bayesian rules of both sides; there is no information set which needs to be judged outside of the equilibrium path. This proves that the strategy combinations and judgments are perfect Bayesian equilibrium.

### 2.2. *Stochastic evolutionary game model between enterprises and government*

Through the equilibrium analysis of signaling game about tax preferences between enterprises and government, this paper has found the key constraints of realizing the separating equilibrium of completely successful market, but the game mainly reflects the case of single period. The method of signal period model can only solve the case of involving certainty factors, but cannot solve the case of involving random factors such as game history, institutional factors, and psychological speculation of enterprises as well as some details of equilibrium process. Because the probability is greater than the game influenced by the random factors, it's necessary to extend the single game research to multiple game researches, so as to analyze the quantitative relationship between the tax preferences given by government and camouflage costs as well as expected costs of risk.

As for the enterprises whose part of the income is eligible to apply for tax preferences, they will only apply for tax preferences for part of the income in single period. But in multiple periods, the possibility of enterprises applying for the tax preferences for all the income or part of the income will change correspondingly because of the interference of random factors. This situation is similar to the interference situation that the enterprises do not meet the standard of applying for tax preferences, which can be analogized out by the same method. In the following analysis, this paper regards the enterprises whose part of the income is eligible to apply for tax preferences as the study objective to explain more specific restrictions among the variables for realizing the separating equilibrium of completely successful market in multiple periods.

#### 2.2.1. *Establishment of stochastic evolution model*

Suppose at time $t$, the probability of the enterprises whose part of income is eligible to apply for tax preferences applying for tax preferences for all the income or applying for tax preferences for part of the income are respectively $x(t)$ or $1 - x(t)$. If the expected benefits of applying for tax preferences for part of the income is greater than those of applying for tax preferences for all the income, $x(t)$ will become smaller. The probability of government granting tax preferences for all the income is $q$. When $x(t)$ becomes smaller, more enterprises will do the application honestly, and the government will reduce the intensity of review, so $q$ will increase. From the perspective of benefits of whole society, when $x(t)$ decreases, $q$ will increase, the whole benefits will increase. Therefore, this paper sets $q = 1 - x$.

Suppose $E_{x(t)}$ is the expected benefits of enterprises whose part of income is eligible to apply for tax preferences applying for tax preferences for all the income, and $E_{x(t)} = q(L_1 - C_1 - kN_1) = (1-x)(L_1 - C_1 - kN_1)$; $E_{1-x(t)}$ is the expected benefits of enterprises whose part of income is eligible to apply for tax preferences applying for tax preferences for part of the income, and $E_{1-x(t)} = L_2$; $\bar{E}$ is the average benefits of $E_{x(t)}$ and $E_{1-x(t)}$,and $\bar{E} = xE_{x(t)} + (1-x)E_{1-x(t)} = x(1-x)(L_1 - C_1 - kN_1) + (1-x)L_2$.

According to the replicator dynamics equation reflecting dynamic change rate of $x(t)$ [26], the change rate can be calculated by (Eq. 5):

$$dx(t) = M(x(t))dt = x(E_{x(t)} - \bar{E})\,dt, t \geq 0 \tag{5}$$

In Eq. (5), $E_{x(t)} - \bar{E}$ only consider the relation between expected benefits of enterprises applying for tax preferences for all the income and average benefits without considering $E_{x(t)} - E_{1-x(t)}$ that is the difference between the expected benefits of enterprises applying for tax preferences for all the income and the expected benefits of enterprises applying for tax preferences for part of the income.

In addition to the main factors such as benefits under stated policy in single period, camouflage costs $C_1$, expected costs of risk $kN_1$ and loss of damaging initiative $J$ , there are still some random factors influencing $x(t)$. We use $\omega(t)$ to represent these random factors. Therefore, referring to the replicator dynamics equation of Xu et al. [27], Eq. (5) can be improved as follows (Eq. 6):

$$dx(t) = M(x(t))dt + d\omega(t) = x(E_{x(t)} - E_{1-x(t)})dt + d\omega(t), t \geq 0 \tag{6}$$

Eq. (6) refers the theory of Ordinary Differential Equations (ODE) to the theory of SDE. $\omega(t)$ is the Brownian motion in one-dimensional Markov process, and $d\omega(t)$ is the Ito differential of random function x(t). Because is random number, it may make $x < 0$ or $x > 1$ in $dx(t)$ which does not match value range of $x$ [0£1]. So we need to introduce random disturbance term to make restrictions on the value range of $x$. Generally taking $\sqrt{x(1-x)}$ in mathematics to reflect the restrictions, we get the final stochastic evolutionary game theory model (Eq. 7):

$$dx(t) = [-(L_1 - C_1 - kN_1)x^2 + (L_1 - C_1 - kN_1 - L_2)x]dt + \sqrt{x(1-x)}d\omega(t) \tag{7}$$

*2.2.2. Analysis of equilibrium solution*

Giving the following type of SDE (Eq. 8):

$$\begin{cases} dx(t) = f(t, x(t))dt + g(t, x(t))d\omega(t) \\ x(t_0) = x_0 \end{cases} \tag{8}$$

$f$ and $g$ are Borel measurable function. Assume certain conditions applied to $f$ and $g$ have ensured the solution of initial value $x(t) = x(0)$ existing uniquely. In order to be facilitate and to avoid loss of generality, when the initial time in game is $t = 0$£we get $x(0) = 0$. Assume $f(t, 0) \equiv 0$, $g(t, 0) \equiv 0$, $t \geq 0$, so $f(t, 0) \equiv 0$ which is the zero solution of Eq. (7).

The zero solution is one kind of equilibrium states for the Eq. (7), which is called equilibrium solution. This indicates that the system initially in zero state will be stay in that state if there is no interference of random factors from outside. Specifically, the game results will remain and keep on going as separating equilibrium of completely successful market in multiple periods if there is no interference of external disturbance factors affecting the game results of tax preferences between enterprises and government in single period. But in the reality of production activities, this kind of best result is rarely achieved because interference of uncertainty factors is always existed. In principle, we should also consider the non-zero equilibrium solution. However, because any non-zero equilibrium solution can be converted into zero solution through a simple translation transformation, it does not lost generality when only considering the zero equilibrium solution.

In order to maximum the overall efficiency of market, namely to realize the separating equilibrium of completely successful market and to stay in stable state in multiple periods, it is necessary to do the stability analysis for SDE in theoretical model (Eq. 7).

*2.2.3. Introduction of the definition of almost surely exponential stability*

The stability of zero solution means: what kind of limit state it is for the solution of SDE when $t \to \infty$; how the limit state depends on the initial value $x_0$. Exponential function has been widely used in the fields of system control with some features such as faster changes and strong convergence and so on. It is a major innovation to use the knowledge of exponential function to solve the problem for the stability of the zero solution at the field of management in recent years. In order to explore the stability problem, this paper use almost surely exponential stability theory of the zero solution of SDE which has much stronger stability than moment exponential stability.

Introducing the definition of the almost surely exponential stability theory of the zero solution:

If $\forall x_0 \in [0,1]$£$x(t)$ has a negative Lyapunov exponent£namely $\overline{\lim_{t\to\infty}} t^{-1} \ln |x(t, x_0)| < 0$ almost surely, we say that it is almost surely exponential stability for the zero solution of Eq. (7).

As for a real random system, almost surely exponential stability is the best stability, because almost all orbits of samples are stable at this time [28]. Different with moment exponential stability, almost surely exponential stability is a kind of orbital stability which depends on the orbit estimate of solution.

*2.2.4. The application of almost surely exponential stability theorem*

In order to get the conclusion of almost surely exponential stability, we need to compare the function $|x(t)|$ with an appropriately chosen function $V(t, x)$ which is Lyapunov function to get the final orbit estimate. According to the almost surely exponential stability theorem given by Hu et al. [29], there is a normal function $V(t, x)$ and the numbers of $k_1$, $k_2$, $k_3$ and $K$, are simultaneously satisfying (Eq. 9):

$$\begin{cases} k_1 |x|^p \le V(t,x) \le k_2 |x|^p \\ LV(t,x) \le -k_3 V(t,x) \\ \left|f(t,x)\right| \vee \left|g(t,x)\right| \le K|x| \end{cases} \tag{9}$$

We get that the zero solution of model (Eq. 8) has features of almost surely exponential stability.

Under the premise of the effectiveness of analytical method for constraints, we assign the following parameters. Set $V(t, x) = x$, $x \in [0, 1]$, $c_1 = c_2 = 1$, and $LV(t, x) = f(t, x)$. Based on theoretical model, Eq. (9) can be expressed by (Eq. 10):

$$-(L_1 - C_1 - kN_1)x^2 + (L_1 - C_1 - kN_1 - L_2)x \le -k_3 x \tag{10}$$

Furthermore, it can be concluded as (Eq. 11):

$$C_1 + kN_1 \le L_1 \le L_2 + C_1 + kN_1 - k_3 \tag{11}$$

In constraints, $L_1 \ge C_1 + kN_1$ means that the sum of camouflage costs $C_1$ and expected costs of risk $kN_1$ is not more than the expected benefits of enterprises whose part of income is eligible to apply for tax preferences applying for tax preferences for all the income. When $L_1 \le L_2 + C_1 + kN_1 - k_3$, the necessary condition $L_1 - C_1 - kN_1 < L_2$ for separating equilibrium is necessarily satisfied in single period of the game, because $k_3$ is a positive number; $L_1 \le L_2 + C_1 + kN_1 - k_3$ namely $C_1 + kN_1 \ge L_1 - L_2 + k_3$ means that the sum of camouflage costs $C_1$ and expected costs of risk $kN_1$ is more than or equal to $L_1 - L_2 + k_3$. The constraint Eq. (11) clears the number size of the camouflage costs and expected risk costs, highlighting that the loss by fraud is the important factor of restricting the enterprise's decision, which can greatly affect the behavior of enterprises. When the constraint conditions are satisfied as an entirety, it can be ensured that the enterprises' behaviors meet the actual situation: P-enterprises will apply for tax preferences for part of the income honestly so that the overall efficiency of market will be maximized. When the constraint conditions are not satisfied, it cannot be ensured that the enterprises' behaviors meet the actual situation, namely may occurring behavior of fraud, causing the efficiency of market reduced greatly.

## 3. Numerical example and simulation experiment

In this section, we use data approximate with reality for numerical example and simulation experiment in order to specifically expound explanatory power of this model for practice. Further analysis will intuitively reflect the correctness of the game theory model in this paper by designing relevant numerical example and simulation experiment.

### 3.1. Analysis of numerical example in single period

The parameters are assigned on the basis of satisfying the constraint conditions in signaling game model of enterprises and government[30]. Considering the following calculation example: enterprise A is an A-enterprise; enterprise B is a P-enterprise; enterprise C is an N-enterprise. In the prerequisite of setting $k = 0.5$, assignment to other parameters are as follows:

Table 1: Parameter assignment for signaling game model (unit: one million Yuan)

| **Variable name** | $L_1$ | $L_2$ | $V_1$ | $V_2$ | $J$ | $C_1$ | $C_2$ | $C_3$ | $N_1$ | $N_2$ | $N_3$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **Parameter assigned** | 11 | 7 | 16 | 10 | 13 | 3 | 4 | 6 | 8 | 10 | 14 |

It will certainly be the most beneficial to apply for all income tax preferences for enterprise A. On the basis of meeting the constraints in separating equilibrium of completely successful market, government can make the judgment below according to the application type of enterprises: As enterprises will certainly apply for tax preference in honest way, enterprise A must satisfy the standard of applying for tax preference for all the income. Therefore, the government will grant the corresponding tax preference to enterprise A. The benefits of enterprise A and government are respectively (11, 5) (unit: one million Yuan).

Similarly, enterprise B will definitely apply for tax preference for part of the income while enterprise C does not apply for tax preference. According to the foregoing constraints, government can also make the judgment below according to the application type of enterprises: enterprise B satisfies the standard of applying for tax preference for part of the income, and enterprise C does not satisfy the standard of applying for any tax preference. Therefore, the benefits of enterprise B and government are respectively (7, 3) (unit: one million Yuan); the benefits of enterprise C and government are (0, 0) (unit: one million Yuan).

In the above analysis about numerical example, the situations of comprehensive utilization of resources are different for the three enterprises: A, B and C. These three enterprises can truthfully apply for tax preferences depending on their own situation without violation and fraud. The reason to achieve the separating equilibrium of completely successful market is that these three kinds of constrains $L_1 - C_1 - kN_1 < L_2$, $L_2 < C_2 + kN_2$ and $L_1 < C_3 + kN_3$ are satisfied at the same time. From the numerical structure of the three conditions, it can be seen that the key parts under different circumstances are the camouflage costs and expected costs of risk $C_1 + kN_1$, $C_2 + kN_2$ and $C_3 + kN_3$. In order to prevent P-enterprises applying for tax preference for all the income, it could be feasible to increase the camouflage and expected risk cost $C_1 + kN_1$ to make it bigger than $L_1 - L_2$ (the difference between tax preference for all the income and tax preference for part of the income). In order to prevent N-enterprises applying for tax preference for part of the income, it could be feasible to increase the camouflage and expected risk cost $C_2 + kN_2$ to make it bigger than $L_2$ (tax preference granted by government for part of the income). In order to prevent N-enterprises applying for tax preference for all the income, it could be feasible to increase the camouflage and expected risk cost $C_3 + kN_3$ to make it bigger than $L_1$ (tax preference granted by government for all the income).

### 3.2. Analysis of simulation experiment in multiple periods

As a method of system modeling, simulation experiments have been widely used in the areas of natural sciences, engineering, social and economic systems. There is great value for simulation experiments in management studies because of its unique research mentality[31]. This paper conducts a simulation experiment for theoretical model in multiple periods by using the latest mathematical software MATLAB, verifying the effectiveness of the theoretical model and the correctness of stability test theorem: regarding the constraint conditions as the precondition, and exploring this precondition can enable the behavior choice of enterprises (the game player) to remain stable state through inverse calculation. Assign the variables of $L_1$, $L_2$ and $C_1 + kN_1$ based constraint conditions:

When $L_1 = 11$, $L_2 = 7$ and $C_1 + kN_1 + 7$,$k_3 \in [1, 3]$ meets the requirement of almost surely exponential stability theorem $C_1+kN_1 \leq L_1 \leq L_2+C_1+kN_1 - k_3$, so $dx(t) = (-4x^2 - 3x)dt + \sqrt{x(1-x)}d\omega(t)$.

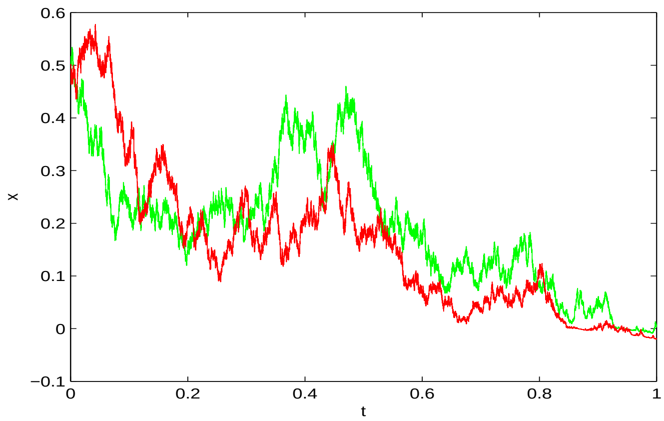


Fig. 2 Simulation result of stochastic evolution

Two curves in the figure above are randomly selected when the constraints are satisfied. According to the change of curves, it can be seen that the two curves show larger fluctuations in the beginning with fluctuation range from 0.2 to 0.7 around. The longer the time is, the smaller the fluctuation range is. In the latter part, the curves are extending to the right side with the trend of less volatility, approaching to zero and keeping relatively stable state to continue at last. This shows that the probability of P-enterprises applying for tax preferences for all the income will be infinitely close to 0 when the time is extended indefinitely. This indicates that behavior of enterprises will be consistent with the actual production situation without choosing the behavior of fraud. The two curves with stabilized tendency ultimately in Fig. 2 meet the constraints, successfully verifying the effectiveness of the theoretical model and the correctness of stability theorem.

In the modern market competition, enterprises put the goal of maximizing their own interests as the basis of adopting strategy. Driven by economic interests, enterprises will choose to adopt operations which can get higher returns when laws and regulations are incomplete or intensity of punishment is insufficient to make enterprises do honest applications. Over time, "be illegal and get high-yield" strategy selection may become inertial behavior of enterprises, which is extremely not conducive to the improvement of healthy market mechanism and environment. However, if the constraint conditions of completely successful market are satisfied in multiple periods, enterprises will take measures which can maximize the benefits of enterprises because of the motivation of "go after profits and avoid loss", namely honestly applying for tax preferences for part of the income. Based on the analysis, government will know enterprises honestly apply for their tax preferences without implementing some unnecessarily complicated review process. This can correspondingly save the cost, achieving genuine maximum of social benefits.

## 4. Conclusions

This paper discusses signaling game model in single period according to the behavior of enterprises applying for enterprise income tax preferences and government giving tax preferences for ECER. By equilibrium analysis of signaling game, this paper found a key condition for achieving separating equilibrium of completely successful market, finding that increasing camouflage cost and expected cost of risk can effectively prevent the generation of enterprise frauds and achieve separating equilibrium of optimal market. Therefore, government should improve reviewed frequency and efficiency, increasing penalties for violations and frauds for the situation of enterprises applying for tax preferences.

On the basis of studying game between enterprises and government to achieve separating equilibrium of completely successful market in single period, the signaling game in single period is promoted and extended to stochastic evolutionary game built up by SDE in multiple periods. This paper regards enterprise benefit in single period as main influencing factors affecting enterprise behavior and random factors as secondary influencing factors affecting enterprise behavior in stochastic evolutionary game model. After analyzing this model, the specific constraints are given to keep operation of tax preference policy continuously efficient in long period of time. And the specific constraints are used to guide government for the development and implementation of tax preference policy, so that enterprises and government can be mutually benefited, maximizing the overall efficiency of market.

In the course of discussing game model between enterprises and government, this paper does not consider the situation that third-party monitoring institution supervises counterfeits of enterprise as well as collusion between government agencies or staff and enterprises. This research point is very significant because of the unique perspective and better approximation of reality. Therefore, in-depth studies can be carried out in this direction in the near future.

**Acknowledgments**

The authors would like to thank the referees for many constructive comments which made this paper more readable.

## References

[1] Chinese Annual Report, Chinese Annual Report for Comprehensive Utilization of Resources, National Development and Reform Commission (2012), Beijing, China.
[2] Twelfth Five-Year Guidance, Twelfth Five-Year Guidance for Comprehensive Utilization of Resources, National Development and Reform Commission (2011), Beijing, China.
[3] Twelfth Five-Year Energy-Saving, Twelfth Five-Year Energy-Saving and Environmental Protection Industry Development Plan, State Council (2012), Beijing, China.
[4] Yi Hu, Dongmei Guo, Mingxi Wang, Xi Zhang, Shouyang Wang, The relationship between energy consumption and economic growth: Evidence from China's industrial sectors, Energies, 2015, 8, 9392-9406.
[5] A.C. Pigou, The economics of welfare, Transaction Publishers (1924), On line at: http://oll.libertyfund.org/titles/1410.
[6] S.L. Hsu, J. Walters, A. Purgas, Pollution tax heuristics: An empirical study of willingness to pay higher gasoline taxes, Energy Policy, 36(2008) 3612-3619.
[7] L. Karp, Nonpoint source pollution taxes and excessive tax burden, Environmental and Resource Economics, 31(2005) 229-251.
[8] D. Fullerton, S.E. West, Tax and subsidy combinations for the control of car pollution, The BE Journal of Economic Analysis and Policy, 10(2010).
[9] A. Baranzini, J. Goldemberg, S. Speck, A future for carbon taxes, Ecological economics, 32 (2000) 395-412.
[10] G.E. Metcalf, Designing a carbon tax to reduce US greenhouse gas emissions, Review of Environmental Economics and Policy, 3(2008) 63-83.
[11] R. Gerlagh, W. Lise, Carbon taxes: a drop in the ocean, or a drop that erodes the stone? The effect of carbon taxes on technological change, Ecological Economics, 54(2005) 241-260.
[12] R.S. Tol, The impact of a carbon tax on international tourism, Transportation Research Part D: Transport and Environment, 12(2007) 129-142.
[13] R. Sathre, L. Gustavsson, Effects of energy and carbon taxes on building material competitiveness, Energy and Buildings, 39(2007) 488-494.
[14] R. Minjares, K. Blumberg, F.P. Sanchez, Alignment of policies to maximize the climate benefits of diesel vehicles through control of particulate matter and black carbon emissions, Energy Policy, 54 (2013) 54-61.

[15] J.M. Cansino, M.D.P. Pablo-Romero, R. Rom¢n, R. Y?iguez, Tax incentives to promote green electricity: an overview of EU-27 countries, Energy Policy, 38(2010) 6000-6008.
[16] G.R. Timilsina, L. Kurdgelashvili, P.A. Narbel, Solar energy: Markets, economics and policies, Renewable and Sustainable Energy Reviews, 16(2012) 449-465.
[17] A. Singh, A market for renewable energy credits in the Indian power sector, Renewable and Sustainable Energy Reviews, 13(2009) 643-652.
[18] A. Markandya, R.A. Ortiz, S. Mudgal, B. Tinetti, Analysis of tax incentives for energy-efficient durables in the EU, Energy Policy, 37(2009) 5662-5674.
[19] A. Minter, E-Recycling in China, Scrap, 62(2005) 58-67.
[20] Z. Liu, X. Zhang, J. Lieu, Design of the incentive mechanism in electricity auction market based on the signaling game theory, Energy, 35(2010) 1813-1819.
[21] X. Xie, Y. Liu, J. Hou, An analysis on behaviors of real estate developers and government in sustainable building decision making", Journal of Industrial Engineering and Management, 7(2014) 491-505.
[22] S.H. Chung, R.D. Weaver, T.L. Friesz, Strategic response to pollution taxes in supply chain networks: Dynamic, spatial, and organizational dimensions, European Journal of Operational Research, 231(2013) 314-327.
[23] Certified Management Approach, Certified Management Approach of Comprehensive Utilization of Resources in Tianjin from 2012 to 2017, Tianjin municipal government (2012), Tianjin, China.
[24] Circular on tax preference, Circular on tax preference of enterprise income management problems for comprehensive utilization of resources, State Taxation Administration (2009), Beijing, China.
[25] Circular of related problems, Circular of related problems on implementation of tax preference of enterprise income directory for comprehensive utilization of resources, Ministry of Finance and State Taxation Administration (2008), Beijing, China.
[26] E. Amann, A. Possajennikov, On the stability of evolutionary dynamics in games with incomplete information, Mathematical Social Sciences, 58(2009) 310-321.
[27] Y. Xu, B. Hu, R. Qian, Analysis on stability of strategic alliances based on stochastic evolutionary game including simulations (in Chinese), Systems Engineering - Theory and Practice, 31(2011) 920-926.
[28] X. Hu, Stability Theory for Stochastic Differential Equations (in Chinese), Nanjing University Press (1986), Nanjing, China.
[29] S. Hu, C. Huang, F. Wu, Stochastic Differential Equation (in Chinese), Science Press (2008), Beijing, China.
[30] F. WEN, X. GONG, Y. CHAO, X. CHEN, The Effects of Prior Outcomes on Risky Choice Evidence from the Stock Market, Mathematical Problems in Engineering, 2014 1-8
[31] Z. DAI, F. WEN, Robust CVaR-based portfolio optimization under a genal affine data perturbation uncertainty set, Journal of Computational Analysis and Applications, 16(2014) 93-103.