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Infinite Matrices and Some Matrix Transformations

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Abstract. The goal of this paper is to define the spaces $V_{\sigma_0}^{\lambda}(p)$ and $V_{\sigma}^{\lambda}(p)$ by using de la Vallée Poussin and invariant mean. Furthermore, we characterize certain matrices in V_{σ}^{λ} which will up a gap in the existing literature.

1. Introduction and Background

Let w denote the set of all real and complex sequences $x = (x_k)$. By l_∞ and c, we denote the Banach spaces of bounded and convergent sequences $x = (x_k)$ normed by $\|x\| = \sup_k |x_k|$, respectively. A linear functional L on l_∞ is said to be a Banach limit [1] if it has the following properties:

1. $L(x) \geq 0$ if $n \geq 0$ (i.e. $x_n \geq 0$ for all n),
2. $L(e) = 1$ where $e = (1, 1, \ldots)$,
3. $L(Dx) = L(x)$, where the shift operator D is defined by $D(x_n) = \{x_{n+1}\}$.

Let B be the set of all Banach limits on l_∞. A sequence $x \in \ell_\infty$ is said to be almost convergent if all Banach limits of x coincide. Let $\hat{c}$ denote the space of almost convergent sequences. Lorentz [3] has shown that

$$\hat{c} = \left\{ x \in l_\infty : \lim_m d_{m,n}(x) \text{ exists uniformly in } n \right\},$$

where

$$d_{m,n}(x) = \frac{x_n + x_{n+1} + x_{n+2} + \cdots + x_{n+m}}{m+1}.$$

If p_k is real and $p_k > 0$, we define (see, Maddox [4])

$$c_0(p) = \left\{ x : \lim_{k\to\infty} |x_k|^{p_k} = 0 \right\}$$

and

$$c(p) = \left\{ x : \lim_{k\to\infty} |x_k - l|^{p_k} = 0, \text{ for some } l \right\}$$

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If p_m is real such that $p_m > 0$ and $\sup p_m < \infty$, we define (see, Nanda [16])

$$\hat{c_0}(p) = \left\{ x : \lim_{m\to\infty} \left| d_{m,n}(x) \right|^{p_m} = 0, \text{ uniformly in } n \right\}$$

and

$$\hat{c}(p) = \left\{ x : \lim_{m\to\infty} \left| d_{m,n}(x - l) \right|^{p_m} = 0, \text{ for some } l, \text{ uniformly in } n \right\}.$$

Shaefer [26] defined the σ-convergence as follows: Let σ be a one-to-one mapping from the set of natural numbers into itself. A continuous linear functional ϕ on l_∞ is said to be an invariant mean or a σ-mean provided that

(i) $\phi(x) \geq 0$ when the sequence $x = (x_k)$ is such that $x_k \geq 0$ for all k,

(ii) $\phi(e) = 1$, where $e = (1, 1, 1, \ldots)$, and

(iii) $\phi(x) = \phi(x_{\sigma(k)})$ for all $x \in l_\infty$.

We denote by V_σ the space of σ-convergent sequences. It is known that $x \in V_\sigma$ if and only if

$$\frac{1}{m} \sum_{k=1}^{m} x_{\sigma^k(n)} \to \text{ a limit}$$

as $m \to \infty$, uniformly in n. Here $\sigma^k(n)$ denotes the k-th iterate of the mapping σ at n.

A σ-mean extends the limit functional on c in the sense that $\phi(x) = \lim x$ for all $x \in c$ if and only if σ has no finite orbits, that is to say, if and only if , for all $n > 0, k \geq 1$ $\sigma^k(n) \neq n$.

In case σ is the translation mapping $n \to n + 1$, a σ-mean reduces to the unique Banach limit and V_σ reduces to $\hat{c}$.

2. (σ, λ)-Convergence

We define the following:

Let $\lambda = (\lambda_m)$ be a non-decreasing sequence of positive numbers tending to ∞ such that

$$\lambda_{m+1} \leq \lambda_m + 1, \quad \lambda_1 = 1.$$

A sequence $x = (x_k)$ of real numbers is said to be (σ, λ)- convergent to a number L if and only if $x \in V_\sigma^\lambda$, where

$$V_\sigma^\lambda = \{x \in l_\infty : \lim_{m\to\infty} t_{mn}(x) = L \text{ uniformly in n}; L = (\sigma, \lambda) - \lim x\},$$

$$t_{mn}(x) = \frac{1}{\lambda_m} \sum_{i \in I_m} x_{\sigma^i(n)},$$

and $I_m = [m - \lambda_m + 1, m]$ (see, [24]). Note that $c \subset V_\sigma^\lambda \subset l_\infty$. For $\sigma(n) = n + 1$, V_σ^λ is reduced to the space $\hat{V}_\lambda$ of almost λ-convergent sequences and if we take $\sigma(n) = n + 1$ and $\lambda = n$, V_σ^λ reduce to $\hat{c}$(see, [16]) .

It is quite natural to expect that the sequence V_σ^λ and $V_{\sigma_0}^\lambda$ can be extended to $V_\sigma^\lambda(p)$ and $V_{\sigma_0}^\lambda(p)$ just as $\hat{c}$ and $\hat{c}_0$ were extended to $\hat{c}(p)$ and $\hat{c}_0(p)$ respectively.

The main object of this paper is to study $V_\sigma^\lambda(p)$ and $V_{\sigma_0}^\lambda(p)$ (the definitions are given below) and characterize certain matrices in $V_\sigma^\lambda(p)$.

If p_m is real such that $p_m > 0$ and $\sup p_m < \infty$, we define

$$V_{\sigma_0}^\lambda(p) = \left\{ x : \lim_{m\to\infty} \left| t_{m,n}(x) \right|^{p_m} = 0, \text{ uniformly in } n \right\}$$

and

$$V_{\sigma}^{\lambda}(p) = \left\{x : \lim_{m\to\infty} \left|t_{m,n}(x - le)\right|^{p_m} = 0, \text{ for some } l \text{ , uniformly in } n\right\}.$$

In particular, if $p_m = p > 0$ for all m, we have $V_{\sigma_0}^{\lambda}(p) = V_{\sigma_0}^{\lambda}$ and $V_{\sigma}^{\lambda}(p) = V_{\sigma}^{\lambda}$. In Theorem 4, we prove that $V_{\sigma_0}^{\lambda}(p)$ and $V_{\sigma}^{\lambda}(p)$ are complete linear topological spaces. Theorem 7 characterizes the matrices in the class $(c_0(p), V_{\sigma_0}(p))$. In Theorem 8 we determine the matrix in the class $(c(p), V_{\sigma}^{\lambda})$. Matrix transformations between sequence spaces have also been discussed by Savas and Mursaleen ([23]), Mursaleen ([7–15]), Savas ([17–22, 25]) and many others.

A linear topological space X is called paranormed space if there exists a subadditive function $g : X \to \mathbb{R}^+$ such that $g(0) = 0, g(x) = g(-x)$ and the multiplication is continuous, that is, $\lambda_n \to \lambda$ and $g(x_n - x) \to 0$ imply that $g(\lambda_n x_n - \lambda x) \to 0$ for λ's $\in \mathbb{C}$ and $x \in X$.

Suppose that $M = \max(1, \sup\ p_m = H)$. Since $p_m/M \leq 1$, we have for all m and n

$$|t_{mn}(x + y)|^{p_m/M} \leq |t_{mn}(x)|^{p_m/M} + |t_{mn}(y)|^{p_m/M} \tag{1}$$

and for all $\lambda \in \mathbb{C}$

$$|\lambda|^{p_m/M} \leq \max(1, |\lambda|). \tag{2}$$

By using (1) and (2) we can see that$V_{\sigma_0}^{\lambda}(p)$ and $V_{\sigma}^{\lambda}(p)$ are linear spaces.

3. Main Results

We first establish a number of lemmas about $V_{\sigma_0}^{\lambda}(p)$ and $V_{\sigma}^{\lambda}(p)$.

Lemma 3.1. *$V_{\sigma_0}^{\lambda}(p)$ is a linear topological space paranormed by g where*

$$g(x) = \sup_{m,n} \left|t_{m,n}(x)\right|^{p_m/M}.$$

Proof. One can easily see that $g(0) = 0$ and $g(x) = g(-x)$. The subadditivity of g follows from (1). It remains to show that the scalar multiplication is continuous. It follows from (2) that for $\mu \in \mathbb{C}$ and $x \in V_{\sigma_0}^{\lambda}(p)$

$$g(\mu x) \leq \max(1, \mu) g(x).$$

Therefore $\mu \to 0, x \to 0 \Rightarrow \mu x \to 0$ and if μ is fixed, $x \to 0 \Rightarrow \mu x \to 0$. If $x \in V_{\sigma_0}^{\theta}(p)$ is fixed, given $\varepsilon > 0$, there exists m_0 such that

$$\sup_{m>m_0} \left|\mu t_{m,n}(x)\right|^{p_m/M} < \varepsilon/2, \tag{3}$$

for all n and we can choose $\delta > 0$ such that for $\left|\mu\right| < \delta$, we have

$$\sup_{m\leq m_0} \left|\mu t_{m,n}(x)\right|^{p_m/M} < \varepsilon/2, \tag{4}$$

for all n. Thus from (3) and (4) we get

$$\left|\mu\right| < \delta \Rightarrow g(\mu x) \leq \varepsilon.$$

This completes the proof. □

Lemma 3.2. *$V_{\sigma}^{\lambda}(p)$ is a linear topological space paranormed by g, if* $\inf p_m > 0$.

Proof. It is enough to show that for fixed $x \in V_{\sigma}^{\lambda}(p)$, $\mu \to 0 \Rightarrow \mu x \to 0$. Let $\inf p_m = p' > 0$, then we have

$$g(\mu x) \leq \max(\left|\mu\right|, \left|\mu\right|^{p'})g(x).$$

The result follows from the above inequality. □

Lemma 3.3. $V_{\sigma_0}^{\lambda}(p)$ *and* $V_{\sigma}^{\theta}(p)(\inf p_m > 0)$ *are complete with respect to their paranorm topologies.*

Proof. Let $\left\{x^i\right\}$ be Cauchy sequence in $V_{\sigma_0}^{\lambda}(p)$. Then $\left\{x_k^i\right\}$ for each k, is Cauchy in $\mathbb{C}$ and hence $x_k^i \to x_k^0$ for each k. Put $x^0 = \left\{x_k^0\right\}$. Given $\varepsilon > 0$, there exists N_0 such that for $i, j > N_0$,

$$\left|t_{m,n}(x^i - x^j)\right|^{p_m/M} < \varepsilon/5 \tag{5}$$

for all m and n. Taking limit as $j \to \infty$ we get

$$\left|t_{m,n}(x^i - x^0)\right|^{p_m/M} < \varepsilon/5, \tag{6}$$

for all m and n. Therefore $(x^i - x^0)$ and by linearity $x^0 \in V_{\sigma_0}^{\lambda}(p)$. If $\left\{x^i\right\}$ be Cauchy sequence in $V_{\sigma}^{\lambda}(p)$ then there exists x^0 such that $x^j \to x^0$. We now show that $x^0 \in V_{\sigma}^{\lambda}$. Since $x^i \in V_{\sigma}^{\lambda}(p)$ there exists $l^i \in \mathbb{C}$ such that

$$\left|t_{m,n}(x^i - l^i e)\right|^{p_m/M} < \varepsilon/5, \tag{7}$$

for all m and n. From that (5), (7) and (1) it follows that

$$\left|t_{m,n}(l^i e - l^j e)\right|^{p_m/M} < 3/5\varepsilon.$$

Thus $\left\{l^i\right\}$ is Cauchy in $\mathbb{C}$ and therefore there exists $l \in \mathbb{C}$ such that

$$\left|t_{m,n}(l^i e - le)\right|^{p_m/M} < 3/5\varepsilon. \tag{8}$$

Now by (1), (6), (7) and (8) we get

$$\left|t_{m,n}(x^0 - le)\right|^{p_m/M} < \varepsilon .$$

This completes the proof. □

Combining the above lemmas we have

Theorem 3.4. $V_{\sigma_0}^{\theta}(p)$ *and* $V_{\sigma}^{\theta}(p)(\inf p_m > 0)$ *are complete linear topological spaces paranormed by* g *as defined in Lemma 1.*

In general g is not a norm. If $p_m = p$ for all m then clearly g is a norm.

The following proposition give inclusion relations among the spaces $V_{\sigma_0}^{\lambda}(p)$ and $V_{\sigma}^{\lambda}(p)$. These are routine verifications and therefore we omit the proofs.

Proposition 3.5. *If* $0 < p_m \leq q_m < \infty$, *then*

(*i*) $V_{\sigma_0}^{\lambda}(p) \subset V_{\sigma_0}^{\lambda}(q)$

(*ii*) $V_{\sigma}^{\lambda}(p) \subset V_{\sigma}^{\lambda}(q)$.

For $r > 0$, a nonempty subset U of a linear space is said to be absolutely r-convex if $x, y \in U$ and $|\alpha|^r + |\mu|^r \leq 1$ together imply that $\alpha x + \mu y \in U$. A linear topological space X is said to be r-convex (see Maddox and Roles[5]) if every neighbourhood of $0 \in X$ contains as absolutely r-convex neighbourhood of $0 \in X$. We have:

Proposition 3.6. *$V^{\lambda}_{\sigma_0}(p)$ and $V^{\lambda}_{\sigma}(p)$ are 1-convex.*

Proof. If $0 < \delta < 1$, then

$$U = \{x : g(x) \leq \delta\}$$

is an absolutely 1-convex set, for let $a, b \in U$ and $|\alpha| + |\mu| \leq 1$, then

$$g(\alpha a + \mu b) \leq \left(|\alpha| + |\mu|\right)^{p_m/M} \delta \leq \delta.$$

This completes the proof. □

Let X and Y be two nonempty subsets of the space w of complex sequences. Let $A = (a_{nk}), (n, k = 1, 2, ...)$ be an infinite matrix of complex numbers. We write $Ax = (A_n(x))$ if $A_n(x) = \sum_k a_{nk} x_k$ converges for each n.(Throughout $\sum_k$ denotes summation over k from $k = 1$ to $k = \infty$). If $x = (x_k) \in X \Rightarrow Ax = (A_n(x)) \in Y$ we say that A defines a matrix transformation from X to Y and we denote it by $A : X \to Y$. By (X, Y) we mean the class of matrices A such that $A : X \to Y$.

We now characterize the matrices in the class $\left(c_0(p), V^{\lambda}_{\sigma_0}(p)\right)$. We write

$$t_{m,n}(Ax) = \sum_k a(n, k, m)\, x_k$$

where

$$a(n, k, m) = \frac{1}{\lambda_m} \sum_{i \in I_n} a_{\sigma^i(n),k}.$$

Theorem 3.7. *$A \in (c_0(p), V^{\lambda}_{\sigma_0}(p)$ if and only if*

(*i*) *there exists an integer $B > 1$ such that*
$$C_n = \sup_m \left\{\textstyle\sum_k |a(n, k, m)|\, B^{-1/p_k}\right\}^{p_m} < \infty, \ (\forall n)$$

(*ii*) $\lim_{m\to\infty} |a(n, k, m)|^{p_m} = 0$ *uniformly in n.*

Proof. Necessity. Suppose that $A \in \left(c_0(p), V^{\lambda}_{\sigma_0}(p)\right)$. Define $e_k = \{\delta_{nk}\}_n$ where $\delta_{nk} = 0\,(n \neq k), = 1\,(n = k)$. Since $e_k \in c_0(p)$, (ii) must hold. Fix $n \in Z^+$. Put $f_{m,n}(x) = \left|t_{m,n}(Ax)\right|^{p_m}$. Now $\{f_{m,n}\}_m$ is a sequence of continuous linear functionals such that $\lim_m f_{m,n}(x)$ exists. Therefore by uniform boundedness principle for $0 < \delta < 1$, there exists $S_\delta[0] \subset c_0(p)$ and a constant K such that $f_{m,n}(x) \leq K$ for each m and $x \in S_\delta[0]$. Define for each r:

$$y^{(r)}_k = \left(\begin{array}{cc} \delta^{K/p_k} sgn(a(n, k, m), & 0 \leq k \leq r; \\ 0, & r < k. \end{array}\right.$$

Now $y^{(r)}_k \in S_\delta[0]$ and

$$\left\{\sum_{k=1}^{r} |a(n, k, m)|\, B^{-1/p_k}\right\}^{p_m} \leq K$$

for each m and each m where $B = \delta^{-K}$. Therefore (i) holds and this proves this necessity.

Sufficiency. Suppose that the conditions (i) and (ii) hold and that $x \in c_0(p)$ .Fix $n \in \mathbb{Z}^+$. Given $\varepsilon > 0$, there exists k_0 such that for k and m both larger than k_0,

$$B^{1/p_k} |x_k| < (\varepsilon/C_n)^{1/p_m}.$$

We have, for $C = \max(1, 2^{H-1})$ the inequality (see Maddox 7, p. 346)

$$\left|t_{m,n}(A(x))\right|^{p_m} \leq C(S_1 + S_2),$$

where

$$S_1 = \left|\sum_{k\leq k_0} a(n,k,m)x_k\right|^{p_m}$$

and

$$S_2 = \left|\sum_{k>k_0} a(n,k,m)x_k\right|^{p_m}.$$

Since (ii) holds there exists $m_0 \in \mathbb{Z}^+$ such that for $m > m_0$, $|a(n,k,m)| < \varepsilon^{1/p_m}$. Therefore for such m,

$$\begin{aligned} S_1 &\leq \left(\sum_{k\leq k_0} |a(n,k,m)x_k|\right)^{p_m} < \varepsilon \left(\sum_{k\leq k_0} |x_k|\right)^{p_m} \\ &< \varepsilon \max\left[1, \left(\sum_{k\leq k_0} |x_k|\right)^M\right]. \end{aligned} \tag{9}$$

Again for $m > m_0$

$$S_2^{1/p_m} \leq \sum_{k>k_0} |a(n,k,m)x_k| < \varepsilon^{1/p_m},$$

and consequently

$$S_2 \leq \varepsilon, (\forall m > m_0). \tag{10}$$

Hence the sufficiency follows from (9) and (10). This completes the proof. □

We now have

Theorem 3.8. *$A \in (c(p), V_\sigma^\lambda)$ if and only if*

(*i*) *there exists some integer $B > 1$ such that*
$D_n = \sup_m \sum_k |a(n,k,m)| B^{-1/p_k} < \infty, (\forall n)$;

(*ii*) *there exists $\alpha_k \in C$ such that $\lim_{m\to\infty} a(n,k,m) = \alpha_k$ uniformly in n;*

(*iii*) *there exists $\alpha \in C$ such that $\lim_{m\to\infty} \sum_k a(n,k,m) = \alpha$ uniformly in n.*

Proof. Necessity. Let $A \in (c(p), V_\sigma^\theta)$. Since e_k and e are in c(p), (ii) and (iii) must hold. Fix $n \in \mathbb{Z}^+$. Put $\sigma_{m,n}(x) = t_{m,n}(Ax)$. Since $(c(p), V_\sigma^\theta) \subset (c_0(p), V_\sigma^\theta), (\sigma_{m,n})_m$ is a sequence of continuous linear functionals on $c_0(p)$, such that $\lim \sigma_{m,n}(x)$ exists uniformly in n. Therefore as in the necessity part of Theorem 7 the result follows from uniform boundedness principle.

Sufficiency. Suppose that conditions $(i)-(iii)$ hold and $x \in c(p)$. Then there exists l such that $|x_k \to l|^{p_k} \to 0$. Hence given $0 < \varepsilon < 1, \exists k_0 : \forall k < k_0$

$$|x_k \to l|^{p_k/M} \leq \frac{\varepsilon}{B(2D_n + 1)} < 1$$

and therefore for $k < k_0$

$$\begin{aligned} B^{1/p_k} |x_k \to l| &< B^{M/p_k} |x_k \to l| \\ &< (\varepsilon/2D_n + 1)^{M/p_k} < \varepsilon/2D_n + 1. \end{aligned}$$

By (i) and (ii) we have

$$\sum_k |a(n,k,m) - \alpha_k| B^{-1/p_k} < 2D_n.$$

Hence

$$\sum_{k>k_0} |(a(n,k,m) - \alpha_k)(x_k - l)| < \varepsilon. \tag{11}$$

Also,

$$\lim \sum_{k \leq k_0} |(a(n,k,m) - \alpha_k)(x_k - l)| = 0, \tag{12}$$

uniformly in n. Therefore by (11) and (12) we get

$$\lim \sum_k a(n,k,m)x_k = l\alpha + \sum_k \alpha_k (x_k - l) \tag{13}$$

uniformly in n. This completes the proof. □

Corollary 3.9. *$A \in (c_0(p), V_\sigma^\lambda)$ if and only if conditions (i) and (ii) of Theorem 7 hold.*

We write $(c(p), V_\sigma^\lambda, P)$ to denote the subset of $(c(p), V_\sigma^\lambda)$ such that Ax is (σ, λ)- convergent to the limit of x in $c(p)$. We now consider the class $(c(p), V_\sigma^\lambda, P)$.

Theorem 3.10. *$A \in (c(p), V_\sigma^\lambda, P)$ if and only if (i) the condition of Theorem 8 holds; (ii) $\lim a(n,k,m) = 0$ uniformly in n; (iii) $\sum_k a(n,k,m) = 1$, uniformly in n.*

Proof. Let $A \in (c(p), V_\sigma^\lambda, P)$. Then the conditions hold by Theorem 3. Let the conditions $(i) - (iii)$ hold. Then by Theorem 8 $A \in (c(p), V_\sigma^\lambda)$ and (13) reduces to

$$\lim \sum_k a(n,k,m)x_k = l$$

uniformly in n. This proves the theorem. □

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