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Using Bezier Curves in Medical Applications

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Abstract. In this paper we survey the $3D$ reconstruction of an object from its $2D$ cross-sections has many applications in different fields of sciences such as medical physics and biomedical applications. The aim of this paper is to give not only the Bezier curves in medical applications, but also by using generating functions for the Bernstein basis functions and their identities, some combinatorial sums involving binomial coefficients are deriven. Finally, we give some comments related to the above areas.

1. Introduction and Main Definition

B-splines have many applications in Computer Aided Geometric Design (CAGD). And also $3D$ reconstruction of an object from its $2D$ cross-sections has many applications in different fields of sciences such as medical physics and biomedical applications. In order to perform $3D$ reconstruction, at first, desired boundaries at each slice are detected and then using a correspondence between points of successive slices surface of desired object is reconstructed. In many applications of medical physics, biomedical engineering, and Computer Aided Design (CAD), an object is often known by a sequence of $2D$ cross sections (slices) (*cf.* [9], [8], [6], [13]; see also the references cited in each of these earlier works). These slices can be obtained using several imaging techniques such as Computed Tomography (CT), Magnetic Resonance Imaging (MRI), Single Photon Emission Tomography (SPECT), and Positron Emission Tomography (PET).

In this paper, we study the properties of the Bezier curves and the Bernstein basis functions. By using generating functions for the Bernstein basis functions, we derive a new identity. By using this identity and the Marsden's identity, we find some combinatorial sums involving binomial coefficients. It is well-known that combinatorial sums and combinatorial problems appear in many areas of mathematics, in statistic, in probability theory, in computer science and also in statistical physics. These sums are used in computer science to derive formulas and also estimates in the analysis of algorithms (*cf.* [2], [7], [12], [16]; see also the references cited in each of these earlier works).

The Bernstein basis functions $B_k^n(x)$ are defined as follows (*cf.* [2], [11]; see also the references cited in each of these earlier works):

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Definition 1.1. *Let $x \in [0,1]$. Let n and k be non-negative integers. The Bernstein basis functions $B_k^n(x)$ can be defined by*

$$B_k^n(x) = \binom{n}{k} x^k (1-x)^{n-k}, \tag{1}$$

where $k = 0, 1, \ldots, n$ and $\binom{n}{k} = \frac{n!}{k!(n-k)!}$.

The Bernstein basis polynomials are used to construct the Bezier curves (*cf.* [2], [16]).

The Bezier curves of degree n can be defined as follows. Given points $P_0, P_1, P_2, \cdots, P_n$, for $0 \leq t \leq 1$, the Bezier curve is defined by

$$B(t) = \sum_{k=0}^{n} P_k B_k^n(t).$$

The points P_j are called control points for the Bezier curve.

Linear Bezier curves (*cf.* [2], [16]):

Let P_0 and P_1 be control points. A linear Bezier curve is simply a straight line between those two points. For $0 \leq t \leq 1$, this curve is given by

$$B(t) = (1-t)P_0 + tP_1.$$

Quadratic Bezier curves (2D Bezier curve)(*cf.* [2], [16]):

Let P_0, P_1 and P_2 be control points. A quadratic Bezier curve is defined by

$$B(t) = (1-t)^2 P_0 + 2t(1-t)P_1 + t^2 P_2.$$

Let $P_0 = (a_0, b_0)$, $P_1 = (a_1, b_1)$ and $P_2 = (a_2, b_2)$. Thus the above curve is given explicitly by

$$B(t) = (x, y),$$

where $x = a_0 + 2(a_1 - a_0)t + (a_0 + a_2 - 2a_1)t^2$ and $y = b_0 + 2(b_1 - b_0)t + (b_0 + b_2 - 2b_1)t^2$.

Cubic Bezier curves (3D Bezier curve)(*cf.* [2], [16]):

Let P_0, P_1, P_2 and P_3 be control points. A quadratic Bezier curve is defined by

$$B(t) = (1-t)^3 P_0 + 3t(1-t)^2 P_1 + 3t^2(1-t)P_2 + t^3 P_3.$$

Let $P_0 = (a_o, b_0, c_0)$, $P_1 = (a_1, b_1, c_1)$, $P_2 = (a_2, b_2, c_2)$ and $P_3 = (a_3, b_3, c_3)$. Thus the above curve is given explicitly as follows:

$$B(t) = (x, y, z),$$

where $x = a_0 + 3(a_1 - a_0)t + (3a_0 + 3a_2 - 6a_1)t^2 + (3a_3 + 3a_1 - a_0 - 3a_2)t^3$, $y = b_0 + 3(b_1 - b_0)t + (3b_0 + 3b_2 - 6b_1)t^2 + (3b_3 + 3b_1 - b_0 - 3b_2)t^3$ and $z = c_0 + 3(c_1 - c_0)t + (3c_0 + 3c_2 - 6c_1)t^2 + (3c_3 + 3c_1 - c_0 - 3c_2)t^3$.

Four points P_0, P_1, P_2 and P_3 in higher-dimensional space define a cubic Bezier curve.

Throughout of this paper, we need the following formulas:

The *Beta function* $B(\alpha, \beta)$ is a function, which is defined by

$$B(\alpha, \beta) = \int_0^1 t^{\alpha-1}(1-t)^{\beta-1} dt = B(\beta, \alpha), \tag{2}$$

where $\Gamma(s)$ denotes the gamma function and α and β are complex numbers with positive real parts (*cf.* [14, p. 9, Eq-(60)]). These two functions have the following elegant relation:

$$B(n, m) = \frac{\Gamma(n)\Gamma(m)}{\Gamma(n+m)} = \frac{(n-1)!(m-1)!}{(n+m-1)!}, \tag{3}$$

where m and n are positive integers (*cf.* [14]).

2. Combinatorial Sums Involving Binomial Coefficients

The combinatorial sums involving the binomial coefficients have been used all branches of Mathematics, particularly including Statistics, Probability, Combinatorics, Computer Algorithm, Discrete mathematics and CAGD. Applying the Riemann integral, the beta function and the gamma function to the identities of the Bernstein basis functions, we derive some new combinatorial sums involving binomial coefficients and sums of inverse binomial coefficients.

The generating functions for the Bernstein basis functions is given by

$$\frac{(xt)^k e^{(1-x)t}}{k!} = \sum_{n=0}^{\infty} B_k^n(x)\frac{t^n}{n!} \tag{4}$$

where t is a complex variable, x is a real number and $k = 0, 1, \cdots, n$ (*cf.* [11]; see also the references cited in each of these earlier work). Replacing x by $-x$ in (4), we have

$$\frac{(-xt)^k e^{(1+x)t}}{k!} = \sum_{n=0}^{\infty} B_k^n(-x)\frac{t^n}{n!}. \tag{5}$$

Combining (4 and (5), we get

$$\left(\sum_{n=0}^{\infty} B_k^n(-x)\frac{t^n}{n!}\right)\left(\sum_{n=0}^{\infty} B_k^n(x)\frac{t^n}{n!}\right) = \frac{\left(-x^2t^2\right)^k e^{2t}}{(k!)^2}.$$

By using Cauchy product and the series representation e^{2t} in the above equation, after comparing the coefficients of $\frac{t^n}{n!}$ on both sides of this equation, we arrive at the following theorems:

Theorem 2.1. *We have*

$$\sum_{j=0}^{n} \binom{n}{j} B_k^j(x) B_k^{n-j}(-x) = \frac{(-1)^k (2k)! 2^n}{(k!)^2} x^{2k} \binom{n}{2k}. \tag{6}$$

Integrate both sides of Equation (6) from 0 to 1, we find the following theorem:

Theorem 2.2. *Let $n, k \in \mathbb{N}$ with $k \leq n$. Then we have*

$$\begin{aligned} &\sum_{j=0}^{n}\sum_{l=0}^{j}\sum_{d=0}^{n-j-k} \frac{(-1)^l}{l+d+2k+1}\binom{n}{j}\binom{n-j}{k}\binom{j}{l}\binom{n-j-k}{d} \\ = \quad & \frac{(-1)^k(2k)!2^n}{(k!)^2(2k+1)}\binom{n}{2k}\binom{n}{k}^{-1}. \end{aligned}$$

By replacing n by $2n$ and n by k in the above theorem, we get the following Corollary:

Corollary 2.3. *We have*

$$\begin{aligned} &\sum_{j=0}^{2n}\sum_{l=0}^{j}\sum_{d=0}^{n-j} \frac{(-1)^l}{l+d+2n+1}\binom{2n}{j}\binom{2n-j}{n}\binom{j}{l}\binom{n-j}{d} \\ = \quad & \frac{(-1)^k(2n)!2^n}{(n!)^2(2n+1)}\binom{2n}{n}^{-1}. \end{aligned}$$

2.1. Marsden's Identity Related to Combinatorial Sums

The Marsden's identity has novel and elegant applications results in the family of the Bezier curves and B-spline. This identity shows that how polynomials are exhibited as linear transformation of B-splines. There are also another methods. For example one of them is the de Boor and Fix, which is equivalent to the Marsde's identity method (*cf.* [2], [7]).

By using the Marsden's identity, we derive some combinatorial sums involving binomial coefficients. The binomial coefficients and their identities are of importance in the theory of the combinatorics and statistics since they provide ready formulas for counting problems. The Marsden's identity is given by the following theorem (*cf.* [2], [12]):

Theorem 2.4.

$$(y-x)^n = \sum_{k=0}^{n} (-1)^k \binom{n}{k}^{-1} B_{n-k}^n(y) B_k^n(x). \tag{7}$$

By using the Marsden's identity, Simsek [12] also gave the following combinatorics identity involving inverse binomial coefficients:

Theorem 2.5. *Let $n \in \mathbb{N}$. Then we have*

$$\sum_{k=0}^{n} (-1)^k \binom{n}{k}^{-1} = (1+(-1)^n)\frac{n+1}{n+2}. \tag{8}$$

Theorem 2.6.

$$\sum_{k=0}^{n} (-1)^k \binom{n}{k}^{-1} = (n+1)\sum_{k=0}^{n} \frac{(-1)^k(2k+3)-1}{(k+1)(k+2)}. \tag{9}$$

Proof. For the proof of this theorem, we need the following the well-known identities:

$$(1-x)^{n+1} = 1 - \sum_{k=0}^{n} x(1-x)^k \tag{10}$$

and

$$(-x)^{n+1} = 1 + \sum_{k=0}^{n} (-1)^{k+1}(1+x)x^k \tag{11}$$

We integrate the polynomials $(y-x)^n$ in (7) with respect to x and y from 0 to 1, we have

$$\int_0^1 \int_0^1 (y-x)^n \, dxdy = \frac{1}{n+1}\int_0^1 \left((1-x)^{n+1} - (-x)^{n+1}\right) dx. \tag{12}$$

If we substitute (10) and (11) the above equation, and also using (2) and (3), we also find the following result:

$$\int_0^1 \int_0^1 (y-x)^n \, dxdy = \frac{1}{n+1}\sum_{k=0}^{n} \frac{(-1)^k(2k+3)-1}{(k+1)(k+2)}. \tag{13}$$

In [12], we have the following relation:

$$\int_0^1 \int_0^1 (y-x)^n \, dxdy = \frac{1}{(n+1)^2}\sum_{k=0}^{n} (-1)^k \binom{n}{k}^{-1}. \tag{14}$$

By combining (13) with (14), and after some elementary calculations, we arrive at the assertion of the theorem. □

Remark 2.7. *Combinatorics identity and Combinatorial sums involving inverse binomial coefficients have been studied by many authors (cf. [15], [12]; see also the references cited in each of these earlier work).*

By combining (8) and (9), we arrive at the following theorem:

Theorem 2.8.

$$\sum_{k=0}^{n} \frac{(-1)^k(2k+3)-1}{(k+1)(k+2)} = (1+(-1)^n)\,\frac{1}{n+2}. \tag{15}$$

$$\int_0^1 \int_0^1 (y-x)^n \, dxdy = \sum_{k=0}^{n} (-1)^{n-k} \binom{n}{k} \int_0^1 \int_0^1 x^{n-k} y^k dxdy.$$

Combining the above equation with (13), we get the following theorem:

Theorem 2.9.

$$\sum_{k=0}^{n} \frac{(-1)^k(2k+3)-1}{(k+1)(k+2)} = (n+1) \sum_{k=0}^{n} (-1)^{n-k} \binom{n}{k} \frac{1}{(k+1)(n-k+2)}.$$

3. Using the Bezier Curve in Medicine Applications

Medical imaging refers to a number of techniques that can be used as non-invasive methods of looking inside the body. This means the body does not have to be opened up surgically for medical practitioners to look at various organs and areas. In an ideal world we would be able to diagnose, treat and cure patients without causing any harmful side effects. Medical imaging also brings scientists from biology, chemistry and physics together and the technologies developed can often be used in many disciplines. Nowadays using of Bezier curves in medicine for medical imaging is getting increases. In the last two decades, imaging procedures have changed medical practice so fundamentally that not only are they indispensable, it is simply impossible to imagine medical practice without them. Several new image-producing modalities have been developed and integrated into the clinical workflow. Today, computer tomography (CT), magnet resonance imaging (MRI), colour Doppler ultrasound (US), single photon emission computed tomography (SPECT) and positron emission tomography (PET) are used in routine applications while the next generation of modalities, such as functional MRI (fMRI) and multidetector tomography, are being introduced into the clinic. These techniques are used across a wide spectrum of patient applications. CT is an X-ray procedure that produces cross-sectional pictures of the patient's body. Because of the defined X-ray spectrum, the grey values of the resulting images are standardised as Houndsfield units. In contrast, the grey values of MR images are not standardised. This technique produces cross-sectional pictures with the help of a powerful magnetic field ($\geq 1.5T$). Resonances of atomic nuclei in different tissues are transformed into a grey value image. MRI differentiates soft tissue very well but liquids like blood and solid material like bones are not visualised. Because MRI is a more expensive procedure with a lower image resolution, CT is still the workhorse of radiological routine, while MRI is only used in special cases. SPECT and PET belong to the family of scintigraphic imaging modalities. Therefore, radioactive material with a very short half-life is applied to the patient and detected by the modality. With this technique metabolic activity can be visualised to locate cancer-affected areas in the patient. In past decade one of the fundamental trends have been recognized is *2D* to *3D* image revolution. While in the past, images were typically *2D* there has been a shift towards reproducing the three dimensionality of human organs. The Bezier curve is popular in computer graphics and computer-aided application. The parametric curve, representing the special case of -spline, has been instrumental for widely diversified applications spanning from industrial shape design to game development. Inherently, the Bezier curve is characterized by convex hull. The curve is contained inside control polygon, guaranteeing that the generated curve will not derail off its control polygon. Because of its versatility and advantages, Bezier curves have been applied in medical image processing. There are many research studies in literature which use the Bezier Curve for getting better medical image. (*cf*. [4]-[1]). Hirokowa and friends have used the Bezier curves in their research study that titled "*3D kinematic estimation of knee prosthesis using X-ray projection images: clinical assessment of the improved algorithm for fluoroscopy image*" for countermeasure to the overlapping between the tibial and femoral silhouettes. Hirokawa has employed two techniques in succession the first one was interpolating the missing parts of tibial and femoral contours by using free form Bezier curves. The Bezier interpolation method did not achieve highly accurate estimation, but neither did it introduce large errors. Recently Foroozandeh and friends were used the cubic Bezier spline curves for 3*D* reconstruction of an object from its 2*D* cross-sections (*cf*. [3]). In order to perform 3*D* reconstruction, at first desired boundaries at each slice were detected and then using a correspondence between points of successive slices surface of desired object was reconstructed. The cubic Bezier spline curves were used to to approximate each of obtained contours and to approximate the corresponding points of different contours at successive slices. The reconstructed surface was a bi-cubic Bezier Spline surface which was smooth with G2 continuity. The method was tested on SPECT data of JASZCZAK phantom and human's left ventricle and results were confirmed their method was accurate, promising, applicable and effective. Hong Lin was proposed at method for modeling the human sipine by 3*D* bezier curves (*cf*. [5]). This simplified 3*D* spine model was based on the bi-planar spinal radiographic images taken from both the coronal plane and sagittal plane. By using these images, the 3*D* bezier curves were fitted on the center of the spinal cord both in coronal plane and sagittal plane images. With this modeling it was possible to analyze and classify the scoliosis spinal deformity, plan and simulate the spinal deformity correction and predict surgery outcome. Iakovodis and friends were tried to improve the quality of portable chest radiographs (*cf*. [1]). They proposed a methodology involves detection of salient points on the anatomic structures around the lung fiealds by subsequent application of simple intensity and edge feature extraction techniques. The Bezier curves were used to detection and interpolation of the salient points for approximating the boundaries of the lung fields.

4. Discussion and Conclusion

Ongoing developments in the field of medical imaging modalities have pushed the frontiers of modern medicine and biomedical engineering, prompting the need for new applications to improve diagnosis, treatment and prevention of diseases. As we have already mentioned before currently, a wide range of medical acquisition modalities are available, such as Computed Tomography (CT) and magnetic resonance imaging (MRI), which are capable of generating accurate portrayals of body interior. Development of medical image based 3*D* computational models that represent human anatomy and physiology have remained an active area of research. Capable of depicting the body interior and the select organs accurately, these models are of great demand in applications such as dosimetry calculations in nuclear medicine, visualization and surgery planning, prosthesis design and manufacturing, and so on. Ideally, these models are not only able to visualize the internal structure of the body, but also to mimic as closely as possible the biological properties of the select organs and tissues. An alternative method to generating a 3*D* bio-CAD model from medical image data is to reconstruct a surface from serial parallel contours extracted from sequential medical images. Digital images are segmented into regions-of-interest and edge points around the boundary are extracted and ordered. The contours are often further thinned to reduce the number of edge points and then closely fitted with closed parametric curves. A set of cross sectional curves extracted from a sequence of evenly spaced planar image slices is then lofted to generate a surface model. The accuracy of the fitted surface model depends largely on the quality of extracted edge points and the *B*-Spline curves fitted to those contours. In order to successfully reconstruct a surface model from contours, it is important to accurately identify and extract features of interest from medical images. Although medical image segmentation is an established and mature field, fully automatic segmentation of medical image data remains an unsolved problem. Segmenting medical data remains a challenging task due to the size of the dataset, the complexity of anatomical shapes, overlap of gray level values between neighboring tissues and organs, lack of consistently distinct boundaries and sampling artifacts and noise. In order to loft through the fitted cross sectional parametric curves, a significant data reduction is required, which is either done manually or by using sub-sampling algorithms which can inadvertently lead to loss of small detail. Consequent contours have to be properly aligned and if adjacent medical images contain multiple contours, a fitting algorithm should be able to determine correspondence between the contours. As a result, a fully automatic process for generating a $3D$ bio-CAD model from cross-sectional images remains difficult to implement and an effective solution has not yet been published in the literature. Besides this $3D$ reconstruction methods encounter different difficulties which some of them related to the time of reconstruction and the smoothness of the final surface. So far, most of the presented studies couldn't address above-mentioned issues. In some researches there is an effort to solve this problems by tracing boundaries using GVF algorithm and approximating obtained contours by cubic Bezier Spline curves yield to final smooth surface is reconstructed in relatively fast procedure. Based on the results of these studies, there is a good trade-off between speed and accuracy in using GVF algorithm and the cubic Bezier Spline curves for approximating the rough obtained edges during a $3D$ reconstruction method. Using cubic Bezier Spline curves instead of other mathematical curves like cubic spline, and B-spline curves yields a decrease time reconstruction.

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