Filomat 30:6 (2016), 1591–1599
DOI 10.2298/FIL1606591S

Published by Faculty of Sciences and Mathematics,
University of Niš, Serbia
Available at: http://www.pmf.ni.ac.rs/filomat

Pick's Theorems for Dissipative Operators

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Abstract. Let $\mathcal{H}$ be a complex Hilbert space and let A be a bounded linear transformation on $\mathcal{H}$. For a complex-valued function f, which is analytic in a domain $\mathbb{D}$ of the complex plane containing the spectrum of A, let $f(A)$ denote the operator on $\mathcal{H}$ defined by means of the *Riesz-Dunford integral*. In the present paper, several (presumably new) versions of Pick's theorems are proved for $f(A)$, where A is a dissipative operator (or a proper contraction) and f is a suitable analytic function in the domain $\mathbb{D}$.

1. Introduction and Definitions

Let $\mathcal{H}$ be a complex Hilbert space. Also let A be an operator (that is, a bounded linear transformation) on $\mathcal{H}$. For a complex-valued function f, which is analytic in a domain $\mathbb{D}$ of the complex plane containing the spectrum $\sigma(A)$ of A, let $f(A)$ denote the operator on $\mathcal{H}$ defined, by the *Riesz-Dunford integral*, as follows [2, p. 568]:

$$f(A) = \frac{1}{2\pi i}\int_C f(z)(zI - A)^{-1}\, dz, \tag{1}$$

where C is a positively-oriented simple closed rectifiable contour containing $\sigma(A)$ in its interior domain Ω and satisfying the condition $C \cup \Omega \subset \mathbb{D}$. Here, as usual, I stands for the identity operator on $\mathcal{H}$. The limit used in defining the integral in (1) is taken in the norm topology (that is, uniform topology) of operators on $\mathcal{H}$.

The real and imaginary parts of an operator A on $\mathcal{H}$ are denoted by $\Re(A)$ and $\Im(A)$, respectively, that is,

$$\Re(A) = \frac{1}{2}(A + A^*) \qquad \text{and} \qquad \Im(A) = \frac{1}{2i}(A - A^*).$$

If A and B are *Hermitian operators* on $\mathcal{H}$, we denote by $A \geqq B$ to mean that $A - B$ is a *positive operator*, that is,

$$\big((A - B)x, x\big) \geqq 0 \qquad (x \in \mathcal{H}).$$

2010 *Mathematics Subject Classification*. Primary 30C80; Secondary 47A56, 47A60, 47B44

Keywords. Analytic functions; Hilbert space; Bounded Linear operators; Proper contraction; Dissipative Operators; Riesz-Dunford integral; Schwarz lemma; Pick's theorems.

Received: 07 May 2014; Accepted: 27 July 2014

Communicated by Dragan S. Djordjević

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The notation $A > B$ indicates that $A - B$ is positive and invertible. A *contraction* is an operator A on $\mathcal{H}$ with $\|A\| \leqq 1$; if $\|A\| < 1$, we say that A is a *proper contraction* (see, for details, [12, p. 76]). It can be readily verified that A is a proper contraction if and only if $I > A^*A$. The operator A on $\mathcal{H}$ is called *dissipative* if $\Im(A) \geqq 0$ (cf. [11, p. 175]) and A is *strictly dissipative* if $\Im(A) > 0$ (cf. [10]).

In *Geometric Function Theory*, we deal with several subclasses of the class $\mathcal{S}$ of *normalized univalent* functions on the open unit disk

$$\mathbb{U} = \{z : z \in \mathbb{C} \quad \text{and} \quad |z| < 1\}.$$

Such *further* subclasses of the class $\mathcal{S}$ as (for example) the classes of *starlike functions*, *convex functions* and *close-to-convex functions* are widely and extensively investigated (cf. [3] and [18]; see also [17] and the references cited therein). We now suppose that f is a member of one of such subclasses of the normalized univalent function class $\mathcal{S}$. Then it is reasonable to expect that its operator range

$$\{f(A) : \|A\| < 1\}$$

inherits some of the geometric properties of the subset

$$\{f(z) : z \in \mathbb{U}\}$$

of the complex plane. Fan [4] proved the following result.

Theorem A (see Fan [4]). *If f maps $\mathbb{U}$ one-to-one onto a star-shaped region with respect to the origin of $\mathbb{C}$, then*

$$\{f(A) : \|A\| < 1\}$$

is a starlike set in the sense that, if $0 < t < 1$ and $\|A\| < 1$, then $tf(A) = f(B)$ for some proper contraction B on $\mathcal{H}$.

On the other hand, by means of a counter-example, Hwang [13] derived the following result.

Theorem B (see Hwang [13]). *If f maps $\mathbb{U}$ univalently onto a convex subset of $\mathbb{C}$, then*

$$\{f(A) : \|A\| < 1\}$$

need not be a convex set.

In a series of papers on the above-mentioned theme, Fan (*cf.* [4] to [9]; see also [1]) investigated various properties and characteristics of analytic functions of proper contractions, which include (among other results) operator versions of the Schwarz lemma, subordination theorems, Julia theorem, Pick-Julia theorem, Harnack's inequalities, Wolff's theorem, growth and distortion theorems, and so on. Mishra [14] also proved a sharpened form of the Schwarz lemma and Harnack's type inequalities for analytic functions of proper contractions. His results were motivated by the celebrated works of von Neumann [15], Potapov [16] and others. In the present sequel to some of these earlier investigations, we propose to prove a number of new Pick's theorems for dissipative operators.

2. A Set of Auxiliary Results

Let

$$\Pi = \{z : z \in \mathbb{C} \quad \text{and} \quad \Im(z) > 0\}.$$

For any point $z_0 \in \Pi$, the Möbius transformation:

$$\zeta = \Phi_{z_0}(z) = \frac{z - z_0}{z - \overline{z_0}} \qquad (z \in \Pi) \tag{2}$$

maps Π univalently onto $\mathbb{U}$ such that $z = z_0$ corresponds to $\zeta = 0$ and its inverse function is given by

$$z = \phi_{z_0}(\zeta) = \frac{z_0 - \overline{z_0}\zeta}{1-\zeta} \qquad (\zeta \in \mathbb{U}). \tag{3}$$

We need the following results in our investigation.

Lemma 1. *Let A be a strictly dissipative operator on $\mathcal{H}$ and $z_0 \in \Pi$. Then*

$$C = \Phi_{z_0}(A)$$

is a proper contraction on $\mathcal{H}$, where Φ_{z_0} is defined by (2). *Furthermore, for every proper contraction C, the operator*

$$A = \phi_{z_0}(C)$$

is a strictly dissipative operator, where ϕ_{z_0} is defined by (3).

Proof. Let A be a strictly dissipative operator on $\mathcal{H}$ and $z_0 \in \Pi$. We first note that

$$C = (A - z_0 I)\,(A - \overline{z_0} I)^{-1}\,.$$

The following verification shows that C is a proper contraction:

$$\begin{aligned} I - C^*C &= I - (A^* - z_0 I)^{-1}(A^* - \overline{z_0} I)(A - z_0 I)(A - \overline{z_0} I)^{-1} \\ &= (A^* - z_0 I)^{-1}\{(A^* - z_0 I)(A - \overline{z_0} I) - (A^* - \overline{z_0} I)(A - z_0 I)\}(A - \overline{z_0} I)^{-1} \\ &= (z_0 - \overline{z_0})(A^* - z_0 I)^{-1}(A^* - A)(A - \overline{z_0} I)^{-1} \\ &= 4\Im(z_0)(A^* - z_0 I)^{-1}\,\Im(A)\,(A - \overline{z_0} I)^{-1}. \end{aligned}$$

Since $\Im(z_0) > 0$ and $\Im(A) > 0$, we get $I - C^*C > 0$. Equivalently, C is a proper contraction on $\mathcal{H}$. This is precisely the assertion of the first part of Lemma 1.

On the other hand, if C is a proper contraction on $\mathcal{H}$, then we write

$$A = \phi_{z_0}(C) = (z_0 I - \overline{z_0} C)(I - C)^{-1}. \tag{4}$$

We now observe that

$$\begin{aligned} A - A^* &= (z_0 I - \overline{z_0} C)(I - C)^{-1} - (I - C^*)^{-1}(\overline{z_0} I - z_0 C^*) \\ &= (I - C^*)^{-1}\{(I - C^*)(z_0 I - \overline{z_0} C) - (\overline{z_0} I - z_0 C^*)(I - C)\}(I - C)^{-1} \\ &= (I - C^*)^{-1}\{(z_0 - \overline{z_0})(I - C^*C)\}(I - C)^{-1}. \end{aligned}$$

Since $\Im(z_0) > 0$ and $I - C^*C > 0$, we have $\Im(A) > 0$. The proof of Lemma 1 is thus completed. □

The following result is an easy consequence of Lemma 1.

Lemma 2. *Let A be a strictly dissipative operator on $\mathcal{H}$ and let z_0 be a point in Π. Then A can be expressed in the form*:

$$A = \phi_{z_0}(C)$$

for some proper contraction C on $\mathcal{H}$, where ϕ_{z_0} is defined by (3). *Moreover, every proper contraction C on $\mathcal{H}$ can be written as follows*:

$$C = \Phi_{z_0}(A)$$

for some strictly dissipative operator A on $\mathcal{H}$, where Φ_{z_0} is defined by (2).

A classical result of von Neumann (*cf.* [15]; see also [4]) states that, *if the complex-valued function f is analytic in an open neighbourhood of the closed unit disk $\overline{\mathbb{U}}$,*

$$f(\overline{\mathbb{U}}) \subset \overline{\mathbb{U}}$$

and A is a contraction of a Hilbert space, then

$$\|f(A)\| \leqq 1.$$

We shall need the following equivalent formulation of this result in our investigation.

Theorem 1 (*cf.* [4]). *Let A be a proper contraction of a Hilbert space. Also let f be a complex-valued analytic function in the open unit disk $\mathbb{U}$. If*

$$f(\mathbb{U}) \subset \mathbb{U},$$

then the following inequality:

$$\|f(A)\| < 1 \tag{5}$$

holds true.

We shall also need the following version of the Schwarz lemma for operators.

Theorem 2 (*cf.* [4]). *Let A be a proper contraction of a Hilbert space. Also let the function $f : \mathbb{U} \longrightarrow \mathbb{U}$ be analytic such that*

$$f(0) = f'(0) = \cdots = f^{(n-1)}(0) = 0 \qquad (n \in \mathbb{N} := \{1, 2, 3, \cdots\}).$$

Then

$$(A^n)^*(A^n) \geqq f(A)^* f(A) \tag{6}$$

and

$$\|A^n\| \geqq \|f(A)\|. \tag{7}$$

Strict inequality holds true in (6) *if and only if* $(A^n)^*(A^n) > 0$ *and f is not of the form*:

$$f(z) = \eta z^n \tag{8}$$

for some constant η $(|\eta| = 1)$. *Equality occurs in* (7) *if and only if either $A^n = 0$ or f is of the form* (8).

3. Analytic Functions of a Dissipative Operator

If f is a complex-valued analytic function on Π and A is a strictly dissipative operator on $\mathcal{H}$, then $f(A)$ is well defined in the sense of the *Riesz-Dunford integral* (1). We first prove another equivalent version of Theorem 1.

Theorem 3. *Let A be a strictly dissipative operator on a Hilbert space $\mathcal{H}$ and let the function $f : \Pi \longrightarrow \Pi$ be analytic. Then $f(A)$ is also a strictly dissipative operator.*

Proof. Suppose that the function $f : \Pi \longrightarrow \Pi$ is analytic and that z_0 be a fixed point in Π. Then the composite function:

$$g(\zeta) = \Phi_{f(z_0)} \circ f \circ \varphi_{z_0}(\zeta) \qquad (\zeta \in \mathbb{U}) \tag{9}$$

is analytic and maps $\mathbb{U}$ into $\mathbb{U}$, where $\Phi_{f(z_0)}$ and φ_{z_0} are defined as in (2) and (3), respectively. We now let A be a strictly dissipative operator on $\mathcal{H}$. Then, by using Lemma 2, we write

$$A = \varphi_{z_0}(C)$$

for some proper contraction C and

$$f(A) = f\Big(\varphi_{z_0}(C)\Big). \tag{10}$$

By an application of Theorem 1, we find that

$$S = g(C) \tag{11}$$

is a proper contraction on $\mathcal{H}$. Moreover, using Lemma 1, we have

$$\Im\Big(\varphi_{f(z_0)}(S)\Big) > 0. \tag{12}$$

We next observe that (9) can be rewritten in the equivalent form:

$$f\Big(\varphi_{z_0}(\zeta)\Big) = \varphi_{f(z_0)}\Big(g(\zeta)\Big) \qquad (\zeta \in \mathbb{U}). \tag{13}$$

Therefore, by the *Spectral Mapping Theorem*, the equations (11) and (13), together, yield

$$\varphi_{f(z_0)}(S) = \varphi_{f(z_0)}\Big(g(C)\Big) = f\Big(\varphi_{z_0}(C)\Big). \tag{14}$$

If we combine (10) and (14), we get

$$f(A) = \varphi_{f(z_0)}(S). \tag{15}$$

This last equation (15) and (12) show that $f(A)$ is a strictly dissipative operator. The proof of Theorem 3 is completed. □

Remark 1. The statements of Theorem 1 and Theorem 3 are equivalent. In fact, we have seen in the proof of Theorem 3 that Theorem 1 implies Theorem 3. In order to see that the converse also holds true, we assume that the function $f : \mathbb{U} \longrightarrow \mathbb{U}$ is analytic and that A is a proper contraction on a Hilbert space $\mathcal{H}$. We suppose also that $z_0 \in \Pi$. Then, by using Lemma 2, we write

$$A = \Phi_{z_0}(C)$$

for some strictly dissipative operator C on $\mathcal{H}$. We now let the function g be defined by

$$g(z) = \varphi_{z_0} \circ f\Big(\Phi_{z_0}(z)\Big) \qquad (z \in \Pi).$$

Then $g : \Pi \longrightarrow \Pi$ is analytic. Moreover, we have

$$g(C) = \varphi_{z_0}\Big(f\Big(\Phi_{z_0}(C)\Big)\Big) = \varphi_{z_0}\Big(f(A)\Big)$$

or, equivalently,

$$f(A) = \Phi_{z_0}(g(C)).$$

Thus, by applying Theorem 3 to the function g, we find that $g(C)$ is a strictly dissipative operator. Therefore, by Lemma 1, we conclude that

$$f(A) = \Phi_{z_0}\Big(g(C)\Big)$$

is a proper contraction as desired.

Remark 2. By suitably modifying the proof of Theorem 3, the following two equivalent forms of Theorem 1 can also be proved.

I. *Suppose that the function $f : \mathbb{U} \longrightarrow \Pi$ is analytic. If A is a proper contraction, then $f(A)$ is a strictly dissipative operator.*

II. *Suppose that the function $f : \Pi \longrightarrow \mathbb{U}$ is analytic. If A is a strictly dissipative operator, then $f(A)$ is a proper contraction.*

We next state and prove the following operator versions of Pick's theorem, which are associated with strictly dissipative operators.

Theorem 4. *Let A be a strictly dissipative operator on a Hilbert space $\mathcal{H}$ and let z_0 be a point in Π. Suppose also that the function $f : \Pi \longrightarrow \Pi$ be analytic. Then*

$$(A^* - z_0 I)^{-1} (A^* - \overline{z_0} I) (A - z_0 I) (A - \overline{z_0} I)^{-1} \geqq \Big(f(A)^* - f(z_0)I\Big)^{-1} \Big(f(A)^* - \overline{f(z_0)} I\Big)\Big(f(A) - f(z_0)I\Big)\Big(f(A) - \overline{f(z_0)})I\Big)^{-1} \tag{16}$$

and

$$\left\|(A - z_0 I) (A - \overline{z_0} I)^{-1}\right\| \geqq \left\|\Big(f(A) - f(z_0)I\Big)\Big(f(A) - (\overline{f(z_0)} I\Big)^{-1}\right\|. \tag{17}$$

Strict inequality occurs in (16) *if and only if*

$$(A^* - \overline{z_0} I)(A - z_0 I) > 0 \tag{18}$$

and f is not of the form:

$$f(z) = \varphi_{z_1}\Big(\eta \Phi_{z_0}(z)\Big) \tag{19}$$

for some complex constant η $(|\eta| = 1)$, some $z_1 \in \Pi$ and for the functions Φ_{z_0} and φ_{z_1} defined as in (2) *and* (3)*, respectively. Equality occurs in* (17) *if and only if $A = z_0 I$ or f is of the form* (19).

Proof. Let the function g be defined by (9). We express g in the following equivalent form:

$$g(\Phi_{z_0}(z)) = \Phi_{f(z_0)}(f(z)) \qquad (z \in \Pi), \tag{20}$$

where Φ_{z_0} is defined by (2). We now let A be a strictly dissipative operator on $\mathcal{H}$. A direct application of Lemma 1 shows that the operator

$$C = \Phi_{z_0}(A) = (A - z_0 I)(A - \overline{z_0} I)^{-1} \tag{21}$$

is a proper contraction. Furthermore, $g(C)$ is well defined in the sense of the *Riesz-Dunford integral* (1). Also, by the *Spectral Mapping Theorem*, we have

$$g(C) = g(\Phi_{z_0}(A)). \tag{22}$$

Similarly, by using Theorem 3, we see that

$$Q = f(A)$$

is a strictly dissipative operator and that

$$\Phi_{f(z_0)}(Q) = \Phi_{f(z_0)}\Big(f(A)\Big) = \Big(f(A) - f(z_0)I\Big)\Big(f(A) - \overline{f(z_0)} I\Big)^{-1}. \tag{23}$$

On the other hand, the equation (20) yields

$$g\Big(\Phi_{z_0}(A)\Big) = \Phi_{f(z_0)}\Big(f(A)\Big).$$

This, together with (22) and (23), would show that

$$g(C) = \Big(f(A) - f(z_0)I\Big)\Big(f(A) - \overline{f(z_0)} I\Big)^{-1}. \tag{24}$$

Since the function g defined by (9) satisfies the conditions of the Schwarz lemma, by using Theorem 2, we get

$$C^* C \geqq g(C)^*\big(g(C)\big) \tag{25}$$

and

$$\|C\| \geqq \|g(C)\|. \tag{26}$$

Lastly, upon replacing the operator C by the expression given by (21) and the operator $g(C)$ by the expression given by (24) in the above inequalities (25) and (26), we get (16) and (17), respectively. The sharpness of (16) and (17) also follows from Theorem 2. The proof of Theorem 4 is thus completed. □

Henceforth we use the notation μ_w $(w \in \mathbb{U})$ to denote the disk automorphism given by

$$\mu_w(z) = \frac{w - z}{1 - \overline{w}z} \qquad (z, w \in \mathbb{U}). \tag{27}$$

It can be readily verified that μ_w is its own inverse. In fact, if

$$\tau = \mu_w(z),$$

then $z = \mu_w(\tau)$. Furthermore, if

$$\eta\mu_w(z) = \tau \qquad (|\eta| = 1),$$

then $z = \mu_w(\overline{\eta}\tau)$.

Theorem 5. *Let the function $f : \mathbb{U} \longrightarrow \Pi$ be analytic and let $w \in \mathbb{U}$. Suppose also that A is a proper contraction on a Hilbert space $\mathcal{H}$. Then*

$$\begin{aligned}(I - wA^*)^{-1}\,(A^* - \overline{w}I)\,(A - wI)\ \ (I - \overline{w}A)^{-1} \\ \geqq \big(f(A)^* - f(w)I\big)^{-1}\big(f(A)^* - \overline{f(w)}I\big)\big(f(A) - f(w)I\big)\big(f(A) - \overline{f(w)}I\big)^{-1}\end{aligned} \tag{28}$$

and

$$\|\,(A - wI)\,(I - \overline{w}A)^{-1}\,\| \geqq \left\|\big(f(A) - f(w)I\big)\big(f(A) - \overline{f(w)}I\big)^{-1}\right\|. \tag{29}$$

Strict inequality holds true in (28) *if and only if*

$$(A^* - \overline{w}I)(A - wI) > 0 \tag{30}$$

and f is not of the form:

$$f(z) = \varphi_{z_1}\big(\eta\mu_w(z)\big), \tag{31}$$

where η is a complex constant with $|\eta| = 1$, $z_1 \in \Pi$, $w \in \mathbb{U}$ and the function φ_{z_1} and μ_w are defined by (3) *and* (27), *respectively. Equality occurs in* (29) *if and only if $A = z_0 I$ or f is of the form* (31).

Proof. Let the function $f : \mathbb{U} \longrightarrow \Pi$ be analytic and let $w \in \mathbb{U}$. Suppose that A is a proper contraction on $\mathcal{H}$. Then $\mu_w(A) = C$ is a proper contraction. Conversely, we have $\mu_w(C) = A$. The following complex-valued composite function:

$$g = \Phi_{f(w)} \circ f \circ \mu_w,$$

where $\Phi_{f(w)}$ and μ_w are defined by (2) and (27), respectively, satisfies the conditions of the Schwarz lemma. Therefore, by applying Theorem 2 to the function g, we have

$$C^*C \geqq g(C)^*g(C) \tag{32}$$

and

$$\|C\| \geqq \|g(C)\| \tag{33}$$

for every proper contraction C on $\mathcal{H}$. Thus, by setting $C = \mu_w(A)$, we have

$$g(C) = \Phi_{f(w)} \circ f(A) = \big(f(A) - f(w)I\big)\big(f(A) - f(w)I\big)^{-1}.$$

Upon replacing the operator C by $\mu_w(A)$ and the operator $g(C)$ by

$$\big(f(A) - f(w)I\big)\big(f(A) - f(w)I\big)^{-1}$$

in the inequalities (32) and (33), we get (28) and (29), respectively. The assertions about sharpness also follow from Theorem 2. The proof of Theorem 5 is completed. □

Theorem 6. *Let the function $f : \Pi \longrightarrow \mathbb{U}$ be analytic and let $z_0 \in \Pi$. Suppose also that A is a strictly dissipative operator on $\mathcal{H}$. Then*

$$\begin{aligned}(A^* - z_0I)^{-1}\,(A^* - \overline{z_0}I)\,(A - z_0I)\,(A - \overline{z_0}I)^{-1}& \\ \geqq \left(I - f(z_0)f(A)^*\right)^{-1}\left(f(A)^* - \overline{f(z_0)}I\right)&\left(f(A) - f(z_0)I\right)\left(I - \overline{f(z_0)}f(A)\right)^{-1}\end{aligned} \tag{34}$$

and

$$\left\|(A - z_0I)\,(A - \overline{z_0}I)^{-1}\right\| \geqq \left\|\left(f(A) - f(z_0)I\right)\left(I - \overline{f(z_0)}f(A)\right)^{-1}\right\|. \tag{35}$$

Strict inequality holds true in (34) *if and only if*

$$(A^* - \overline{z_0}I)(A - z_0I) > 0$$

and f is not of the form:

$$f(z) = \mu_{z_1}\big(\eta\Phi_{z_0}(z)\big), \tag{36}$$

where η is a complex number with $|\eta| = 1$, $z_0 \in \Pi$ and $z_1 \in \mathbb{U}$, and Φ_{z_0} and μ_w are defined by (2) *and* (27), *respectively. Equality occurs in* (35) *if and only if $A = z_0I$ or f is of the form* (36).

Proof. The proof of Theorem 6 is similar to that of Theorem 5. In this case, we consider the function g given by

$$g = \mu_{f(z_0)} \circ f \circ \varphi_{z_0},$$

where $z_0 \in \Pi$, $\mu_{f(z_0)}$ and φ_{z_0} are defined by (27) and (3), respectively. We now suppose that A is a strictly dissipative operator on $\mathcal{H}$. Then, by Lemma 2, we have

$$A = \varphi_{z_0}(C)$$

for some proper contraction C on $\mathcal{H}$. Moreover, we have

$$g(C) = \left(f(A) - f(z_0)I\right)\left(I - \overline{f(z_0)}f(A)\right)^{-1}$$

and

$$C = (A - z_0I)(I - \overline{z_0}I)^{-1}.$$

Thus, by applying Theorem 2 to $g(C)$, we get (34) and (35), respectively. The assertions about sharpness also follow from Theorem 2. The proof of Theorem 6 is completed. □

Remark 3. The works by Fan [4] and by Ando and Fan [1] contain several other versions of Pick's theorems for operators.

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