Filomat 30:9 (2016), 2397–2403
DOI 10.2298/FIL1609397C

Published by Faculty of Sciences and Mathematics,
University of Niš, Serbia
Available at: http://www.pmf.ni.ac.rs/filomat

Matrix Constructions of Centroid Sets for Classification Systems

M. Chowdhury[a], J. Abawajy[a], H. F. Jelinek[b], A. V. Kelarev[a], J. Ryan[c]

[a] *School of Information Technology, Deakin University, Geelong, Victoria, Australia*
[b] *Centre for Research in Complex Systems and School of Community Health, Charles Sturt University, Albury, NSW 2640, Australia*
[c] *School of Electrical Engineering and Computer Science, University of Newcastle, University Drive, Callaghan, NSW 2308, Australia*

Abstract. This article continues the investigation of matrix constructions motivated by their applications to the design of classification systems. Our main theorems strengthen and generalize previous results by describing all centroid sets for classification systems that can be generated as one-sided ideals with the largest weight in structural matrix semirings. Centroid sets are well known in data mining, where they are used for the design of centroid-based classification systems, as well as for the design of multiple classification systems combining several individual classifiers.

1. Introduction

This article continues the investigation of matrix constructions motivated by applications for classification. Here we strengthen and generalize previous results obtained in the recent article [24] devoted to centroid sets in matrix semirings. Centroid sets are very well known, since they are used for the design of centroid-based classification systems, also called classifiers, as well as for the design of multiple classification systems combining several individual initial classifiers (cf. [1, 18, 26].

Many interesting results on structural matrix rings have been obtained in the literature, for example, see [9–11]. Let us refer the readers to [9, 24] and the monograph [17] for a comprehensive bibliography on structural matrix rings. More general structural matrix semirings were introduced in [24], where the authors investigated centroid sets that can be generated as two-sided ideals in this construction. This study is important, because semirings have valuable applications in computer science (cf. [7, 8, 12]).

The present article strengthens previous results by describing optimal sets of centroids that can be generated for the design of classification systems as one-sided ideals of the largest weight in structural matrix semirings. The concept of an ideal is very important and has many applications in several branches of modern mathematics. The class of one-sided ideals is larger than that of two-sided ideals. It is important to handle this larger class for several reasons. First, considering the larger class of ideals may lead to the design of classification systems with better properties. Secondly, it turns out that the results we obtain in the present paper not only generalize previous formulas, but also make it possible to simplify them.

We refer to the book [26] for more information on the design of classifiers and their roles in data mining. More details are also given in Section 2 below. In particular, special sets satisfying certain optimal

2010 *Mathematics Subject Classification*. Primary 68T05; Secondary 68T10
Keywords. Matrix semirings, Ideals, Classification
Received: 16 June 2014; Accepted: 28 November 2015
Communicated by Dragan S. Djordjević
Email addresses: morshed.chowdhury@deakin.edu.au (M. Chowdhury), jemal.abawajy@deakin.edu.au (J. Abawajy), hjelinek@csu.edu.au (H. F. Jelinek), andreikelarev-deakinuniversity@yahoo.com (A. V. Kelarev), joe.ryan@newcastle.edu.au (J. Ryan)

properties are required for the design of centroid-based classifiers, as well as for the design of multiple classifiers combining several individual or initial classifiers, see [1, 23]. Such classifiers were also used, for example, in [2, 3, 21, 25]. Our main theorems give complete descriptions of centroid sets with largest weights that can be generated as one-sided ideals in structural matrix semirings.

As mentioned above, it is interesting that considering the larger class of one-sided ideals in the present paper has made it possible to obtain descriptions involving simpler formulas that may be easier to use in applications as compared to the formulas obtained in the literature previously. It is also essential to handle all one-sided ideals, since considering the more general type of centroid sets may lead to the design of classification systems with better properties.

The paper is organised as follows. Background information and preliminaries on the applications of matrix constructions for the design of classification systems in data mining is given in Section 2. The main results of the present paper are Theorems 3.1 and 3.2 presented in Section 3. These theorems describe all centroid sets that can be generated as right ideals with largest weight among all right ideals in structural matrix semirings, and all centroid sets that can be generated as left ideals with largest weight among all left ideals in structural matrix semirings, respectively. Complete proofs are included in Section 4.

2. Motivation and Preliminaries

This section contains a concise review of the main definitions required for our new theorems. We use standard notions and terminology and refer to [4–6, 12, 14, 15, 17] for preliminaries, background information, more detailed explanations and illustrating examples explaining these concepts and notation.

The design of efficient classifiers is very important in data mining, see [26]. Matrix semirings can be used in order to generate convenient sets of centroids for centroid-based classifiers and to design combined multiple classifiers capable of correcting the errors of individual initial classifiers. Classification deals with known classes of data. These classes are represented by given samples of data. The samples are used for supervised training of the classifier to enable it to recognize new elements of the same known classes. The classification process begins with feature extraction and representation of data in a standard vector space F^n, where F can be regarded as a semifield. Recall that a *semifield* is a semiring, where the set of nonzero elements forms a group with respect to multiplication.

Every centroid-based classifier selects special elements $c_1, \ldots, c_k$ in F^n, called *centroids* (see [26]). For $i = 1, \ldots, k$, each centroid c_i defines its class $K(c_i)$ consisting of all vectors v such that c_i is the nearest centroid of v. Every vector is assigned to the class of its nearest centroid.

On the other hand, multiple classifiers are often used in analysis of data to combine individual initial classifiers (see, for example, [2, 3, 20, 25]). A well-known method for the design of multiple classifiers consists in designing several simpler initial or individual classifiers, and then combining them into one multiple classification scheme with several classes. This method is very effective, and is often recommended for various applications, see [26], Section 7.5 and [13]. The main advantage of using combined multiple classifiers is in their ability to correct errors of individual classifiers and produce correct classifications despite individual classification errors.

Denote the number of initial classifiers being combined by n. If $x_1, \ldots, x_n$ are the outputs of the initial classifiers, then the sequence $(x_1, \ldots, x_n)$ is called a *vector of outputs* of the initial classifiers. In order to define the multiple classifier and enable correction of errors of the initial classifiers, a set of centroids $c_1, \ldots, c_k$ is again selected in F^n. For $i = 1, \ldots, k$, the class $K(c_i)$ of the centroid c_i is again defined as the set of all observations with the vector outputs of the initial classifiers having c_i as its nearest centroid.

The design of multiple classifiers by combining individual classifiers is quite common in the literature. We refer to [24] and [26] for a list of properties required of the sets of centroids. In particular, it is essential to find sets of centroids with large weights and small numbers of generators. The *weight* $\mathrm{wt}(v)$ of $v \in F^n$ is the number of nonzero components or coordinates in v. The *weight* of a set $C \subseteq F^n$ is the minimum weight of a nonzero element in C. For additional references and discussion related to these properties we refer the readers to [1, 13, 16, 22, 23]. In particular, it is essential to find sets of centroids with large weights and small numbers of generators, see [24].

Recall that a *semiring* is a set Q with two binary operations, addition $+$ and multiplication $\cdot$, such that the following conditions are satisfied:

(S1) $(Q, +)$ is a commutative semigroup with zero 0,

(S2) $(Q, \cdot)$ is a semigroup,

(S3) multiplication distributes over addition,

(S4) zero 0 annihilates Q, i.e., $0 \cdot Q = Q \cdot 0 = 0$.

If the multiplicative semigroups $(Q, \cdot)$ has an identity element 1, then Q is called a *semiring with identity element*, see [12, 24].

Let F be a semiring. For $m \in \mathbb{N}$, consider the semiring $M_m(F)$ of all $m \times m$ matrices over F. Denote the set of all positive integers by $\mathbb{N}$ and put $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$. For $i, j \in [1 : m]$ denote by $e_{i,j}$ the standard elementary matrix in $M_m(F)$ with 1 in the intersection of i-th row and j-th column and zeros in all other entries. Let ϱ be a binary relation on the set $[1 : m] = \{1, \dots, m\}$. It is well known and easy to verify that the set $M_\varrho(F) = \bigoplus_{(i,j)\in\varrho} Fe_{i,j}$ is a subsemiring of $M_m(F)$ if and only if the relation ϱ is transitive, i.e., $(i, j), (j, k) \in \varrho$ implies $(i, k) \in \varrho$ for all i, j, k. In this case $M_\varrho(F)$ is called a *structural matrix semiring*. Clearly, $M_m(F) = 0$ if and only if $\varrho = \emptyset$. Throughout we assume that $M_m(F) \neq 0$ and $\varrho \neq \emptyset$. Many valuable results on structural matrix rings have been obtained in the literature (see, for example, [11, 19]). Known facts and references concerning structural matrix rings can be also found in [17].

If $|\varrho| = n$, then the additive semigroup of $M_\varrho(F)$ is isomorphic to F^n and we can introduce multiplication in F^n by identifying it with $M_\varrho(F)$. Further we consider sets of centroids as subsets generated in $M_\varrho(F)$.

Here we deal with centroid sets that can be generated as one-sided ideals in the semiring $M_\varrho(F)$. Let us recall the definitions of ideals and one-sided ideals. Suppose that G is a subset of $M_\varrho(F)$. An *ideal* generated by G in $M_\varrho(F)$ is the set

$$C(G) = \left\{ \sum_{i=1}^{k} \ell_i g_i r_i \;\middle|\; k \in \mathbb{N}_0, g_i \in G,\ \ell_i, r_i \in M_\varrho(F) \cup \mathbb{N} \right\}, \tag{1}$$

where it is assumed that the identity element 1 of $\mathbb{N}$ acts as an identity on the whole $M_\varrho(F)$ too. A *right ideal* generated by G is the set

$$C_r(G) = \left\{ \sum_{i=1}^{k} g_i r_i \;\middle|\; k \in \mathbb{N}_0, g_i \in G,\ r_i \in M_\varrho(F) \cup \mathbb{N} \right\}, \tag{2}$$

and a *left ideal* generated by G is the set

$$C_\ell(G) = \left\{ \sum_{i=1}^{k} \ell_i g_i \;\middle|\; k \in \mathbb{N}_0, g_i \in G,\ \ell_i \in M_\varrho(F) \cup \mathbb{N} \right\}. \tag{3}$$

The set G is called a *generating set*. A *finitely generated ideal* (resp., *right ideal*, *left ideal*) is an ideal (resp., right ideal, left ideal) that has a finite set of generators. A *one-sided ideal* is a set that is a right ideal or a left ideal.

3. Main Results

Let ϱ be a nonempty binary relation on the set $[1 : m]$. We introduce the following binary relations

$$\varrho_r = \{(i, j) \in \varrho \mid \exists k \in [1 : m] : (j, k) \in \varrho\}, \tag{4}$$

$$\varrho_\ell = \{(i, j) \in \varrho \mid \exists k \in [1 : m] : (k, i) \in \varrho\}. \tag{5}$$

and subsets in the semiring $M_\varrho(F)$:

$$\mathcal{K}_R = \left\{ x = \sum_{(i,j)\in\varrho\setminus\varrho_r} x_{i,j}e_{i,j} \;\middle|\; \text{where } 0 \neq x_{i,j} \in F \text{ for all } i,j \right\}, \tag{6}$$

$$\mathcal{K}_L = \left\{ x = \sum_{(i,j)\in\varrho\setminus\varrho_\ell} x_{i,j}e_{i,j} \;\middle|\; \text{where } 0 \neq x_{i,j} \in F \text{ for all } i,j \right\}. \tag{7}$$

Let us also define the sets

$$R_j = \{i \mid (i,j) \in \varrho_r\}, \tag{8}$$

$$L_i = \{j \mid (i,j) \in \varrho_\ell\}, \tag{9}$$

nonnegative integers

$$N_R = \max\{|R_j| \;:\; j = 1,\dots,m\}, \tag{10}$$

$$N_L = \max\{|L_i| \;:\; i = 1,\dots,m\} \tag{11}$$

and the following subsets of the semiring $M_\varrho(F)$:

$$\mathcal{H}_R = \left\{ h = \sum_{i\in R_j} h_{i,j}e_{i,j} \;\middle|\; j \in [1:m], |R_j| = N_R, 0 \neq h_{i,j} \in F \right\}, \tag{12}$$

$$\mathcal{H}_L = \left\{ h = \sum_{j\in L_i} h_{i,j}e_{i,j} \;\middle|\; i \in [1:m], |L_i| = N_L, 0 \neq h_{i,j} \in F \right\}. \tag{13}$$

Theorem 3.1. *Let $M_\varrho(F)$ be a structural matrix semiring over a semifield F, and let C_r be a centroid set that can be generated as a right ideal of the largest possible weight among all right ideals of $M_\varrho(F)$. Then the weight of C_r is given by the formula*

$$\mathrm{wt}(C_r) = \max\{|\varrho \setminus \varrho_r|, N_R\} \tag{14}$$

and C_r contains an element of weight $\mathrm{wt}(C_r)$ *that belongs to $\mathcal{K}_R \cup \mathcal{H}_R$.*

Theorem 3.2. *Let $M_\varrho(F)$ be a structural matrix semiring over a semifield F, and let C_ℓ be a centroid set that can be generated as a left ideal of the largest possible weight among all left ideals in $M_\varrho(F)$. Then the weight of C_ℓ is given by the formula*

$$\mathrm{wt}(C_\ell) = \max\{|\varrho \setminus \varrho_\ell|, N_L\} \tag{15}$$

and C_ℓ contains an element of weight $\mathrm{wt}(C_\ell)$ *that belongs to $\mathcal{K}_L \cup \mathcal{H}_L$.*

4. Proofs

For any $i \in [1:m]$, let us define the sets

$$\varrho(i) = \{j \mid (i,j) \in \varrho\}, \tag{16}$$

$$\varrho^{-1}(i) = \{j \mid (j,i) \in \varrho\}. \tag{17}$$

For any semiring Q, the *left annihilator* of Q is the set

$$\mathrm{Ann}_\ell(Q) = \{x \in Q \mid xQ = 0\}, \tag{18}$$

and the *right annihilator* of Q is the set

$$\mathrm{Ann}_r(Q) = \{x \in Q \mid Qx = 0\}. \tag{19}$$

Lemma 4.1. ([24]) *For any structural matrix semiring $M_\varrho(F)$ over a semifield F, the following equalities are satisfied:*

$$\begin{aligned} \mathrm{Ann}_r(M_\varrho(F)) &= M_{\varrho\setminus\varrho_\ell}(F), && (20)\\ \mathrm{Ann}_\ell(M_\varrho(F)) &= M_{\varrho\setminus\varrho_r}(F). && (21)\end{aligned}$$

Lemma 4.2. *For any structural matrix semiring $M_\varrho(F)$ over a semifield F, the following inclusions hold:*

(i) $\mathcal{K}_R \subseteq \mathrm{Ann}_\ell(M_\varrho(F))$,

(ii) $\mathcal{K}_L \subseteq \mathrm{Ann}_r(M_\varrho(F))$.

Proof. Condition (i) follows from (6) and Lemma 4.1(i). Likewise, condition (ii) follows from (7) and Lemma 4.1(ii). □

Lemma 4.3. *For any structural matrix semiring $M_\varrho(F)$ over a semifield F, the following conditions hold:*

(i) $\mathrm{wt}(C_r(x)) = \mathrm{wt}(x) = |\varrho \setminus \varrho_r|$, *for every* $x \in \mathcal{K}_R$;

(ii) $\mathrm{wt}(C_r(x)) = \mathrm{wt}(x) = N_R$, *for every* $x \in \mathcal{H}_R$;

(iii) $\mathrm{wt}(C_\ell(x)) = \mathrm{wt}(x) = |\varrho \setminus \varrho_\ell|$, *for every* $x \in \mathcal{K}_L$;

(iv) $\mathrm{wt}(C_\ell(x)) = \mathrm{wt}(x) = N_L$, *for every* $x \in \mathcal{H}_L$.

Proof. **(i):** Pick an arbitrary nonzero element $x = \sum_{(i,j)\in\varrho\setminus\varrho_r} x_{i,j}e_{i,j} \in \mathcal{K}_R$. Since $x_{i,j} \neq 0$ for all $(i, j) \in \varrho \setminus \varrho_r$ by (6), it follows that $\mathrm{wt}(x) = |\varrho \setminus \varrho_r|$. Obviously, $\mathrm{wt}(x) \geq \mathrm{wt}(C_r(x)$. To verify that $\mathrm{wt}(x) \leq \mathrm{wt}(C_r(x)$, let us choose a nonzero element y of minimal weight in $C_r(x)$. It follows from (2) that we can represent y as $y = sx+\sum_{t=1}^{k} xr_t$, where $s \in \mathbb{N}_0$ and $0 \neq r_t \in M_\varrho(F)$ for all t. Condition (i) of Lemma 4.2 implies that $xM_\varrho(F) = 0$. Hence we get $y = sx$. Therefore $s \neq 0$ and $\mathrm{wt}(C_r(x)) = \mathrm{wt}(y) = \mathrm{wt}(x)$, which means that condition (i) holds true.

(ii): Choose an arbitrary nonzero element $h \in \mathcal{H}_R$. By (12), we can represent it in the form

$$h = \sum_{i\in R_j} h_{i,j}e_{i,j}, \tag{22}$$

where j is an element of $[1 : m]$ such that $|R_j| = N_R$, and where $0 \neq h_{i,j} \in F$ for all i, j. Therefore $\mathrm{wt}(h) = N_R$. Obviously, $\mathrm{wt}(h) \geq \mathrm{wt}(C_r(h)$. To verify the reversed inequality, pick a nonzero element $y \in C_r(h)$. It follows from (2) that there exists $s \in \mathbb{N}_0$ such that

$$y = sh + \sum_{t=1}^{k} hr_t, \tag{23}$$

where $0 \neq r_t \in M_\varrho(F)$. In view of the distributive law we may assume that all the r_t are homogeneous elements of $M_\varrho(F)$, i.e., $r_t = f_te_{i_t,j_t}$, for $f_t \in F^*$. We can remove all zero products hr_t from (23) and assume that $i_t = j$ for all t, so that

$$y = sh + \sum_{t=1}^{k} h\left(f_te_{j,j_t}\right). \tag{24}$$

Substituting (22) in (24), we get

$$y = sh + \sum_{t=1}^{k} \left(\sum_{i\in R_j} (h_{i,j}f_t)e_{i,j_t}\right). \tag{25}$$

The weight of each summand $\sum_{i\in R_j}(h_{i,j}f_t)e_{i,j_t}$ in (25) is equal to $|R_j| = N_R$, and these summands do not cancel with each other, since without loss of generality we may assume from the very beginning that $j_{t_1} \neq j_{t_2}$ for

$t_1 \neq t_2$ in (24). Therefore $\mathrm{wt}(y) \geq N_R = \mathrm{wt}(h)$. Thus $\mathrm{wt}(C_r(h)) = \mathrm{wt}(h)$, which means that condition (ii) holds true.

(iii), (iv): The proofs of conditions (iii) and (iv) are dual to those of (i) and (iv), respectively, and so we omit them. □

Proof of Theorem 3.1. Denote the maximum that occurs in the right-hand side of equality (14) by W_r. Lemma 4.3(i) tells us that $M_\varrho(F)$ always contains elements generating right ideals of weight $|\varrho \setminus \varrho_r|$. Therefore the maximality of the weight of C_r shows that $\mathrm{wt}(C_r) \geq |\varrho \setminus \varrho_r|$. Similarly, Lemma 4.3(i) implies that $M_\varrho(F)$ always has elements generating right ideals of weight N_r. Hence $\mathrm{wt}(C_r) \geq N_r$. Therefore we get

$$\mathrm{wt}(C_r) \geq W_r. \tag{26}$$

To prove the reversed inequality, choose a nonzero element

$$x = \sum_{(i,j)\in\varrho} x_{i,j} e_{i,j} \tag{27}$$

of minimal weight in C_r. The following two cases are possible.

Case 1: There exists $(i, j) \in \varrho_r$ such that $x_{i,j} \neq 0$.

Then (4) implies that $(j, k) \in \varrho$ for some $k \in [1 : m]$. Therefore $e_{j,k} \in M_\varrho(F)$, and so $xe_{j,k} \in C_r$. Since

$$Xe_{j,k} = \sum_{i\in R_j} xi,\, je_{i,k} \tag{28}$$

we see that $xe_{j,k}$ It follows that $\mathrm{wt}(xe_{j,k}) \leq |R_j| \leq N_R \leq W_r$. Hence we get $\mathrm{wt}(C_r) = \mathrm{wt}(x) \leq \mathrm{wt}(xe_{j,k}) \leq W_r$.

Case 2: $x_{i,j} = 0$ for all $(i, j) \in \varrho$.

Then x belongs to $\bigoplus_{(i,j)\in\varrho\setminus\varrho_r} Fe_{i,j}$; whence $\mathrm{wt}(x) \leq \varrho \setminus \varrho_r \leq W_r$.

Thus, we see that $\mathrm{wt}(C_r) \leq W_r$ in both cases. Hence (26) yields us that $\mathrm{wt}(C_r) = W_r$. Therefore equality (14) always holds.

In view of equality (14) there are two possible cases. First, the equality $\mathrm{wt}(C_r) = |\varrho \setminus \varrho_r|$ may hold. In this case we can use Lemma 4.3(i) and find an element x in $\mathcal{K}_R$ such that $\mathrm{wt}(x) = |\varrho \setminus \varrho_r| = \mathrm{wt}(C_r)$. Second, the equality $\mathrm{wt}(C_r) = N_R$ may hold true. In this case, Lemma 4.3(i) tells us that $M_\varrho(F)$ always contains an element x in $\mathcal{H}_R$ such that $\mathrm{wt}(x) = N_R = \mathrm{wt}(C_r)$. Thus C_r always contains an element of weight $\mathrm{wt}(C_r)$ that belongs to $\mathcal{K}_R \cup \mathcal{H}_R$. This completes the proof. □

Proof of Theorem 3.2 is dual to the proof of Theorem 3.1, and so we omit it. □

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