



Classification of Totally Umbilical Slant Submanifolds of a Kenmotsu Manifold

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Abstract. The purpose of this paper is to classify totally umbilical slant submanifolds of a Kenmotsu manifold. We prove that a totally umbilical slant submanifold M of a Kenmotsu manifold \bar{M} is either invariant or anti-invariant or $\dim M = 1$ or the mean curvature vector H of M lies in the invariant normal subbundle. Moreover, we find with an example that every totally umbilical proper slant submanifold is totally geodesic.

1. Introduction

Slant submanifolds of an almost Hermitian manifold were defined by Chen as a natural generalization of both holomorphic and totally real submanifolds [6]. On the other hand, A. Lotta [13] has introduced the notion of slant immersions into almost contact metric manifolds and obtained the results of fundamental importance. He has also studied the intrinsic geometry of 3–dimensional non anti-invariant slant submanifolds of K –contact manifolds [14]. Later on, Cabrerizo et. al [3] studied the geometry of slant submanifolds in more specialized settings of K –contact and Sasakian manifolds and obtained many interesting results.

On the other hand, in 1954, J.A. Schouten studied the totally umbilical submanifolds and proved that every totally umbilical submanifold of $\dim \geq 4$ in a conformally flat space is conformally flat [15]. After that many authors studied the geometrical aspects of these submanifolds in different settings, including those of [1, 4, 5, 7, 8, 16]. In this paper, we consider M , a totally umbilical slant submanifold tangent to the structure vector field ξ of a Kenmotsu manifold \bar{M} and obtain a classification result that either (i) M is anti-invariant or (ii) $\dim M = 1$ or (iii) $H \in \Gamma(\mu)$, where μ is the invariant normal subbundle under ϕ . We also prove that every totally umbilical proper slant submanifold is totally geodesic. To, this end, we provide an example to justify our results.

2. Preliminaries

A $(2n + 1)$ –dimensional manifold (\bar{M}, g) is said to be an *almost contact metric manifold* if it admits an endomorphism ϕ of its tangent bundle $T\bar{M}$, a vector field ξ , called *structure vector field* and η , the dual 1–form of ξ satisfying the following [2]:

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$$\phi^2 = -I + \eta \otimes \xi, \quad \eta(\xi) = 1, \quad \phi(\xi) = 0, \quad \eta \circ \phi = 0 \quad (1)$$

and

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y), \quad \eta(X) = g(X, \xi) \quad (2)$$

for any X, Y tangent to \bar{M} . An almost contact metric manifold is known to be *Kenmotsu manifold* [11] if

$$(\bar{\nabla}_X \phi)Y = g(\phi X, Y)\xi - \eta(Y)\phi X \quad (3)$$

consequently, we also have

$$\bar{\nabla}_X \xi = X - \eta(X)\xi \quad (4)$$

for any vector fields X, Y on \bar{M} , where $\bar{\nabla}$ denotes the Riemannian connection with respect to g .

Now, let M be a submanifold of \bar{M} . We will denote by ∇ , the induced Riemannian connection on M and g , the Riemannian metric on \bar{M} as well as the metric induced on M . Let TM and $T^\perp M$ be the Lie algebras of vector fields tangent to M and normal to M , respectively and ∇^\perp the induced connection on $T^\perp M$. Denote by $\mathcal{F}(M)$ the algebra of smooth functions on M and by $\Gamma(TM)$ the $\mathcal{F}(M)$ -module of smooth sections of TM over M . Then the Gauss and Weingarten formulas are given by

$$\bar{\nabla}_X Y = \nabla_X Y + h(X, Y) \quad (5)$$

$$\bar{\nabla}_X N = -A_N X + \nabla_X^\perp N, \quad (6)$$

for each $X, Y \in \Gamma(TM)$ and $N \in \Gamma(T^\perp M)$, where h and A_N are the second fundamental form and the shape operator (corresponding to the normal vector field N) respectively for the immersion of M into \bar{M} . They are related as

$$g(h(X, Y), N) = g(A_N X, Y). \quad (7)$$

Now, for any $X \in \Gamma(TM)$, we write

$$\phi X = TX + FX, \quad (8)$$

where TX and FX are the tangential and normal components of ϕX , respectively. Similarly for any $N \in \Gamma(T^\perp M)$, we have

$$\phi N = tN + fN, \quad (9)$$

where tN (resp. fN) is the tangential (resp. normal) component of ϕN .

$$g(TX, Y) = -g(X, TY). \quad (10)$$

The covariant derivatives of the endomorphisms ϕ , T and F are defined respectively as

$$(\bar{\nabla}_X \phi)Y = \bar{\nabla}_X \phi Y - \phi \bar{\nabla}_X Y, \quad \forall X, Y \in \Gamma(T\bar{M}) \quad (11)$$

$$(\nabla_X T)Y = \nabla_X TY - T\nabla_X Y, \quad \forall X, Y \in \Gamma(TM) \quad (12)$$

$$(\nabla_X F)Y = \nabla_X^\perp FY - F\nabla_X Y, \quad \forall X, Y \in \Gamma(TM). \quad (13)$$

Throughout, the structure vector field ξ assumed to be tangential to M , otherwise M is simply anti-invariant [13]. For any $X \in \Gamma(TM)$, on using (4) and (5), we may obtain

$$(a) \nabla_X \xi = X - \eta(X)\xi, \quad (b) h(X, \xi) = 0. \quad (14)$$

On using (3), (5), (6), (8), (9) and (11)-(13), we obtain

$$(\nabla_X T)Y = g(TX, Y)\xi - \eta(Y)TX + A_{FY}X + th(X, Y) \quad (15)$$

$$(\nabla_X F)Y = fh(X, Y) - h(X, TY) - \eta(Y)FX. \quad (16)$$

A submanifold M of an almost contact metric manifold \bar{M} is said to be *totally umbilical* if

$$h(X, Y) = g(X, Y)H, \quad (17)$$

where H is the mean curvature vector of M . Furthermore, if $h(X, Y) = 0$, for all $X, Y \in \Gamma(TM)$, then M is said to be *totally geodesic* and if $H = 0$, then M is *minimal* in \bar{M} .

For a totally umbilical submanifold M tangent to the structure vector field ξ of a Kenmotsu manifold \bar{M} , we have

$$g(X, \xi)H = 0, \quad \forall X \in \Gamma(TM). \quad (18)$$

There are two possible cases arise, hence we conclude the following:

Case (i): When X and ξ are linearly dependent, i.e., $X = \alpha\xi$, for some non-zero $\alpha \in \mathbb{R}$, then $g(X, \xi) = \alpha$. In this case, from (18), we get $H = 0$ with $\dim M = 1$, which is trivial case of totally geodesic 1-dimensional submanifold.

Case (ii): When X and ξ are orthogonal, then from (18), it is not necessary that $H = 0$, which is the case has to be discussed for totally umbilical submanifolds.

In the following section, we will discuss all possible cases of totally umbilical slant submanifolds.

3. Slant Submanifolds

A submanifold M tangent to the structure vector field ξ of an almost contact metric manifold \bar{M} is said to be *slant submanifold* if for any $x \in M$ and $X \in T_x M - \langle \xi \rangle$, the angle between ϕX and $T_x M$ is constant. The constant angle $\theta \in [0, \pi/2]$ is then called *slant angle* of M in \bar{M} . Thus, for a slant submanifold M , the tangent bundle TM is decomposed as

$$TM = D \oplus \langle \xi \rangle$$

where the orthogonal complementary distribution D of $\langle \xi \rangle$ is known as *slant distribution* on M . The normal bundle $T^\perp M$ of M is decomposed as

$$T^\perp M = F(TM) \oplus \mu,$$

where μ is the invariant normal subbundle with respect to ϕ orthogonal to $F(TM)$.

For a proper slant submanifold M of an almost contact metric manifold \bar{M} with the slant angle θ , Lotta [13] proved that

$$T^2 X = -\cos^2 \theta (X - \eta(X)\xi) \quad (19)$$

for any $X \in \Gamma(TM)$.

Recently, Cabrerizo et. al [3] extended the above result into a characterization for a slant submanifold in a contact metric manifold. In fact, they have obtained the following theorem.

Theorem 3.1. [3] *Let M be a submanifold of an almost contact metric manifold \bar{M} such that $\xi \in TM$. Then M is slant if and only if there exists a constant $\lambda \in [0, 1]$ such that*

$$T^2 = \lambda(-I + \eta \otimes \xi).$$

Furthermore, in such a case, if θ is slant angle, then it satisfies that $\lambda = \cos^2 \theta$.

Hence, for a slant submanifold M of an almost contact metric manifold \bar{M} , the following relations are consequences of the above theorem.

$$g(TX, TY) = \cos^2 \theta [g(X, Y) - \eta(X)\eta(Y)] \quad (20)$$

$$g(FX, FY) = \sin^2 \theta [g(X, Y) - \eta(X)\eta(Y)] \quad (21)$$

for any $X, Y \in \Gamma(TM)$.

In the following theorem we consider M as a totally umbilical slant submanifold of a Kenmotsu manifold \bar{M} .

Theorem 3.2. *Let M be a totally umbilical slant submanifold of a Kenmotsu manifold \bar{M} . Then at least one of the following statements is true*

- (i) M is invariant
- (ii) M is anti-invariant
- (iii) M is totally geodesic
- (iv) $\dim M = 1$
- (v) If M is proper slant, then $H \in \Gamma(\mu)$

where H is the mean curvature vector of M .

Proof. As M is a totally umbilical slant submanifold, then we have

$$h(TX, TX) = g(TX, TX)H = \cos^2 \theta (\|X\|^2 - \eta^2(X))H.$$

Using (5), we obtain

$$\cos^2 \theta (\|X\|^2 - \eta^2(X))H = \bar{\nabla}_{TX}TX - \nabla_{TX}TX.$$

Then from (8), we get

$$\cos^2 \theta (\|X\|^2 - \eta^2(X))H = \bar{\nabla}_{TX}\phi X - \bar{\nabla}_{TX}FX - \nabla_{TX}TX.$$

By (6) and (11), we derive

$$\cos^2 \theta (\|X\|^2 - \eta^2(X))H = (\bar{\nabla}_{TX}\phi)X + \phi\bar{\nabla}_{TX}X + A_{FX}TX - \nabla_{TX}^\perp FX - \nabla_{TX}TX.$$

Using (3) and (5), we obtain

$$\begin{aligned} \cos^2 \theta (\|X\|^2 - \eta^2(X))H &= g(\phi TX, X)\xi - \eta(X)\phi TX + \phi(\nabla_{TX}X + h(X, TX)) \\ &\quad + A_{FX}TX - \nabla_{TX}^\perp FX - \nabla_{TX}TX. \end{aligned}$$

From (8), (10), (17) and the fact that X and TX are orthogonal vector fields on M , we arrive at

$$\begin{aligned} \cos^2 \theta (\|X\|^2 - \eta^2(X))H &= -g(TX, TX)\xi - \eta(X)T^2X - \eta(X)FTX + T\nabla_{TX}X \\ &\quad + F\nabla_{TX}X + A_{FX}TX - \nabla_{TX}^\perp FX - \nabla_{TX}TX. \end{aligned}$$

Then, by Theorem 3.1 and the relation (20), we get

$$\begin{aligned} \cos^2 \theta (\|X\|^2 - \eta^2(X))H &= -\cos^2 \theta (\|X\|^2 - \eta^2(X))\xi - \cos^2 \theta \eta(X)\{-X + \eta(X)\xi\} \\ &\quad - \eta(X)FTX + T\nabla_{TX}X + F\nabla_{TX}X + A_{FX}TX - \nabla_{TX}^\perp FX - \nabla_{TX}TX. \end{aligned} \quad (22)$$

Taking the inner product with TX in (22), for any $X \in \Gamma(TM)$, we obtain

$$0 = g(T\nabla_{TX}X, TX) + g(A_{FX}TX, TX) - g(\nabla_{TX}TX, TX). \quad (23)$$

Now, we compute the first and last term of (23) as follows

$$g(T\nabla_{TX}X, TX) = \cos^2 \theta \{g(\nabla_{TX}X, X) - \eta(X)g(\nabla_{TX}X, \xi)\}. \quad (24)$$

Also, we have

$$g(\nabla_{TX}TX, TX) = g(\bar{\nabla}_{TX}TX, TX).$$

Using the property of Riemannian connection the above equation will be

$$g(\nabla_{TX}TX, TX) = \frac{1}{2}TXg(TX, TX) = \frac{1}{2}TX\{\cos^2 \theta(g(X, X) - \eta(X)\eta(X))\}.$$

Again by the property of Riemannian connection, we derive

$$g(\nabla_{TX}TX, TX) = \cos^2 \theta \{g(\bar{\nabla}_{TX}X, X) - \eta(X)g(\bar{\nabla}_{TX}X, \xi)\} - \cos^2 \theta \eta(X)g(\bar{\nabla}_{TX}\xi, X). \quad (25)$$

Using (4) and the fact that X and TX are orthogonal vector fields on M , the last term of (25) is identically zero, then by (5), we obtain

$$g(\nabla_{TX}TX, TX) = \cos^2 \theta \{g(\nabla_{TX}X, X) - \eta(X)g(\nabla_{TX}X, \xi)\}. \quad (26)$$

Thus, from (24) and (26), we get

$$g(T\nabla_{TX}X, TX) = g(\nabla_{TX}TX, TX). \quad (27)$$

Using this fact in (23), we obtain

$$0 = g(A_{FX}X, TX) = g(h(TX, TX), FX).$$

As M is totally umbilical slant, then from (2.17) and (3.2), we get

$$0 = \cos^2 \theta \{\|X\|^2 - \eta^2(X)\}g(H, FX). \quad (28)$$

Thus, from (28), we conclude that either $\theta = \pi/2$, that is M is anti-invariant which is a part (ii) or the vector field X is parallel to the structure vector field ξ , i.e., M is 1-dimensional submanifold which is fourth part of the theorem or $H \perp FX$, for all $X \in \Gamma(TM)$, i.e., $H \in \Gamma(\mu)$ which is the last part of the thorem or $H = 0$, i.e., M is totally geodesic which is (iii) or $FX = 0$, $\forall X \in \Gamma(TM)$, i.e., M is invariant which is part (i). This proves the theorem completely. \square

Now, if we consider M , a proper slant submanifold of a Kenmotsu manifold \bar{M} , then neither M is invariant nor anti-invariant (by definition of proper slant) and also neither $\dim M = 1$. Hence, by the above result, only possibility is that $H \in \Gamma(\mu)$ for a totally umbilical proper slant submanifold. Thus, we prove the following main result.

Theorem 3.3. *Every totally umbilical proper slant submanifold of a Kenmotsu manifold is totally geodesic.*

Proof. Let M be a totally umbilical proper slant submanifold of a Kenmotsu manifold \bar{M} , then for any $X, Y \in \Gamma(TM)$, we have

$$\bar{\nabla}_X \phi Y - \phi \bar{\nabla}_X Y = g(\phi X, Y)\xi - \eta(Y)\phi X.$$

From (5) and (8), we obtain

$$\bar{\nabla}_X TY + \bar{\nabla}_X FY - \phi(\nabla_X Y + h(X, Y)) = g(TX, Y)\xi - \eta(Y)TX - \eta(Y)FX.$$

Again using (5), (6) and (8), we get

$$g(TX, Y)\xi - \eta(Y)TX - \eta(Y)FX = \nabla_X TY + h(X, TY) - A_{FY}X + \nabla_X^\perp FY - T\nabla_X Y - F\nabla_X Y - \phi h(X, Y).$$

As M is totally umbilical, then

$$g(TX, Y)\xi - \eta(Y)TX - \eta(Y)FX = \nabla_X TY + g(X, TY)H - A_{FY}X + \nabla_X^\perp FY - T\nabla_X Y - F\nabla_X Y - g(X, Y)\phi H. \quad (29)$$

Taking the inner product with ϕH in (29) and using the fact that $H \in \Gamma(\mu)$ (by Theorem 3.2 (v)), we obtain

$$g(\nabla_X^\perp FY, \phi H) = g(X, Y)\|H\|^2.$$

Using (6) and the property of Riemannian connection, the above equation takes the form

$$g(FY, \nabla_X^\perp \phi H) = -g(X, Y)\|H\|^2. \quad (30)$$

Now, for any $X \in \Gamma(TM)$, we have

$$\bar{\nabla}_X \phi H = (\bar{\nabla}_X \phi)H + \phi \bar{\nabla}_X H.$$

Using (3), (6), (8) and the fact that $H \in \Gamma(\mu)$, we obtain

$$-A_{\phi H}X + \nabla_X^\perp \phi H = -TA_H X - FA_H X + \phi \nabla_X^\perp H. \quad (31)$$

Also, for any $X \in \Gamma(TM)$, we have

$$\begin{aligned} g(\nabla_X^\perp H, FX) &= g(\bar{\nabla}_X H, FX) \\ &= -g(H, \bar{\nabla}_X FX). \end{aligned}$$

Using (8), we get

$$g(\nabla_X^\perp H, FX) = -g(H, \bar{\nabla}_X \phi X) + g(H, \bar{\nabla}_X TX).$$

Then from (5) and (11), we derive

$$g(\nabla_X^\perp H, FX) = -g(H, (\bar{\nabla}_X \phi)X) - g(H, \phi \bar{\nabla}_X X) + g(H, h(X, TX)).$$

Using (3) and (17), the first and last term of right hand side of the above equation are identically zero and hence by (2), the second term gives

$$g(\nabla_X^\perp H, FX) = g(\phi H, \bar{\nabla}_X X).$$

Again, using (5) and (17), finally we obtain

$$g(\nabla_X^\perp H, FX) = g(\phi H, H)\|X\|^2 = 0.$$

This means that

$$\nabla_X^\perp H \in \Gamma(\mu). \quad (32)$$

Now, taking the inner product in (31) with FY , for any $Y \in \Gamma(TM)$, we get

$$g(\nabla_X^\perp \phi H, FY) = -g(FA_H X, FY) + g(\phi \nabla_X^\perp H, FY).$$

Using (32), the last term of the right hand side of the above equation will be zero and then from (21), (30), we obtain

$$g(X, Y)\|H\|^2 = \sin^2 \theta \{g(A_H X, Y) - \eta(Y)g(A_H X, \xi)\}. \quad (33)$$

Hence, by (7) and (17), the above equation reduces to

$$g(X, Y)\|H\|^2 = \sin^2 \theta \{g(X, Y)\|H\|^2 - \eta(Y)g(h(X, \xi), H)\}. \quad (34)$$

Since, for a Kenmotsu manifold \bar{M} , $h(X, \xi) = 0$, for any X tangent to \bar{M} , thus we obtain

$$g(X, Y)\|H\|^2 = \sin^2 \theta g(X, Y)\|H\|^2.$$

Therefore, the above equation can be written as

$$\cos^2 \theta g(X, Y)\|H\|^2 = 0. \quad (35)$$

Since, M is proper slant, thus from (35), we conclude that $H = 0$ i.e., M is totally geodesic in \bar{M} . This completes the proof of the theorem. \square

We now give the following example of a proper slant, totally geodesic submanifold in \mathbb{R}^5 with its standard Kenmotsu structure.

Example 3.4. Consider the 3–dimensional proper slant submanifold with the slant angle $\theta \in (0, \pi/2)$ of \mathbb{R}^5 defined by

$$x(u, v, t) = 2(u \cos \theta, u \sin \theta, v, 0, t)$$

with its usual Kenmotsu structure $\mathbb{R}^5 = \mathbb{C}^2 \times \mathbb{R}$, (ϕ, ξ, η, g)

$$\phi \left\{ \sum_{i=1}^2 \left(X_i \frac{\partial}{\partial x^i} + Y_i \frac{\partial}{\partial y^i} \right) + Z \frac{\partial}{\partial t} \right\} = \sum_{i=1}^2 \left(-Y_i \frac{\partial}{\partial x^i} + X_i \frac{\partial}{\partial y^i} \right),$$

$$\xi = 2 \frac{\partial}{\partial t}, \quad \eta = \frac{1}{2} dt \quad \text{and} \quad g = \eta \otimes \eta + \frac{e^{2t}}{4} \sum_{i=1}^2 (dx^i \otimes dx^i + dy^i \otimes dy^i)$$

where (x^i, y^i, t) , $i = 1, 2$ are cartesian coordinates. If we denote by M a slant submanifold, then its tangent space TM span by the vectors

$$e_1 = \frac{1}{e^t} \left\{ 2 \left\{ \cos \theta \left(\frac{\partial}{\partial x^1} + y^1 \frac{\partial}{\partial t} \right) + 2 \sin \theta \left(\frac{\partial}{\partial x^2} + y^2 \frac{\partial}{\partial t} \right) \right\} \right\},$$

$$e_2 = \frac{2}{e^t} \frac{\partial}{\partial y^1}, \quad e_3 = 2 \frac{\partial}{\partial t} = \xi.$$

Clearly, we have

$$\phi e_1 = \frac{1}{e^t} \left\{ 2 \left\{ \cos \theta \left(\frac{\partial}{\partial y^1} \right) + 2 \sin \theta \left(\frac{\partial}{\partial y^2} \right) \right\} \right\},$$

$$\phi e_2 = -\frac{2}{e^t} \frac{\partial}{\partial x^1}, \quad \phi e_3 = 0.$$

Furthermore, using Koszul's formula, we get $\bar{\nabla}_{e_i} e_i = -e_3 = -\xi$, $i = 1, 2$ and when $i \neq j$, then $\bar{\nabla}_{e_i} e_j = 0$, for $i, j = 1, 2, 3$. Also, $\bar{\nabla}_{e_3} e_3 = 0$, thus, from Gauss formula and (2.14), we obtain

$$h(e_1, e_1) = 0, \quad h(e_2, e_2) = 0, \quad h(e_3, e_3) = 0$$

and

$$h(e_1, e_2) = 0, \quad h(e_1, e_3) = 0, \quad h(e_2, e_3) = 0,$$

hence we conclude that M is totally geodesic.

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