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Soft Sets and Soft Topology on Nearness Approximation Spaces

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Abstract. Near set theory presents a fundamental basis for observation, comparison and classification of perceptual granules. Soft set theory is proposed as a general framework to model vagueness. The purpose of this paper is to combine these two theories in what are known near soft sets in defining near soft topology based on a nearness approximation space.

1. Introduction

In 1980's, Pawlak introduced rough set theory to deal the problem of vagueness and uncertainty. The main aim of that theory was to introduce some approximations of sets. Near sets have been introduced by Peters [12, 13] where objects with affinities are considered perceptually near to each other, i.e., objects with similar descriptions to some degree. Any rough set has a non-empty boundary region. In fact, this is the main difference between a rough set and a near set.

The other notion of soft set, initiated by Molodtsov [8], has been studied by many scientists and offers a new approach for uncertainty [1, 2, 7, 14]. A soft set is a parameterized family of subsets of the universal set. Afterwards, Feng and Li [5] have investigated the problem of combining soft sets with rough sets, and introduced the notion of rough soft sets. In this paper, we aim to combine near sets approach with soft set theory and to define near soft topology with some properties.

2. Preliminary

2.1. Rough Sets

Let a finite set of objects U and R be an equivalance relation defined as an indiscernibility relation on the universe U. For $x, y \in U$, x and y are said to be $R-$indiscernible, if $(x, y) \in R$. An equivalance class of x is denoted by $[x]_R$. The pair (U, R) is called an approximation space. Using the indiscernibility relation R, we can now define the following two operations; for every subset $X \subseteq U$,

$$\begin{aligned} R_*X &= \{x \in U : [x]_R \subseteq X\}, \\ R^*X &= \{x \in U : [x]_R \cap X \neq \emptyset\}. \end{aligned}$$

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The sets R_*X, R^*X are called the lower and upper approximations of X with respect to (U, R), respectively. A subset $X \subseteq U$ is called definable if $R_*X = R^*X$; and if $Bnd_R X = R^*X - R_*X \neq \emptyset$ and X is called a rough set (or inexact set) [11].

2.2. *Near Sets*

In this section, we will give some definitions and properties regarding near sets as given in [12].

Table 1: Description Symbols [13].

Symbol	Interpretation
$\mathbb{R}$	Set of real numbers,
$\mathcal{O}$	Set of perceptual objects,
X	$X \subseteq \mathcal{O}$, set of sample objects,
x	$x \in \mathcal{O}$, sample objects,
$\mathcal{F}$	*a* set of functions representing object features,
B Φ	$B \subseteq \mathcal{F}$, $\Phi : \mathcal{O} \to \mathbb{R}^L$, object description,
L	L is a description length,
i	$1 \leq i \leq L$,
$\Phi(x)$	$\Phi(x) = (\phi_1(x), \phi_2(x), \phi_3(x), ..., \phi_i(x), ..., \phi_L(x))$.

Objects are known by their descriptions [13]. An object description is defined by means of a tuple of function values $\Phi(x)$ associated with an object $x \in X$. The important thing to notice is the choice of functions $\phi_i \in B$ used to describe an object of interest. Assume that $B \subseteq F$ (see Table 1) is a given set of functions representing features of sample objects $X \subseteq \mathcal{O}$. Let $\phi_i \in B$, where $\phi_i : \mathcal{O} \to \mathbb{R}$. In combination, the functions representing object features provide a basis for an object description $\Phi_i : \mathcal{O} \to \mathbb{R}^L$, a vector containing measurements associated with each functional value $\phi_i(x)$, where the description length $|\Phi| = L$. Object Description $\Phi(x) = (\phi_1(x), \phi_2(x), \phi_3(x), ..., \phi_i(x), ..., \phi_L(x))$. The intuition underlying a description $\Phi(x)$ is a recording of measurements from sensors, where each sensor is modelled by a function ϕ_i [6].

Now we give two tables which contain basic concepts for nearness.

Table 2: Relation and partition symbols [13].

Symbol	Interpretation
$\sim_B$	$\sim_B = \{(x, x^{'}) \in \mathcal{O} \times \mathcal{O} \mid \forall \phi_i \in B, \Delta_{\phi_i} = 0\}$, indiscernibility relation,
$[x]_B$	$[x]_B = \{x\prime \in X \mid x \sim_B x\prime\}$, elementary set (class),
$\mathcal{O}/\sim_B$	$\mathcal{O}/\sim_B = \{[x]_B \mid x \in \mathcal{O}\}$, quotient set,
ξ_B	partition $\xi_B = \mathcal{O}/\sim_B$
Δ_{ϕ_i}	$\Delta_{\phi_i} = \mid \phi_i(x) - \phi_i(x') \mid$ probe function difference.

Definition 2.1. Let $B \subseteq \mathcal{F}$ be a set of functions representing features of objects $x, x^{'} \in \mathcal{O}$. Objects $x, x^{'}$ are called *minimally near* each other, if there exists $\phi_i \in B$ such that $x \sim_{\phi_i} x^{'} \iff \Delta_{\phi_i} = 0$. It is called the "Nearness Description Principle - *NDP*". In fact, the objects in a class $[x]_B \in \xi_B$ are near objects [6].

The basic idea of the near set approach to object recognition is to compare object descriptions. Sets of objects $X, X^{'}$ are considered near to each other, if the sets contain objects with at least partial matching descriptions.

Definition 2.2. Let $X \subseteq \mathcal{O}$ and $x, x^{'} \in X$. If x is near to $x^{'}$, then X is called a near set relative to itself or the reflexive nearness of X [6].

Theorem 2.3. ([6]) *A class in a partition ξ_B and ξ_B are near sets.*

2.3. Fundamental Approximation Space (FAS)

This subsection proposes a number of near sets resulting from the approximation of one set by another set. Approximations are carried out within the context of a fundamental approximation space $FAS = (\mathcal{O}, \mathcal{F}, \sim_B)$, where $\mathcal{O}$ is a set of perceived objects, $\mathcal{F}$ is a set of probe functions representing object features, and $\sim_B$ is an indiscernibility relation defined relative to $B \subseteq \mathcal{F}$. The FAS is considered fundamental, because it provides a framework for the original rough set theory [12]. It has also been observed that an approximation space is the formal counterpart of perception. Approximation starts with the partition ξ_B of $\mathcal{O}$ is defined by the relation $\sim_B$. Next, any set $X \subseteq \mathcal{O}$ is approximated by considering the relation between X and the classes $[x]_B \in \xi_B$, $x \in \mathcal{O}$. To see this, first consider the lower approximation of a set [6].

Table 3: Approximation notation [13].

Symbol	Interpretation
$(\mathcal{O}, \mathcal{F}, \sim_B)$	Fundamental approximation space (FAS), $B \subseteq \mathcal{F}$,
B_*X	$\bigcup\limits_{x \in \mathcal{O}} \{[x]_B : [x]_B \subseteq X\}$, B-lower approximation of X,
B^*X	$\bigcup\limits_{x \in \mathcal{O}} \{[x]_B : [x]_B \cap X \neq \emptyset\}$, B-upper approximation of X,
$Bnd_B X$	$B^*X \backslash B_*X = \{x \mid x \in B^*X \text{and } x \notin B_*X\}$.

Theorem 2.4. ([13]) *The lower approximation B_*X of a set X is a near set.*

Theorem 2.5. ([6]) *If a set X has a non-empty lower approximation B_*X, then X is a near set.*

Theorem 2.6. ([6]) *The upper approximation B^*X and the set X are near sets.*

Theorem 2.7. ([6]) *A set X with an approximation boundary $|Bnd_B X| \geq 0$ is a near set.*

2.4. Nearness Approximation Spaces (NAS)

Table 4: Nearness Approximation Space Symbols [13].

Symbol	Interpretation
B	$B \subseteq \mathcal{F}$,
B_r	$r \leq \lvert B \rvert$ probe functions in B,
$\sim_{B_r}$	B_r indiscernibility relation defined using B_r,
$[x]_{B_r}$	$[x]_{B_r} = \{x' \in \mathcal{O} \mid x \sim_{B_r} x'\}$, equivalence class,
$\mathcal{O}/\sim_{B_r}$	$\mathcal{O}/\sim_{B_r} = \{[x]_{B_r} \mid x \in \mathcal{O}\}$,quotient set,
$\xi_{\mathcal{O},B_r}$	Partition $\xi_{\mathcal{O},B_r} = \mathcal{O}/\sim_{B_r}$,
r	i.e. $\binom{\lvert B \rvert}{r}$ probe functions $\phi_i \in B$ taken r at a time,
$N_r(B)$	$N_r(B) = \{\xi_{\mathcal{O},B_r} \mid B_r \subseteq B\}$, set of partitions,
ν_{N_r}	$\nu_{N_r} : P(\mathcal{O}) \times P(\mathcal{O}) \to [0,1]$, overlap function,
$N_r(B)_*X$	$\bigcup\limits_{x \in \mathcal{O}} \{[x]_{B_r} : [x]_{B_r} \subseteq X\}$, B-lower approximation of X,
$N_r(B)^*X$	$\bigcup\limits_{x \in \mathcal{O}} \{[x]_{B_r} : [x]_{B_r} \cap X \neq \emptyset\}$, B-upper approximation of X,
$Bnd_{N_r(B)}(X)$	$N_r(B)^*X \backslash N_r(B)_*X = \{x \mid x \in N_r(B)^*X \text{and} x \notin N_r(B)_*X\}$.

A nearness approximation space (NAS) is denoted by $NAS = (\mathcal{O}, \mathcal{F}, \sim_{B_r}, N_r, \nu_{N_r})$ which is defined with a set of perceived objects $\mathcal{O}$, a set of probe functions $\mathcal{F}$ representing object features, an indiscernibility relation $\sim_{B_r}$ defined relative to $B_r \subseteq B \subseteq \mathcal{F}$, a collection of partitions (families of neighbourhoods) $N_r(B)$, and a neighbourhood overlap function N_r. The relation $\sim_{B_r}$ is the usual indiscernibility relation from rough set theory restricted to a subset $B_r \subseteq B$. The subscript r denotes the cardinality of the restricted subset B_r, where we consider $\binom{\lvert B \rvert}{r}$, i.e., $|B|$ functions $i \in \mathcal{F}$ taken r at a time to define the relation $\sim_{B_r}$.

This relation defines a partition of $\mathcal{O}$ into non-empty, pairwise disjoint subsets that are equivalence classes denoted by $[x]_{B_r}$, where $[x]_{B_r} = \{x\prime \in O \mid x \sim_{B_r} x'\}$. These classes form a new set called the quotient set

$\mathcal{O}/\sim_{Br}$, where $\mathcal{O}/\sim_{B_r}= \{[x]_{B_r}|x \in \mathcal{O}\}$. In effect, each choice of probe functions Br defines a partition $\xi_{\mathcal{O},B_r}$ on a set of objects $\mathcal{O}$, namely, $\xi_{\mathcal{O},B_r} = \mathcal{O}/\sim_{B_r}$. Every choice of the set B_r leads to a new partition of $\mathcal{O}$. Let $\mathcal{F}$ denote a set of features for objects in a set X, where each $i \in \mathcal{F}$ maps X to some value set $V_{\phi i}$ (range of i). The value of $\phi_i(x)$ is a measurement associated with a feature of an object $x \in X$. The overlap function v_{N_r} is defined by $v_{N_r} : \mathcal{P}(\mathcal{O}) \times \mathcal{P}(\mathcal{O}) \to [0, 1]$, where $\mathcal{P}(\mathcal{O})$ is the powerset of $\mathcal{O}$. The overlap function v_{N_r} maps a pair of sets to a number in $[0, 1]$, representing the degree of overlap between sets of objects with features defined by he probe functions $B_r \subseteq B$ [23]. For each subset $B_r \subseteq B$ of probe functions, define the binary relation $\sim_{B_r}= \{(x, x') \in \mathcal{O} \times \mathcal{O}|\forall i \in B_r, i(x) = i(x')\}$. Since each $\sim_{B_r}$ is the usual indiscernibility relation [4], for $B_r \subseteq B$ and $x \in O$, let $[x]_{B_r}$ denote the equivalence class containing x. If $(x, x') \in\sim_{B_r}$, then x and x' are said to be B-indiscernible with respect to all feature probe functions in B_r. Then define a collection of partitions $N_r(B)$, where $N_r(B) = \{\xi_{\mathcal{O},B_r}|B_r \subseteq B\}$. Families of neighborhoods are constructed for each combination of probe functions in B using $\binom{|B|}{r}$, i.e. the probe functions $|B|$ taken r at a time [6].

Theorem 2.8. ([13]) *A set X with an approximation boundary* $|Bnd_{N_r(B)}(X)| \geq 0$ *is a near set.*

3. Soft Sets

Let U be an initial universe set and E be a collection of all possible parameters with respect to U, where parameters are the characteristics or properties of objects in U. Then we will call E the universe set of parameters with respect to U.

Definition 3.1. A pair (F, A) is called a soft set over U, if $A \subseteq E$ and $F : A \to \mathcal{P}(U)$, where $\mathcal{P}(U)$ is the set of all subsets of U [4].

In other words, a soft set over U is a parameterized family of subsets of the universe U. For $\varepsilon \in A$, $F(\varepsilon)$ may be considered us the set of $\varepsilon-$ elements of the soft set (F, A) [1].

Example 3.2. Let $\mathcal{U} = \{x_1, x_2, x_3, x_4\}$, $A = \{h_3\} \subseteq E = \{h_1, h_2, h_3, h_4\}$ denote a initial universe set and a set of parameters, respectively. Sample values of the h_i, $i = 1, 2, 3, 4$. Let (F, A) be a soft set is defined by $A = \{h_3\}$, $F(h_3) = \{x_2, x_3, x_4\}$, $(F, A) = \{h_3, (x_2, x_3, x_4)\}$.

Definition 3.3. Assume that two soft sets (F, A) and (G, B) are defined over U. Then (G, B) is called a soft subset of (F, A) denoted by $(F, A) \subseteq (G, B)$, if $B \subseteq A$ and $G(\phi) \subset F(\phi)$ for all $\phi \in B$. Two soft sets (F, A) and (G, B) over U are said to be equal, denoted by $(F, A) = (G, B)$ if $(F, A) \subseteq (G, B)$ and $(G, B) \subseteq (F, A)$ [1].

Definition 3.4. Let (F, A) and (G, B) be two soft sets over a common universe U.

1. The extended intersection of (F, A) and (G, B) denoted by $(F, A) \sqcap_{\varepsilon} (G, B)$ is defined as the soft set (H, C), where $C = A \cup B$ and $\forall e \in C$

$$H(e) = \begin{cases} F(e), & e \in A - B \\ G(e), & e \in B - A \\ F(e) \cap G(e), & e \in A \cap B \end{cases} \tag{1}$$

2. The restricted intersection of (F, A) and (G, B) denoted by $(F, A) \Cap (G, B)$ is defined as the soft set (H, C), where $C = A \cap B$ and $H(c) = F(c) \cap G(c)$ for all $c \in C$.
3. The extended union of (F, A) and (G, B) denoted by $(F, A) \tilde{\cup} (G, B)$ is defined as the soft set (H, C), where $C = A \cup B$ and $\forall\ e \in C$

$$H(e) = \begin{cases} F(e), & e \in A - B \\ G(e), & e \in B - A \\ F(e) \cup G(e), & e \in A \cap B \end{cases} \tag{2}$$

4. The restricted union of (F, A) and (G, B) denoted by $(F, A) \cup_{\mathcal{R}} (G, B)$ is defined as the soft set (H, C), where $C = A \cap B$ and $H(c) = F(c) \cup G(c)$ for all $c \in C$ [5].

Definition 3.5. Let τ be the collection of soft sets over X and let A be the nonempty set of parameters. Then τ is said to be a soft topology on X if the following conditions are satisfied:

(i) $(\emptyset, A), (X, A) \in \tau$, where $\emptyset(\phi) = \emptyset$ and $X(\phi) = X$, for all $\phi \in A$.

(ii) The intersection of any two soft sets in τ belongs to τ.

(iii) The union of any number of soft sets in τ belongs to τ.

The triplet (X, A, τ) is called a soft topological space over X [9].

Definition 3.6. Let (U, R) be an approximation space and $\sigma = (F, A)$ be a soft set over U. The lower and upper rough approximations of $\sigma = (F, A)$ with respect to (U, R) denoted by $R_*(\sigma) = (F_*, A)$ and $R^*(\sigma) = (F^*, A)$, which are soft sets over U with the set-valued mappings given by

$$F_*(x) = R_*(F(x)) \tag{3}$$

and

$$F^*(x) = R^*(F(x)) \tag{4}$$

where $x \in A$. The operators R_* and R^* are called the lower and upper rough approximation operators on soft sets. If $R_*(\sigma) = R^*(\sigma)$, the soft set σ is said to be definable; otherwise σ is called a rough soft set [5].

4. Near Soft Sets

In this section, we will obtain near soft set by combining two concepts soft set and near sets. We introduce the concept of a near soft sets and near soft topology and give some of their properties.

Let $N_r(B)(X)$ be a family of neighbourhoods of a set X.

Proposition 4.1. *Every family of neighbourhoods may be considered a soft set.*

Proof. Let $N_r(B)(X)$ be a family of neighbourhoods of X in the universe $\mathcal{O}$ with respect to the equivalance relation R and probe functions r. The neighbourhoods of X is defined by a lower approximation $N_{r*}(B)(X) \neq \emptyset$ and upper approximation $N_r^*(B)(X)$. Consider the predicates $p_1(x) = \{h_1, h_2, ..., h_{|B|}\}$ which stands for $[x]_R \subseteq X$ is not empty for $r \leq | B |$ and $p_2(x)$ which stands for $[x]_R \cap X \neq \emptyset$. Also $Bnd_{N_r(B)}(X) \geq 0$.

The conditions $p_1(x)$ and $p_2(x)$ may be treated as elements of a parameter set; that is $E = \{p_1(x), p_2(x)\}$. Then we can write the function

$$F : E \to P(U),\ F(p_i(x)) = \{x \in U : p_i(x) \text{ is true}\},\ i = 1, 2.$$

Thus every family of neighbourhoods $N_r(B)(X)$ of X may also be considered a soft set with the representation

$$(F, E) = \{(p_1(x), N_{r*}(B)(X)), (p_2(x), N_r^*(B)(X))\}.$$

□

Example 4.2. Let us take the following information table.

$\mathcal{O} = \{x_1, x_2, x_3, x_4, x_5\}$, $B = \{\phi_1, \phi_2\} \subset \mathcal{F} = \{\phi_1, \phi_2, \phi_3, \phi_4\}$ with $r = 1$. For $X = \{x_1, x_2, x_3\}$,

$$\begin{aligned} [x_1]_{\phi_1} &= \{x_1, x_4\}, [x_2]_{\phi_1} = \{x_2, x_3, x_5\}, \\ [x_1]_{\phi_2} &= \{x_1, x_4\}, [x_2]_{\phi_2} = \{x_2, x_3\}, [x_4]_{\phi_2} = \{x_4\}, \end{aligned}$$

$$\begin{aligned} N_{1*}(B)(X) &= \{x_2, x_3\}, \\ N_1^*(B)(X) &= \mathcal{O} \end{aligned}$$

Then X is a near set. Every near set $N_r(B)(X)$ of X may also be considered as a soft set with representation

$$(F, B) = \{(p_1(x) = \phi_2, \{x_2, x_3\}), (p_2(x) = \phi_1, \phi_2, \{x_1, x_2, x_3, x_4, x_5\})\}.$$

Table 5:

	x_1	x_2	x_3	x_4	x_5
ϕ_1	0.1	0.2	0.2	0.1	0.2
ϕ_2	0.2	0.3	0.3	0.2	0.5
ϕ_3	0.3	0.4	0.3	0.3	0.4
ϕ_4	0.4	0.2	0.2	0.4	0.2

Inan and Ozturk [10] combine the soft sets approach with near set theory giving rise to the new concepts of soft nearness approximation space.

We consider the lower and upper approximations of a soft set in a near approximation space, which give rise to the following notions in a natural way. Let $\mathcal{O}$ is an initial universe set in soft set and $\mathcal{F}$ is a collection of all possible parameters with respect to $\mathcal{O}$. Now we can define concept of near soft sets.

Definition 4.3. Let $NAS = (\mathcal{O}, \mathcal{F}, \sim_{Br}, N_r, \nu_{N_r})$ be a nearness approximation space and $\sigma = (F, B)$ be a soft set over $\mathcal{O}$. The lower and upper near approximation of $\sigma = (F, B)$ with respect to NAS are denoted by $N_r*(\sigma) = (F_*, B)$ and $N_r^*(\sigma) = (F^*, B)$, which are soft sets over with the set-valued mappings given by

$$\begin{aligned} F_*(\phi) = N_r*(F(\phi)) &= \cup\{x \in \mathcal{O} : [x]_{Br} \subseteq F(\phi)\}, \\ F^*(\phi) = N_r^*(F(\phi)) &= \cup\{x \in \mathcal{O} : [x]_{Br} \cap F(\phi) \neq \emptyset\} \end{aligned}$$

where all $\phi \in B$. The operators N_r* and N_r^* are called the lower and upper near approximation operators on soft sets, respectively. If $Bnd_{N_r(B)}(\sigma) \geqslant 0$, then the soft set σ is called a near soft set.

Example 4.4. Let $\mathcal{O} = \{x_1, x_2, x_3, x_4, x_5\}$, $B = \{\phi_1, \phi_2\} \subseteq \mathcal{F} = \{\phi_1, \phi_2, \phi_3, \phi_4\}$ be denote a set of perceptual objects and a set of functions, respectively. Sample values of the ϕ_i, $i = 1, 2, 3, 4$ functions are shown in Table 5.

Let $\sigma = (F, B)$, $B = \{\phi_1, \phi_2\}$ be a soft set defined by

$$F(\phi_1) = \{x_2, x_3\}, F(\phi_2) = \{x_2, x_3\}. \tag{5}$$

Then (F, B) is a near soft set with r=1;

$$\begin{aligned} [x_1]_{\phi_1} &= \{x_1, x_4\}, [x_2]_{\phi_1} = \{x_2, x_3, x_5\}, \\ [x_1]_{\phi_2} &= \{x_1, x_4\}, [x_2]_{\phi_2} = \{x_2, x_3\}, [x_4]_{\phi_2} = \{x_4\}, \end{aligned}$$

$$N_*(\sigma) = N_*(F(\phi), B) = (N_*F(\phi), B) = (F_*(\phi), B), \tag{6}$$

for $\phi_2 \in B$, $N_*(\sigma) = (F_*(\phi_2), B) = \{(\phi_2, \{x_2, x_3\})\}$

$$N^*(\sigma) = (F^*(\phi), B) \tag{7}$$

for $\phi_1, \phi_2 \in B$, $N^*(\sigma) = \{(\phi_1, \{x_2, x_3, x_5\}), (\phi_2, \{x_2, x_3\})\}$

$Bnd_N(\sigma) \geq 0$,then (F, B) is a near soft set.

Then (F, B) is a near soft set with r=2;

$$\begin{aligned} [x_1]_{\phi_1,\phi_2} &= \{x_1, x_4\}, \\ [x_2]_{\phi_1,\phi_2} &= \{x_2, x_3\}, \\ [x_5]_{\phi_1,\phi_2} &= \{x_5\}, \end{aligned}$$

$$N_*(\sigma) = \{(\phi_1, \{x_2, x_3\}), (\phi_2, \{x_2, x_3\})\},$$
$$N^*(\sigma) = \{(\phi_1, \{x_2, x_3\}), (\phi_2, \{x_2, x_3\})\}$$

$Bnd_N(\sigma) \geq 0$, then (F, B) is a near soft set. But
$\mu = (F_k, B) = \{(\phi_1, \{x_2, x_3\})\}$ is not a near soft set because $N_*(\mu) = \emptyset$.

Theorem 4.5. *A collection of partitions (families of neighbourhoods) $N(\sigma)$ is a near soft set.*

Proof. Given a collection of partitions $N(\sigma)$. A partition $\xi_{O,B}(\sigma) \in N(\sigma)$ consists of classes $(F, B) = ([x]_B, B)$. These classes for $\phi \in B$ are near sets. Hence $\xi_{O,B}(\sigma)$ is a near set and $N(\sigma)$ is a near soft set. □

Theorem 4.6. *A lower approximation $N_*(\sigma)$ of a set $\sigma = (F, B) \subseteq (O, B)$ is a near soft set.*

Proof. Given a lower approximation $N_*(\sigma) = N_*(F(\phi), B)$ of a set (F, B). By definition $N_*(\sigma) \subseteq \sigma$ and $N_*(\sigma)$ consist of classes that are subsets of σ. $N_*(\sigma)$ consists of classes $([x]_B, B)$. These classes are near soft sets. Hence $N_*(\sigma)$ is a near soft set. □

Theorem 4.7. *An upper approximation $N^*(\sigma)$ of a set σ is a near soft set.*

Theorem 4.8. *A set with an approximation boundary $| Bnd_N(\sigma) | \geqslant 0$ is a near soft set.*

Proof. There are two cases to consider:

1. $| Bnd_N(\sigma) | > 0$. Given an approximation boundary $| Bnd_N(\sigma) | > 0$ for a set σ. Then $N_*(\sigma) \subseteq N^*(\sigma)$, i.e,the lower approximation $N_*(\sigma)$ is a non-empty subset of the upper approximation $N^*(\sigma)$ and $N_*(\sigma)$ is also a subset of σ. Hence σ is a near soft set.
2. $| Bnd_N(\sigma) | = 0$. Given $| Bnd_N(\sigma) | = 0$ for a set σ. Then from Theorem 7 $N^*(\sigma) = N_*(\sigma)$ and $N_*(\sigma)$ is also a subset of σ.

□

Theorem 4.9. *Every rough soft set is a near soft set.*

Proof. Set σ is termed a near set relative to a chosen collection of partitions $N(\sigma)$ iff $| Bnd_N(\sigma) | \geqslant 0$. In the case where $| Bnd_N(\sigma) | > 0$ $(R^*(\sigma) \neq R_*(\sigma))$ the set X has been roughly approximated , i.e., X is a rough soft set as well as a near soft set. In the case where $| Bnd_N(\sigma) | = 0$,the set X is considered a near soft set but not a rough soft set. In effect, every rough soft set is a near soft set, but otherwise is not true. □

Theorem 4.10. *Let $\sigma = (F, A)$ and $\mu = (G, B)$ be two soft sets over a common universe O and $N(\sigma) = N(F, A)$ and $N(\mu) = N(G, B)$ be two near soft sets over O. Let for $\forall e \in C$, $H(e) \neq \emptyset$ and $N_*(H, C) \neq \emptyset$*

1. The extended intersection of $N(F, A)$ and $N(G, B)$ denoted by $N(F, A) \sqcap_\varepsilon N(G, B)$, is defined as the near soft set $N(\gamma) = N(H, C)$, where $C = A \cup B$ and $\gamma = (H, C)$,$\forall e \in C$ and also $N_*(F, A) \neq \emptyset$, $N_*(G, B) \neq \emptyset, N_*(H, C) \neq \emptyset$

$$N(\gamma) = N(H, C) = \begin{cases} N(F, A), & e \in A - B \\ N(G, B), & e \in B - A \\ N(F, A) \cap N(G, B), & e \in A \cap B \end{cases} \tag{8}$$

2. The restricted intersection of $N(F, A)$ and $N(G, B)$ denoted by $N(F, A) \Cap N(G, B)$ is defined as the near soft set $N(H, C)$, where $C = A \cap B$ and $N(H, C) = N(F, A) \cap N(G, B)$ for all $c \in C$.
3. The extended union of $N(F, A)$ and $N(G, B)$ denoted by $N(F, A) \tilde{\cup} N(G, B)$ is defined as the near soft set $N(H, C)$, where $C = A \cup B$ and $\gamma = (H, C)$,$\forall$ $e \in C$

$$N(\gamma) = N(H, C) = \begin{cases} N(F, A), & e \in A - B \\ N(G, B), & e \in B - A \\ N(F, A) \cup N(G, B), & e \in A \cap B \end{cases}$$

4. The restricted union of $N(F, A)$ and $N(G, B)$ denoted by $N(F, A) \cup_{\mathcal{R}} N(G, B)$ is defined as the near soft set $N(H, C)$, where $C = A \cap B$ and $N(H, C) = N(F, A) \cup N(G, B)$ for all $c \in C$.

Proof. 1. Let $C = A \cup B$ and $\gamma = (H, C)$,$\forall e \in C$

$$N(\gamma) = N(H, A \cup B) = N(H, C)$$

Using definition 11, we obtain

$$H(e) = \begin{cases} F(e), & e \in A - B \\ G(e), & e \in B - A \\ F(e) \cap G(e), & e \in A \cap B \end{cases} \Rightarrow N(\gamma) = N(H, C)$$

$$= \begin{cases} N(F, A), & e \in A - B \\ N(G, B), & e \in B - A \\ N(F, A) \cap N(G, B), & e \in A \cap B \end{cases}$$

2. This is similar to the proof of (1).

□

Definition 4.11. Let O be an initial universe set, E be the universe set of parameters and $A, B \subseteq E$

1. (K, A) is called a relative null near soft set (with respect to the parameters of A) if $K(\phi) = \emptyset$, for all $\phi \in A$.
2. (W, B) is called a relative whole near soft set (with respect to the parameters of B) if $W(\phi) = O$, for all $\phi \in B$.

Definition 4.12. The relative complement of a near soft set (F, A) denoted by $(F, A)^c$, is defined as the near soft set (F^c, A) where $F^c(\phi) = O - F(\phi)$ for all $\phi \in A$.

Definition 4.13. (F, B) and (G, B) two near soft sets on (O, B).(F, B) is defined to be near soft subset of (G, B) if $N_*(F, B) \subseteq N_*(G, B)$ for all $\phi \in B$,i.e., $N_*(F(\phi), B) \subseteq N_*(G(\phi), B)$ for all $\phi \in B$ and is denoted by $(F, B) \sqsubseteq (G, B)$.

Theorem 4.14. *Let (F, B) and (G, B) be two near soft sets on (O, B). Then the following holds:*

1. $(F, B)^c \sqcap (G, B)^c = [(F, A) \sqcup (G, B)]^c$
2. $((F, B) \sqcap (G, B))^c = (F, A)^c \sqcup (G, B)^c$

Theorem 4.15. *Let (F_i, B) be a family of near soft sets on (O, B). Then the following holds:*

1. $\sqcap_i (F_i, B)^c = (\sqcup_i (F_i, B))^c$
2. $\sqcup_i (F_i, B)^c = (\sqcap_i (F_i, B))^c$

Definition 4.16. Let $\sigma = (F, B)$ be a near soft set over (O, B) , τ be the collection of near soft subsets of σ, B is the nonempty set of parameters, and then (O, B) is said to be a near soft topology on σ if the following conditions are met:

(i) $(\emptyset, B), (O, B) \in \tau$ where $\emptyset(\phi) = \emptyset$ and $F(\phi) = F$, for all $\phi \in B$.

(ii) The intersection of any two near soft sets in τ belongs to τ.

(iii) The union of any number of near soft sets in τ belongs to τ.

The pair (O, τ) is called a near soft topological space.

Example 4.17. Let $O = \{x_1, x_2, x_3, x_4, x_5\}$, $\mathcal{F} = \{\phi_1, \phi_2, \phi_3, \phi_4\}$, $B = \{\phi_1, \phi_2\}$ and Table 5 is the tabular representation of (O, B). Then
$\sigma = (F, B) = \{(\phi_1, \{x_1, x_2, x_3, x_4\}), (\phi_2, \{x_1, x_2, x_3, x_4\}), (\phi_3, \{x_1, x_3, x_4\})\}$ be a near soft set.
$(F_1, B) = \{(\phi_1, \{x_1\}), (\phi_2, \{x_2, x_3\})\}$,
$(F_2, B) = \{(\phi_1, \{x_1\}), (\phi_2, \{x_2, x_3, x_4\})\}$,
$\tau = \{(\emptyset, B), (F, B), (F_1, B), (F_2, B)\}$. Then τ is a near soft topology on (O, B).

Definition 4.18. Let $(\mathcal{O}, \tau)$ be a near soft topological space over $(\mathcal{O}, B)$. A near soft subset of $(\mathcal{O}, B)$ is called near soft closed if its complement is open and a member of τ.

Theorem 4.19. *Let $(\mathcal{O}, B, \tau)$ be a near soft topological space over $(\mathcal{O}, B)$. Then,*

1. $(\emptyset, B)^c, (\mathcal{O}, B)^c$ *are near soft closed sets.*
2. *The arbitrary intersection of near soft closed sets are near soft closed.*
3. *The union of two near soft closed sets is a near soft closed sets.*

5. Conclusions

In this study, we introduce near soft set, near soft topological space. In our future work we will go on studying near soft interior and near soft closure.

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