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Coefficient Bounds for Certain Subclasses of Close-To-Convex Functions of Complex Order

Serap Bulut[a]

[a] *Kocaeli University, Faculty of Aviation and Space Sciences, Arslanbey Campus, TR-41285 Kartepe-Kocaeli, TURKEY*

Abstract. In this paper, we determine the coefficient bounds for functions in certain subclasses of close-to-convex functions of complex order, which are introduced here by means of a certain non-homogeneous Cauchy-Euler-type differential equation of order m. Relevant connections of some of the results obtained with those in earlier works are also provided.

1. Introduction, Definitions and Preliminaries

Let $\mathbb{R} = (-\infty, \infty)$ be the set of real numbers, $\mathbb{C} := \mathbb{C}^* \cup \{0\}$ be the set of complex numbers,

$$\mathbb{N} := \{1, 2, 3, \ldots\} = \mathbb{N}_0 \setminus \{0\}$$

be the set of positive integers and

$$\mathbb{N}^* := \mathbb{N} \setminus \{1\} = \{2, 3, 4, \ldots\}.$$

Let $\mathcal{A}$ denote the class of functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \tag{1}$$

which are analytic in the open unit disk

$$\mathbb{U} = \{z : z \in \mathbb{C} \quad \text{and} \quad |z| < 1\}.$$

Recently Xu *et al.* [12] introduced the subclasses $\mathcal{S}_\varphi(\lambda, \gamma)$ and $\mathcal{K}_\varphi(\lambda, \gamma, m; u)$ of analytic functions of complex order $\gamma \in \mathbb{C}^*$, and obtained the coefficient bounds for the Taylor-Maclaurin coefficients for functions in each of these new sublasses $\mathcal{S}_\varphi(\lambda, \gamma)$ and $\mathcal{K}_\varphi(\lambda, \gamma, m; u)$ of complex order $\gamma \in \mathbb{C}^*$, which is given by Definitions 1.1 and 1.2 below.

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Email address: serap.bulut@kocaeli.edu.tr (Serap Bulut)

Definition 1.1. *(see [12]) Let $\varphi : \mathbb{U} \to \mathbb{C}$ be a convex function such that*

$$\varphi(0) = 1 \qquad \textit{and} \qquad \Re\left(\varphi\left(z\right)\right) > 0 \qquad \left(z \in \mathbb{U}\right).$$

We denote by $\mathcal{S}_{\varphi}\left(\lambda, \gamma\right)$ the class of functions $f \in \mathcal{A}$ satisfying

$$1 + \frac{1}{\gamma}\left(\frac{z\left[\left(1-\lambda\right)f\left(z\right) + \lambda z f'\left(z\right)\right]'}{\left(1-\lambda\right)f\left(z\right) + \lambda z f'\left(z\right)} - 1\right) \in \varphi\left(\mathbb{U}\right) \qquad \left(z \in \mathbb{U}\right),$$

where $0 \le \lambda \le 1;\ \gamma \in \mathbb{C}^{*}$.

Definition 1.2. *(see [12]) A function $f \in \mathcal{A}$ is said to be in the class $\mathcal{K}_{\varphi}\left(\lambda, \gamma, m; u\right)$ if it satisfies the following non-homogenous Cauchy-Euler differential equation:*

$$z^{m}\frac{d^{m}w}{dz^{m}} + \binom{m}{1}\left(u + m - 1\right)z^{m-1}\frac{d^{m-1}w}{dz^{m-1}} + \cdots + \binom{m}{m}w\prod_{j=0}^{m-1}\left(u + j\right) = h(z)\prod_{j=0}^{m-1}\left(u + j + 1\right)$$

$$\left(w = f(z) \in \mathcal{A};\ h \in \mathcal{S}_{\varphi}\left(\lambda, \gamma\right);\ m \in \mathbb{N}^{*};\ u \in \mathbb{R}\backslash\left(-\infty, -1\right]\right).$$

Making use of Definitions 1.1 and 1.2, Xu *et al.* [12] proved the following coefficient bounds for the Taylor-Maclaurin coefficients for functions in the sublasses $\mathcal{S}_{\varphi}\left(\lambda, \gamma\right)$ and $\mathcal{K}_{\varphi}\left(\lambda, \gamma, m; u\right)$ of analytic functions of complex order $\gamma \in \mathbb{C}^{*}$.

Theorem 1.3. (*see [12]*) *Let the function $f \in \mathcal{A}$ be defined by* (1). *If $f \in \mathcal{S}_{\varphi}\left(\lambda, \gamma\right)$, then*

$$\left|a_{n}\right| \le \frac{\prod\limits_{k=0}^{n-2}\left[k + \left|\varphi'\left(0\right)\right| \cdot \left|\gamma\right|\right]}{\left(n-1\right)!\ \left[1 + \lambda\left(n-1\right)\right]} \qquad \left(n \in \mathbb{N}^{*}\right).$$

Theorem 1.4. (*see [12]*) *Let the function $f \in \mathcal{A}$ be defined by* (1). *If $f \in \mathcal{K}_{\varphi}\left(\lambda, \gamma, m; u\right)$, then*

$$\left|a_{n}\right| \le \frac{\prod\limits_{k=0}^{n-2}\left[k + \left|\varphi'\left(0\right)\right| \cdot \left|\gamma\right|\right]\prod\limits_{j=0}^{m-1}\left(u + j + 1\right)}{\left(n-1\right)!\ \left[1 + \lambda\left(n-1\right)\right]\prod\limits_{j=0}^{m-1}\left(u + j + n\right)} \qquad \left(m, n \in \mathbb{N}^{*}\right),$$

$$\left(0 \le \lambda \le 1;\ \gamma \in \mathbb{C}^{*};\ u \in \mathbb{R}\backslash\left(-\infty, -1\right]\right).$$

Here, in our present sequel to some of the aforecited works (especially [12]), we first introduce the following subclasses of analytic functions of complex order $\gamma \in \mathbb{C}^{*}$.

Definition 1.5. *Let $\varphi : \mathbb{U} \to \mathbb{C}$ be a convex function such that*

$$\varphi(0) = 1 \qquad \textit{and} \qquad \Re\left(\varphi\left(z\right)\right) > 0 \qquad \left(z \in \mathbb{U}\right).$$

We denote by $\mathcal{SQ}_{\varphi}\left(\lambda, \gamma, \delta, \tau\right)$ the class of functions $f \in \mathcal{A}$ satisfying

$$1 + \frac{1}{\gamma}\left(\frac{z\left[\left(1-\lambda\right)f\left(z\right) + \lambda z f'\left(z\right)\right]'}{\left(1-\lambda\right)g\left(z\right) + \lambda z g'\left(z\right)} - 1\right) \in \varphi\left(\mathbb{U}\right) \qquad \left(z \in \mathbb{U}\right),$$

where $g \in \mathcal{S}_{\varphi}\left(\delta, \tau\right);\ 0 \le \lambda, \delta \le 1;\ \gamma, \tau \in \mathbb{C}^{*}$.

Definition 1.6. *A function $f \in \mathcal{A}$ is said to be in the class $\mathcal{KQ}_{\varphi}(\lambda, \gamma, \delta, \tau, m; u)$ if it satisfies the following non-homogenous Cauchy-Euler differential equation of order m :*

$$z^m \frac{d^m w}{dz^m} + \binom{m}{1}(u+m-1) z^{m-1} \frac{d^{m-1} w}{dz^{m-1}} + \cdots + \binom{m}{m} w \prod_{j=0}^{m-1}(u+j) = h(z) \prod_{j=0}^{m-1}(u+j+1)$$

$$\left(w = f(z) \in \mathcal{A};\ h \in \mathcal{SQ}_{\varphi}(\lambda, \gamma, \delta, \tau);\ m \in \mathbb{N}^*;\ u \in \mathbb{R} \backslash (-\infty, -1]\right).$$

Remark 1. There are many choices of the function φ which would provide interesting subclasses of analytic functions of complex order $\gamma \in \mathbb{C}^*$. In particular,
(**i**) if we let

$$\varphi(z) = \frac{1+Az}{1+Bz} \qquad (-1 \leq B < A \leq 1;\ z \in \mathbb{U}),$$

then it is easy to verify that φ is a convex function in $\mathbb{U}$ and satisfies the hypotheses of Definition 1.5. Therefore we obtain the new classes

$$\mathcal{SQ}_{\varphi}(\lambda, \gamma, \delta, \tau) = \mathcal{KQ}(\lambda, \gamma, \delta, \tau, A, B) \qquad \text{and} \qquad \mathcal{KQ}_{\varphi}(\lambda, \gamma, \delta, \tau, m; u) = \mathcal{DK}(\lambda, \gamma, \delta, \tau, A, B, m; u).$$

For $\delta = \lambda$ and $\tau = 1$, these classes introduced and studied by Ul-Haq *et al.* [10].
(**ii**) if we let

$$\varphi(z) = \frac{1+(1-2\beta) z}{1-z} \qquad (0 \leq \beta < 1;\ z \in \mathbb{U}),$$

then we obtain the new classes

$$\mathcal{SQ}_{\varphi}(\lambda, \gamma, \delta, \tau) = \mathcal{KQ}(\lambda, \gamma, \delta, \tau, \beta) \qquad \text{and} \qquad \mathcal{KQ}_{\varphi}(\lambda, \gamma, \delta, \tau, m; u) = \mathcal{BK}(\lambda, \gamma, \delta, \tau, \beta; u).$$

For $\delta = \lambda$, $\tau = 1$ and $m = 2$, these classes are introduced and studied by Ul-Haq *et al.* [9].

In this paper, by using the subordination principle between analytic functions, we obtain coefficient bounds for the Taylor-Maclaurin coefficients for functions in the substantially more general function classes $\mathcal{SQ}_{\varphi}(\lambda, \gamma, \delta, \tau)$ and $\mathcal{KQ}_{\varphi}(\lambda, \gamma, \delta, \tau, m; u)$ of analytic functions of complex order $\gamma \in \mathbb{C}^*$, which we have introduced here.

Our results presented here would generalize and improve the corresponding results obtained earlier by (for example) Altıntaş *et al.* [1], Nasr and Aouf [4], Robertson [5], Srivastava *et al.* [7] and Ul-Haq *et al.* [9, 10], (see also [2, 3, 8, 11]).

In our investigation, we shall make use of the principle of subordination between analytic functions, which is explained in Definition 1.7 below.

Definition 1.7. *For two functions f and g, analytic in $\mathbb{U}$, we say that the function f is subordinate to g in $\mathbb{U}$, and write*

$$f(z) \prec g(z) \qquad (z \in \mathbb{U}),$$

if there exists a Schwarz function ω, analytic in $\mathbb{U}$, with

$$\omega(0) = 0 \qquad \text{and} \qquad |\omega(z)| < 1 \qquad (z \in \mathbb{U})$$

such that

$$f(z) = g(\omega(z)) \qquad (z \in \mathbb{U}).$$

Indeed, it is known that

$$f(z) \prec g(z) \quad (z \in \mathbb{U}) \Rightarrow f(0) = g(0) \qquad \text{and} \qquad f(\mathbb{U}) \subset g(\mathbb{U}).$$

Furthermore, if the function g is univalent in $\mathbb{U}$, then we have the following equivalence

$$f(z) \prec g(z) \quad (z \in \mathbb{U}) \Leftrightarrow f(0) = g(0) \qquad \text{and} \qquad f(\mathbb{U}) \subset g(\mathbb{U}).$$

2. Main Results and their Demonstration

In order to prove our main results (Theorems 2.2 and 2.3 below), we first recall the following lemma due to Rogosinski [6].

Lemma 2.1. *Let the function* $\mathfrak{g}$ *given by*

$$\mathfrak{g}(z) = \sum_{k=1}^{\infty} \mathfrak{b}_k z^k \qquad (z \in \mathbb{U})$$

be convex in $\mathbb{U}$. *Also let the function* $\mathfrak{f}$ *given by*

$$\mathfrak{f}(z) = \sum_{k=1}^{\infty} \mathfrak{a}_k z^k \qquad (z \in \mathbb{U})$$

be holomorphic in $\mathbb{U}$. *If*

$$\mathfrak{f}(z) \prec \mathfrak{g}(z) \qquad (z \in \mathbb{U}),$$

then

$$|\mathfrak{a}_k| \leq |\mathfrak{b}_1| \qquad (k \in \mathbb{N}).$$

We now state and prove each of our main results given by Theorems 2.2 and 2.3 below.

Theorem 2.2. *Let the function* $f \in \mathcal{A}$ *be defined by* (1). *If* $f \in \mathcal{SQ}_{\varphi}(\lambda, \gamma, \delta, \tau)$, *then*

$$\begin{aligned} |a_n| \quad \leq \quad & \frac{\prod_{k=0}^{n-2}\left[k + \left|\varphi'(0)\right| \cdot |\tau|\right]}{n!\,[1 + \delta(n-1)]} \\ & + \frac{\left|\varphi'(0)\right| \cdot |\gamma|}{n\,[1 + \lambda(n-1)]} \left(1 + \sum_{j=1}^{n-2} \frac{[1 + \lambda(n-j-1)] \prod_{k=0}^{n-j-2}\left[k + \left|\varphi'(0)\right| \cdot |\tau|\right]}{(n-j-1)!\,[1 + \delta(n-j-1)]}\right) \quad (n \in \mathbb{N}^*), \end{aligned}$$

$$\left(g \in \mathcal{S}_{\varphi}(\delta, \tau);\ 0 \leq \lambda, \delta \leq 1;\ \gamma, \tau \in \mathbb{C}^*\right).$$

Proof. Let the function $f \in \mathcal{SQ}_{\varphi}(\lambda, \gamma, \delta, \tau)$ be of the form (1). Therefore, there exists a function

$$g(z) = z + \sum_{n=2}^{\infty} b_n z^n \in \mathcal{S}_{\varphi}(\delta, \tau) \qquad (\tau \in \mathbb{C}^*) \tag{2}$$

so that

$$1 + \frac{1}{\gamma}\left(\frac{z\left[(1-\lambda) f(z) + \lambda z f'(z)\right]'}{(1-\lambda) g(z) + \lambda z g'(z)} - 1\right) \in \varphi(\mathbb{U}). \tag{3}$$

Note that by Theorem 1.3, we have

$$|b_n| \leq \frac{\prod_{k=0}^{n-2}\left[k + \left|\varphi'(0)\right| \cdot |\tau|\right]}{(n-1)!\,[1 + \delta(n-1)]} \qquad (n \in \mathbb{N}^*). \tag{4}$$

Let

$$F(z) = (1-\lambda) f(z) + \lambda z f'(z) = z + \sum_{n=2}^{\infty} A_n z^n, \qquad A_n = [1+\lambda(n-1)] a_n \tag{5}$$

$$G(z) = (1-\lambda) g(z) + \lambda z g'(z) = z + \sum_{n=2}^{\infty} B_n z^n, \qquad B_n = [1+\lambda(n-1)] b_n. \tag{6}$$

Then (3) is of the form

$$1 + \frac{1}{\gamma}\left(\frac{zF'(z)}{G(z)} - 1\right) \in \varphi(\mathbb{U}). \tag{7}$$

Let us define the function $p(z)$ by

$$p(z) = 1 + \frac{1}{\gamma}\left(\frac{zF'(z)}{G(z)} - 1\right) \qquad (z \in \mathbb{U}). \tag{8}$$

Therefore, we deduce that

$$p(0) = \varphi(0) = 1 \qquad \text{and} \qquad p(z) \in \varphi(\mathbb{U}) \qquad (z \in \mathbb{U}).$$

So we have

$$p(z) \prec \varphi(z) \qquad (z \in \mathbb{U}).$$

Hence, by Lemma 2.1, we obtain

$$\left|\frac{p^{(m)}(0)}{m!}\right| = |c_m| \le \left|\varphi'(0)\right| \qquad (m \in \mathbb{N}), \tag{9}$$

where

$$p(z) = 1 + c_1 z + c_2 z^2 + \cdots \qquad (z \in \mathbb{U}). \tag{10}$$

Also from (8), we find

$$zF'(z) - G(z) = \gamma(p(z) - 1) G(z). \tag{11}$$

Since $A_1 = B_1 = 1$, in view of (11), we obtain

$$nA_n - B_n = \gamma\{c_{n-1} + c_{n-2}B_2 + \cdots + c_1 B_{n-1}\} = \gamma\left(c_{n-1} + \sum_{j=1}^{n-2} c_j B_{n-j}\right) \quad (n \in \mathbb{N}^*). \tag{12}$$

Now we get from (4), (5), (6), (9) and (12),

$$|a_n| \le \frac{\prod_{k=0}^{n-2}\left[k + \left|\varphi'(0)\right| \cdot |\tau|\right]}{n!\,[1+\delta(n-1)]} + \frac{\left|\varphi'(0)\right| \cdot |\gamma|}{n\,[1+\lambda(n-1)]}\left(1 + \sum_{j=1}^{n-2} \frac{[1+\lambda(n-j-1)] \prod_{k=0}^{n-j-2}\left[k + \left|\varphi'(0)\right| \cdot |\tau|\right]}{(n-j-1)!\,[1+\delta(n-j-1)]}\right) \quad (n \in \mathbb{N}^*).$$

This evidently completes the proof of Theorem 2.2. □

Theorem 2.3. *Let the function $f \in \mathcal{A}$ be defined by* (1). *If $f \in \mathcal{KQ}_\varphi(\lambda,\gamma,\delta,\tau,m;u)$, then*

$$
\begin{aligned}
|a_n| \leq & \left\{ \frac{\prod_{k=0}^{n-2}\left[k+\left|\varphi'(0)\right|\cdot|\tau|\right]}{n!\,[1+\delta(n-1)]} \right. \\
& \left. + \frac{\left|\varphi'(0)\right|\cdot\left|\gamma\right|}{n\,[1+\lambda(n-1)]}\left(1+\sum_{j=1}^{n-2}\frac{[1+\lambda(n-j-1)]\prod_{k=0}^{n-j-2}\left[k+\left|\varphi'(0)\right|\cdot|\tau|\right]}{(n-j-1)!\,[1+\delta(n-j-1)]}\right)\right\} \\
& \times \frac{\prod_{j=0}^{m-1}(u+j+1)}{\prod_{j=0}^{m-1}(u+j+n)} \quad (n \in \mathbb{N}^*), \qquad (13)
\end{aligned}
$$

$$(0 \leq \lambda,\delta \leq 1;\ \gamma,\tau \in \mathbb{C}^*;\ m \in \mathbb{N}^*;\ u \in \mathbb{R}\backslash(-\infty,-1]).$$

Proof. Let the function $f \in \mathcal{A}$ be given by (1). Also let

$$h(z) = z + \sum_{n=2}^{\infty} h_n z^n \in \mathcal{SQ}_\varphi(\lambda,\gamma,\delta,\tau).$$

We then deduce from Definition 1.6 that

$$a_n = \frac{\prod_{j=0}^{m-1}(u+j+1)}{\prod_{j=0}^{m-1}(u+j+n)} h_n \qquad (n \in \mathbb{N}^*,\ u \in \mathbb{R}\backslash(-\infty,-1]).$$

Thus, by using Theorem 2.2 in conjunction with the above equality, we have assertion (13) of Theorem 2.3. This completes the proof of Theorem 2.3. □

3. Corollaries and consequences

In this section, we apply our main results (Theorems 2.2 and 2.3) in order to deduce each of the following corollaries and consequences.

Setting

$$\varphi(z) = \frac{1+Az}{1+Bz} \qquad (-1 \leq B < A \leq 1;\ z \in \mathbb{U}),$$

in Theorems 2.2 and 2.3, we get Corollaries 3.1 and 3.2, respectively.

Corollary 3.1. *Let the function $f \in \mathcal{A}$ be defined by* (1). *If $f \in \mathcal{KQ}(\lambda,\gamma,\delta,\tau,A,B)$, then*

$$
\begin{aligned}
|a_n| \leq & \frac{\prod_{k=0}^{n-2}[k+|\tau|(A-B)]}{n!\,[1+\delta(n-1)]} \\
& + \frac{\left|\gamma\right|(A-B)}{n\,[1+\lambda(n-1)]}\left(1+\sum_{j=1}^{n-2}\frac{[1+\lambda(n-j-1)]\prod_{k=0}^{n-j-2}[k+|\tau|(A-B)]}{(n-j-1)!\,[1+\delta(n-j-1)]}\right) \quad (n \in \mathbb{N}^*),
\end{aligned}
$$

$$\left(g \in \mathcal{S}_\varphi(\delta,\tau);\ 0 \le \lambda,\delta \le 1;\ \gamma,\tau \in \mathbb{C}^*;\ -1 \le B < A \le 1\right).$$

Corollary 3.2. *Let the function $f \in \mathcal{A}$ be defined by* (1)*. If $f \in \mathcal{DK}(\lambda,\gamma,\delta,\tau,A,B,m;u)$, then*

$$|a_n| \le \left\{ \frac{\prod_{k=0}^{n-2}[k+|\tau|(A-B)]}{n!\,[1+\delta(n-1)]} + \frac{|\gamma|(A-B)}{n\,[1+\lambda(n-1)]}\left(1+\sum_{j=1}^{n-2}\frac{[1+\lambda(n-j-1)]\prod_{k=0}^{n-j-2}[k+|\tau|(A-B)]}{(n-j-1)!\,[1+\delta(n-j-1)]}\right)\right\}\frac{\prod_{j=0}^{m-1}(u+j+1)}{\prod_{j=0}^{m-1}(u+j+n)} \qquad (n \in \mathbb{N}^*),$$

$$(0 \le \lambda,\delta \le 1;\ \gamma,\tau \in \mathbb{C}^*;\ -1 \le B < A \le 1;\ m \in \mathbb{N}^*;\ u \in \mathbb{R}\setminus(-\infty,-1]).$$

Remark 2. It is easy to see that

$$k+|\tau|(A-B) \le k+\frac{2|\tau|(A-B)}{1-B} \qquad (k \in \mathbb{N}^*,\ -1 \le B < A \le 1,\ \tau \in \mathbb{C}^*),$$

which would obviously yield significant improvements over [10, Theorems 1 and 2], with $\delta = \lambda$ and $\tau = 1$ in Corollaries 3.1 and 3.2, respectively.

Setting

$$\varphi(z) = \frac{1+(1-2\beta)z}{1-z} \qquad (0 \le \beta < 1;\ z \in \mathbb{U}),$$

in Theorems 2.2 and 2.3, we get Corollaries 3.3 and 3.4, respectively.

Corollary 3.3. *Let the function $f \in \mathcal{A}$ be defined by* (1)*. If $f \in \mathcal{KQ}(\lambda,\gamma,\delta,\tau,\beta)$, then*

$$|a_n| \le \frac{\prod_{k=0}^{n-2}[k+2|\tau|(1-\beta)]}{n!\,[1+\delta(n-1)]} + \frac{2|\gamma|(1-\beta)}{n\,[1+\lambda(n-1)]}\left(1+\sum_{j=1}^{n-2}\frac{[1+\lambda(n-j-1)]\prod_{k=0}^{n-j-2}[k+2|\tau|(1-\beta)]}{(n-j-1)!\,[1+\delta(n-j-1)]}\right) \qquad (n \in \mathbb{N}^*),$$

$$\left(g \in \mathcal{S}_\varphi(\delta,\tau);\ 0 \le \lambda,\delta \le 1;\ \gamma,\tau \in \mathbb{C}^*;\ 0 \le \beta < 1\right).$$

Corollary 3.4. *Let the function* $f \in \mathcal{A}$ *be defined by* (1)*. If* $f \in \mathcal{BK}(\lambda, \gamma, \delta, \tau, \beta; u)$*, then*

$$
\begin{aligned}
|a_n| \quad \leq \quad & \left\{ \frac{\prod_{k=0}^{n-2} [k + 2|\tau|(1-\beta)]}{n!\,[1 + \delta(n-1)]} \right. \\
& \left. + \frac{2|\gamma|(1-\beta)}{n\,[1 + \lambda(n-1)]} \left(1 + \sum_{j=1}^{n-2} \frac{[1 + \lambda(n-j-1)] \prod_{k=0}^{n-j-2} [k + 2|\tau|(1-\beta)]}{(n-j-1)!\,[1 + \delta(n-j-1)]} \right) \right\} \\
& \times \frac{\prod_{j=0}^{m-1} (u + j + 1)}{\prod_{j=0}^{m-1} (u + j + n)} \qquad (n \in \mathbb{N}^*),
\end{aligned}
$$

$$
(0 \leq \lambda, \delta \leq 1;\ \gamma, \tau \in \mathbb{C}^*;\ 0 \leq \beta < 1;\ m \in \mathbb{N}^*;\ u \in \mathbb{R} \backslash (-\infty, -1]).
$$

Remark 3. Taking $\delta = \lambda$, $\tau = 1$ and $m = 2$ in Corollaries 3.3 and 3.4, we have [9, Theorems 1 and 2], respectively.

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