Filomat 32:12 (2018), 4275–4284
https://doi.org/10.2298/FIL1812275D

Published by Faculty of Sciences and Mathematics, University of Niš, Serbia
Available at: http://www.pmf.ni.ac.rs/filomat

Generalized Cascade Orthogonal Filters based on Symmetric Bilinear Transformation with Application to Modeling of Dynamic Systems

Nikola Danković[a], Dragan Antić[a], Saša Nikolić[a], Staniša Perić[a], Miodrag Spasić[a]

[a] *Department of Control Systems, Faculty of Electronic Engineering, University of Niš, 18000 Niš, Serbia*

Abstract. This paper presents generalized orthogonal cascade filters based on bilinear transformation for mapping poles to zeroes and zeroes to poles in transfer functions. The filters are orthogonal with respect to a new generalized inner product. Actually, they represent a generalization of several classes of existing traditional filters: the ones obtained by using linear transformation of poles to zeroes, and the ones obtained by reciprocal transformation of poles to zeroes. Generalized filters provide obtaining more precise models of dynamic systems. This is verified by comparison between models based on new filters and models based on classical filters. Practical realization of these filters with adjusting parameters of bilinear transformation and transfer function is performed. An application in modeling continuous-time systems as a complex industrial process is given, and it is shown that in that way we obtain more quality models than by using the classical filters.

1. Introduction

In the second half of the 20th century several classes of rational orthogonal basis functions, which were used for designing new classes of orthogonal filters, were developed [5, 10, 11, 20, 21]. From these classes of rational functions orthogonal Müntz polynomials were derived [4, 14, 22].

The method of obtaining a class of Müntz polynomials orthogonal with respect to a special inner product from orthogonal rational Malmquist function is presented by [18]. A generalization of this method is given by [9, 13].

It can be noticed that the most of already existing classes of orthogonal rational functions are obtained by using either a linear transformation of poles to zeroes $s \to as + b$ or reciprocal transformation $s \to b/(cs)$ [6, 12].

This class of orthogonal rational functions is used for designing a new class of orthogonal filters [7, 9, 19].

In this paper, the orthogonal filters which represent a generalization of filters obtained by using linear and reciprocal mapping poles to zeroes are presented. These filters are designed using a sequence of rational functions where zeroes are obtained via symmetric bilinear transformation $s \to (as + b)/(cs - a)$. Filters

2010 *Mathematics Subject Classification*. Primary 42C05; Secondary 33C47
Keywords. Orthogonal rational function, inner product, bilinear transformation, orthogonal filter, Müntz polynomial
Received: 30 November 2017; Accepted: 05 July 2018
Communicated by Dragan S. Djordjević
This paper was supported by the Ministry of Education, Science and Technological Development of the Republic of Serbia within the framework of technological development, integrated and interdisciplinary research for the period 2011-2017 (projects TR 35005, III 43007 and III 44006).
Email addresses: nikola.dankovic@elfak.ni.ac.rs (Nikola Danković), dragan.antic@elfak.ni.ac.rs (Dragan Antić), sasa.s.nikolic@elfak.ni.ac.rs (Saša Nikolić), stanisa.peric@elfak.ni.ac.rs (Staniša Perić), miodrag.spasic@elfak.ni.ac.rs (Miodrag Spasić)

designed in this way are more general than classical Laguerre, Legendre ($c = 0$), and Takenaka-Malmquist ($a = 0, b \neq 1$) and generalized Malmquist filters ($a = 0, b \neq 1$). The filters obtained by using bilinear transformation are orthogonal with respect to a new generalized inner product which is degenerated into the classical inner product for $c = 0$.

The method for mapping poles to zeroes was also used for design of generalized cascade orthogonal filters [19], quasi-orthogonal filters [2, 16], almost orthogonal filters [1, 3, 8, 17], and the filters which are orthogonal with respect to a special inner product [9].

The filters obtained by mapping poles to zeroes using symmetric bilinear transformation are a generalization of all the mentioned filters. By adjusting the parameters of bilinear transformation, in some cases, the obtained filters could be superior to the classical ones with respect to specific criteria functions for estimation of matching between the system which is being identified and its model.

2. Orthogonal Filters Based on Mapping Poles to Zeroes Using Symmetric Bilinear Transformation

By using Legendre orthogonal functions

$$W_n(x) = \prod_{j=0}^{n-1} \frac{s + \alpha_j + 1}{s - \alpha_j} \frac{1}{s - \alpha_n}, \tag{1}$$

Müntz-Legendre polynomials, which are orthogonal on the interval $(0, 1)$, were obtained by [6, 14, 22]

$$P_n(x) = \frac{1}{2\pi i} \oint_\Gamma \prod_{j=0}^{n-1} \frac{s + \alpha_j + 1}{s - \alpha_j} \frac{x^s}{s - \alpha_n} ds, \tag{2}$$

where the contour Γ surrounds all the poles of the integrand. Functions $W_n(x)$ are used for designing orthogonal Legendre filters. MüntzLegendre polynomials are used for obtaining outputs from these filters. Let us notice that in these filters the zeroes are obtained by linear transformation of the poles.

Orthogonal Laguerre functions where the zeroes have reciprocal values of the poles

$$W_n(s) = \frac{\sqrt{1 - \alpha^2}}{1 - \alpha} \left(\frac{1 - \alpha s}{s - \alpha} \right)^{n-1} \tag{3}$$

are used for design of orthogonal Laguerre filters [12, p. 319].

The Malmquist (Takenaka-Malmquist) rational function system [10, 11, 15, 18]

$$W_n(s) = \frac{\sqrt{1 - \alpha_n{}^2}}{1 - \alpha_n s} \prod_{k=0}^{n-1} \frac{\alpha_k - s}{1 - \alpha_k s}, n = 0, 1, 2, ... \tag{4}$$

is used for designing appropriate orthogonal filters (Takenaka-Malmquist filters, [7, 12, 18][1)]). Let us notice that the zeroes of have reciprocal values of the poles.

The generalization of Malmquist functions was performed in [9] by using mapping $s \to b/s$

$$W(s) = b_0 \frac{1}{s + \alpha_0} + \sum_{l=1}^{3} b_l \frac{1}{s + \alpha_0} \prod_{k=1}^{l} \frac{s + \dfrac{b}{\alpha_k - 1}}{s + a_k}. \tag{5}$$

By using these functions and the following relation

$$P_n(x) = \frac{1}{2\pi i} \oint_\Gamma W_n(s) x^s ds \tag{6}$$

[1)] [12, p. 32]

a class of orthogonal Müntz polynomials which are orthogonal with respect to the special inner product was derived. In this way, generalized Takenaka-Malmquist filters are designed. Outputs of these filters are obtained using (6). A new class of Müntz polynomials derived from a sequence of orthogonal Malmquist functions, is introduced by [18]. These Müntz polynomials are orthogonal with respect to the special inner product. They have been applied for design of a new class of filters (generalized Malmquist type) which are orthogonal with respect to a new special inner product [9].

Hence, the zeroes in the above mentioned rational orthogonal basis functions are obtained either by linear transformation ($s \to as + b$) or reciprocal transformation ($s \to b/(cs)$) of poles. Transfer functions of the orthogonal filters are $W_n(s)$, and the outputs are obtained using Müntz polynomials derived by (4). In this paper, a class of orthogonal cascade filters which represents a generalization of all the mentioned filters obtained by using linear and reciprocal mapping poles to zeroes, will be introduced. The generalization is performed by using symmetric bilinear transformation

$$s \to \frac{as+b}{cs-a}, (a^2+bc>0), (a,b,c \in R). \tag{7}$$

By using this transformation for mapping poles α_k to zeroes α_k^* we obtain

$$\alpha_k^* = \frac{a\overline{\alpha_k}+b}{c\overline{\alpha_k}-a}. \tag{8}$$

For design of orthogonal cascade filters with real poles, with taking into account that the bilinear transformation is symmetric, we have

$$\alpha_k^* = \frac{a\alpha_k+b}{c\alpha_k-a}, \alpha_k = \frac{a\alpha_k^*+b}{c\alpha_k^*-a}. \tag{9}$$

The sequence of the appropriate rational functions has the following form now

$$W_n(s) = \frac{\prod_{k=0}^{n-1}\left(s - \dfrac{a\alpha_k+b}{c\alpha_k-a}\right)}{\prod_{k=0}^{n}(s-\alpha_k)}. \tag{10}$$

If we apply transformation (7) onto $W_n(s)$ we obtain

$$W_n^*(s) = \frac{\prod_{k=0}^{n-1}(s-\alpha_k)}{\prod_{k=0}^{n}\left(s - \dfrac{a\alpha_k+b}{c\alpha_k-a}\right)}. \tag{11}$$

Let us consider the inner product

$$(W_n, W_m) = \frac{1}{2\pi i}\oint_\Gamma W_n(s)W_m^*(s)ds, \tag{12}$$

where the contour Γ surrounds all the poles of $W_n(s)$. If $m \neq n$ due to symmetry of the bilinear transformation, all the poles of the integrand (12) that lie inside the contour Γ are annulled with appropriate zeros of $W_m^*(s)$, so the contour integral (12) is equal to zero. In the case of $m = n$, there exists one first-order pole inside the contour Γ. After applying Cauchy theorem, we can obtain the following expression: $(W_n(s), W_m(s)) = N_n^2 \neq 0$. Finally, all the expressions stated above imply

$$(W_n, W_m) = N_n^2\delta_{n,m} \tag{13}$$

where $\delta_{n,m}$ represents Kronecker symbol, and poles α_k lie inside the contour Γ , while all the zeroes α_k^* (9) lie outside the contour Γ. Thus, the sequence of the rational functions $W_n(s)$ is orthogonal in the complex domain bordered by the contour Γ with respect to the inner product (13).

Using the sequence $W_n(s)$ we can obtain a class of orthogonal Müntz polynomials based on (6) in the following way [13, 18]

$$P_n(x) = \frac{1}{2\pi i} \oint_{\Gamma} \frac{\prod_{k=0}^{n-1} \left(s - \dfrac{a\overline{\alpha_k} + b}{c\overline{\alpha_k} - a} \right)}{\prod_{k=0}^{n} (s - a_k)} ds. \tag{14}$$

These polynomials can be written as

$$P_n(x) = \sum_{k=0}^{n} A_{n.k} x^{\alpha_k}, \tag{15}$$

where

$$A_{n,k} = \frac{\prod_{j=0}^{n-1} \left(\alpha_k - \dfrac{a\overline{\alpha}_j + b}{c\overline{\alpha}_j - a} \right)}{\prod_{j=0, j\neq k}^{n} \left(\alpha_k - \alpha_j \right)}, k = 0, 1, 2, ..., n.$$

It is shown that these Müntz polynomials are orthogonal with respect to an inner product which is defined below.

First, the operation $\otimes$ on monoms x^α and x^β is defined in the following way [13]

$$x^\alpha \otimes x^\beta = x^{c\alpha\beta - a(\alpha+\beta) - b}. \tag{16}$$

Using this operation, the product of two Müntz polynomials, $P_n(x) = \sum_{k=0}^{n} p_k x^{\alpha_k}$ and $P_m(x) = \sum_{j=0}^{m} q_j x^{\alpha_j}$ can be defined in the following manner

$$P_n(x) \otimes Q_m(x) = \sum_{k=0}^{n} \sum_{j=0}^{m} p_k q_j x^{c\alpha_k \alpha_j} - a(\alpha_k + \alpha_j)^{-b}. \tag{17}$$

Using this product of Müntz polynomials, a new inner product can be defined as

$$(P_n(x), P_m(x))_\otimes = \int_0^1 P_n(x) \otimes \overline{P_n}(x) \frac{dx}{x^2}. \tag{18}$$

Finally, by using (12) and (18) we obtain [13]

$$(P_n(x), P_m(x))_\otimes = \frac{\left(a^2 + bc \right)^n}{\prod_{k=0}^{n-1} |c\alpha_k - a|^2} \frac{\delta_{n,m}}{c|\alpha_n|^2 - 2aRe\alpha_n - b} = N_n^2 \delta_{n,m}. \tag{19}$$

Hence, Müntz polynomials (6) derived from orthogonal rational functions $W_n(s)$ are orthogonal on interval $(0, 1)$ with respect to the inner product (18). If rational functions $W_n(s)$ have real poles, then Müntz polynomials $P_n(x)$ are with real exponents. In that case, substituting $x = e^{-t}$ into $P_n(x)$, we obtain exponential functions

$$\varphi_n(t) = P_n(e^{-t}) = \sum_{k=0}^{n} A_{n,k} e^{-\alpha_k t}. \tag{20}$$

Using(13), (16), and (20) we obtain

$$(\varphi_n(t), \varphi_m(t))_\otimes = \int_0^\infty \varphi_n(t) \otimes \varphi_m(t) e^{-t} dt = N_n^2 \delta_{n,m}, \tag{21}$$

where: $(\varphi_n(t)\varphi_m(t)) \sum_{k=0}^{n} A_{n,k}e^{-\alpha_k t} \otimes \sum_{j=0}^{m} A_{m,j}e^{-\alpha_j t} = \sum_{k=0}^{n}\sum_{j=0}^{m} A_{n,k}A_{m,j}e^{-[c\alpha_k\alpha_j - a(\alpha_k+\alpha_j)-b]}$. This is a new generalized inner product based on the bilinear transformation of poles to zeroes.

Functions $W_n(s)$ and $\varphi_n(t)$ will be used for designing the new class of cascade filters which are orthogonal with respect to the new inner product (21). In order to obtain the transfer function of the filters which are designed, we need to rewrite the expression (10) as

$$W_n(s) \to \frac{1}{s-\alpha_0}\prod_{k=1}^{n}\frac{s-\dfrac{a\alpha_{k-1}+b}{c\alpha_{k-1}-a}}{s-\alpha_k}. \tag{22}$$

Complete transfer function of the filter fits the Fig. 1.

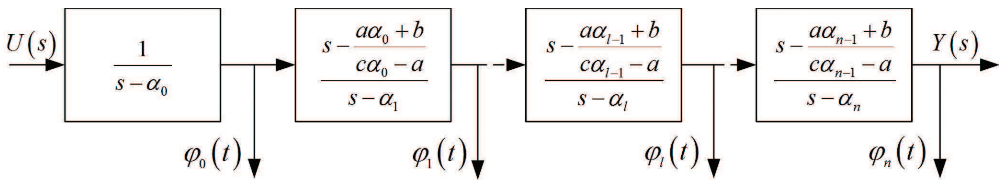

Figure 1: Block diagram of the designed filter.

Outputs of this filter can be determined on the basis of (12) and (18), i.e.

$$\varphi_l(t) = \sum_{k=0}^{l} A_{lk}e^{-\alpha_k t}, l = 1, 2, ..., n. \tag{23}$$

In this way, we obtain a generalized orthogonal filter by using the bilinear transformation. In complex s-domain, outputs of the filter are orthogonal with respect to the inner product (13). Outputs of the filter in time domain are

$$\varphi_l(t) = \mathcal{L}^{-1}\{W_l(s)\} = \frac{1}{2\pi i}\oint_{\Gamma} W_l(s)e^{st}ds = P_l(e^t), \tag{24}$$

where: $W_l(s) = \prod_{k=0}^{l}\dfrac{s-\dfrac{a\alpha_{k-1}+b}{c\alpha_{k-1}-a}}{s-\alpha_k}, l = 0, 1, 2, ..., n$, P_l are Müntz polynomials derived from $W_l(s)$ by using (10). Filter outputs $\varphi_l(t)$ are orthogonal in the time domain on $(0,\infty)$ with respect to the inner product (21).

A new class of orthogonal filters derived above represents a generalization of classical orthogonal filters. Therefore, filters of Legendre type are obtained for the following values of parameters of bilinear transformation $a \neq 0, b \neq 0, c = 0$ and where poles a_k are integer, while filters of Laguerre type are obtained when poles have the same values. Filters of generalized Legendre type are obtained in the case when poles are arbitrary real numbers and $a \neq 0, b \neq 0, c = 0$. Takenaka-Malmquist filters are obtained for $a = 0, b = 1, c = 1$, while generalized Malmquist filters are obtained for $a = 0, b \neq 0, c \neq 0$. New filters proposed in this paper when $a \neq 0, b \neq 0, c \neq 0$ include all the mentioned classes and have all the good performances of them.

3. Practical Realization of the New Orthogonal Filter Based on Bilinear Transformation for Mapping Poles to Zeroes

The sequence of orthogonal rational functions (22) given in the previous section is used for designing a new class of orthogonal filters

$$W_n(s) \to \frac{1}{s-\alpha_0}\prod_{k=1}^{n}\frac{s-\dfrac{a\alpha_{k-1}+b}{c\alpha_{k-1}-a}}{s-\alpha_k}, \alpha_k \in R, \alpha_k \geq 0. \tag{25}$$

Relation (25) represents a transfer function of the new orthogonal filter, and it is a base for its practical realization (Fig. 2).

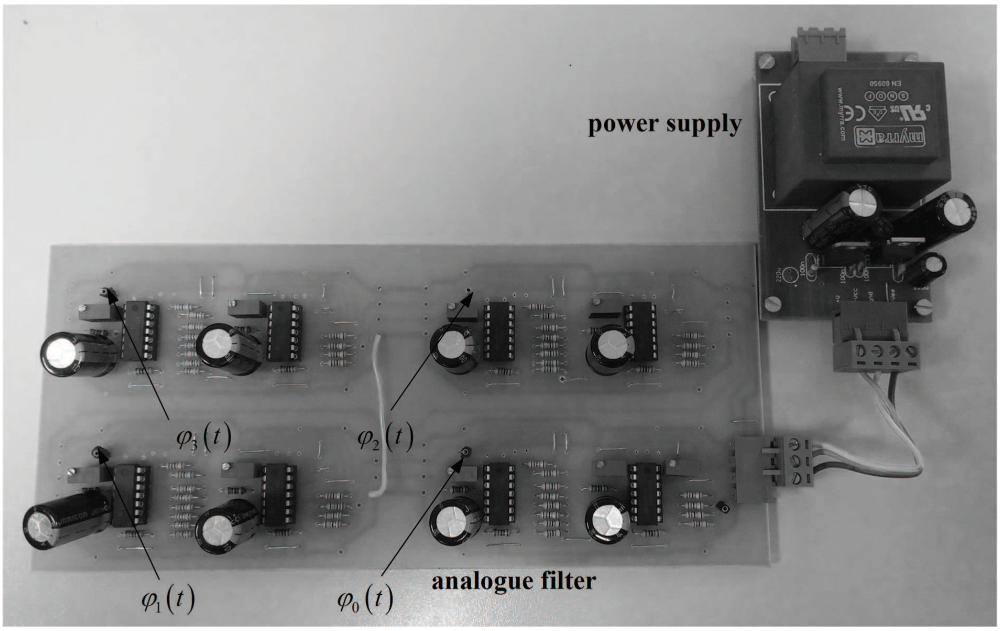

Figure 2: An orthogonal filter based on bilinear transformation, a printed circuit board.

This filter is orthogonal in the complex domain with respect to the new inner product (19) on the contour Γ and in the time domain with respect to the new inner product (21).

For illustrative purposes, a sequence of functions on the outputs in the time domain of the first few cascades of the proposed analogue filter (25) for the following values of poles $\alpha_0 = -2, \alpha_1 = -3, \alpha_2 = -4, \alpha_3 = -5, \alpha_4 = -6$ and parameters of bilinear transformation $a = 1, b = 1, c = 1$ is shown. Mathematically obtained outputs using Müntz polynomials (20) are

$$\begin{aligned}
\varphi_0(t) &= e^{-2t}\\
\varphi_1(t) &= \frac{10}{3}e^{-3t} - \frac{7}{3}e^{-2t}\\
\varphi_2(t) &= \frac{39}{4}e^{-4t} - \frac{35}{3}e^{-3t} - \frac{35}{12}e^{-2t}\\
\varphi_3(t) &= \frac{1232}{45}e^{-5t} - \frac{897}{20}e^{-4t} - 21e^{-3t} - \frac{9}{36}e^{-2t}\\
\varphi_4(t) &= \frac{2717}{36}e^{-6t} - \frac{20944}{135}e^{-5t} - \frac{2093}{20}e^{-4t} - \frac{77}{3}e^{-3t} - \frac{91}{54}e^{-2t}
\end{aligned} \tag{26}$$

The outputs from the realized filter are shown in Fig. 3.

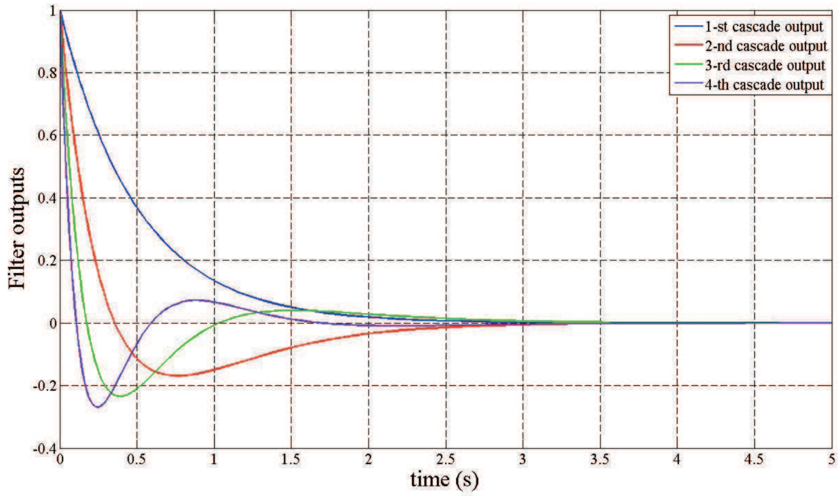

Figure 3: Outputs of the analogue filter, signals sensed on a printed circuit board.

Orthogonality with respect to the new inner product of these outputs can be verified using relation (19), i.e.: $\int_0^\infty \varphi_n(t) \otimes \varphi_m(t)dt = N_n^2\delta_{n,m}$, and relation $e^{\alpha_k t} \otimes e^{\alpha_j t} = e^{-[\alpha_l\alpha_j-(\alpha_k+\alpha_j)-1]}$.

4. Case Study Application in Modeling of a Protector Cooling System

The newly designed cascade orthogonal filter has been applied in modeling of one technological process in tire industry (Tigar Tyres, a radial tire factory). It is a process of protector (a rubber strip which forms the outer part of a tire) cooling [2, 19, 23]. This is a complex electromechanical and thermodynamic system which consists of $5-16$ about $12-20m$ long cascades. There are about several hundred systems like that in the world. The cascade modeling is performed successively, starting with the first one using the genetic algorithm. We used the adjustable model shown in Fig. 4. The outputs of cascades in the model are given by

$$y_{M,i} = \sum_{l=0}^{3} b_l\varphi_l(t), i = 1, 2, ..., N, \tag{27}$$

where N is a number of cascades of the system, and b_l are summation coefficients.

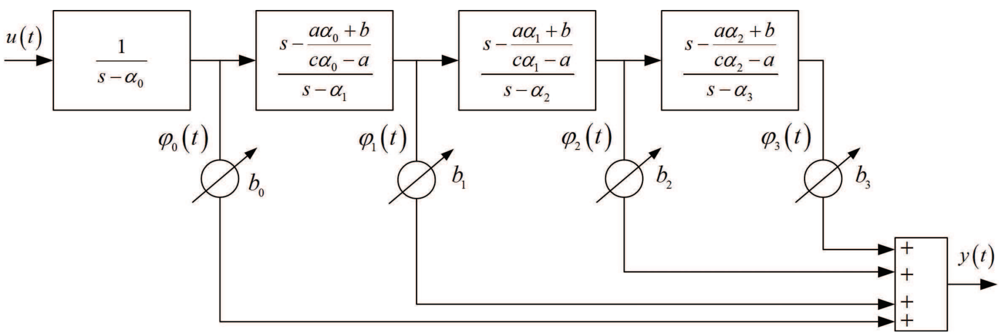

Figure 4: Block diagram of an adjustable model with the proposed orthogonal filter.

Step input was used for determining a model of the first cascade, while for modeling of other cascades, we used the output of the previous cascade (e.g. for i-th cascade's input, (i-1)-th cascade's output is used). The outputs of the first cascade in the process, $y_S(t)$, and the model based on the proposed filter, $y_M(t)$, are given in Fig. 5.

Using a genetic algorithm [2, 19] with minimization of the mean squared error

$$J = \frac{1}{T}\int_0^T (y_S(t) - y_M(t))^2\, dt,$$

we obtained poles of the process α_i, summation coefficients b_i, and parameters of bilinear transformation a, b and c. The genetic algorithm used in the experiments has the following parameters: an initial population of 800 individuals, a number of generations of 400, a stochastic uniform selection, a reproduction with ten elite individuals, and Gaussian mutation with shrinking. The used structure of chromosome was with 11 parameters coded by real numbers: $\alpha_0, \alpha_1, \alpha_2, \alpha_3, a, b$, and c. The main goal of the experiments was to obtain the best model of the unknown system in regard to the given criteria function J. The obtained results are shown in Table 1.

In order to verify the quality of the model based on the new filter with four sections, a comparison with the models based on the some already existing orthogonal filters (Laguerre (3), generalized Legendre ($W_n(s)\dfrac{1}{s+\alpha_0}\prod_{i=1}^{3}\dfrac{s-\alpha_{i-1}+\lambda}{s+\alpha_i}$) and generalized Malmquist filters (5) is performed. The criteria function is J again and the number of sections is four. The output of the models based on these filters are calculated as $y_M(t) = \sum_{l=0}^{3} b_l \varphi_l(t)$. In all the experiments the structure of chromosome that is used is with 8 standard parameters coded by real numbers: $\alpha_0, \alpha_1, \alpha_2, \alpha_3, \beta_0, \beta_1, \beta_2$, and β_3 and two additional parameters: λ for generalized Legendre filters, and b for generalized Malmquist filters).

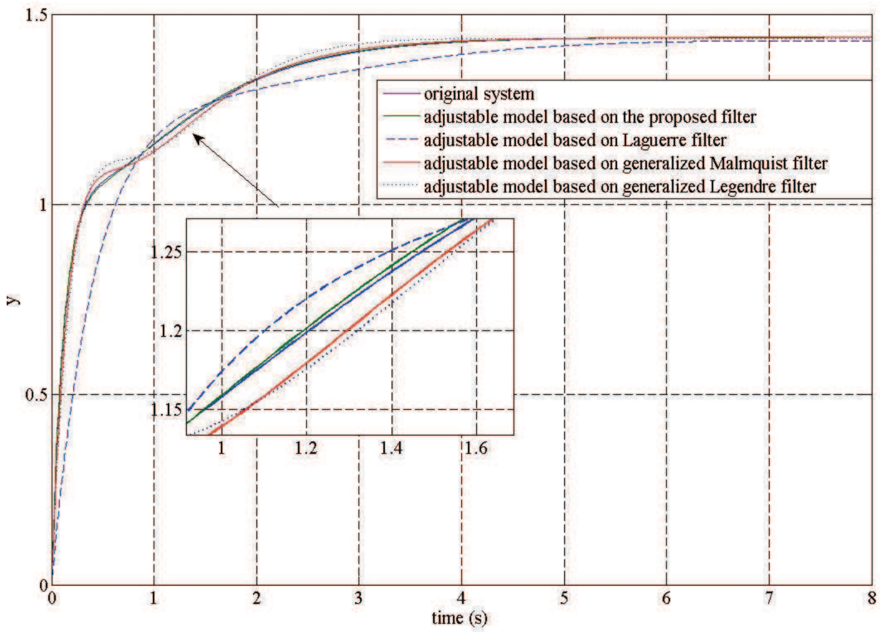

Figure 5: Outputs of the first cascade of a protector cooling system and the adjustable models.

Criteria function / orthogonal model	i	0	1	2	3
$J = 0.0331 * 10^{-3}$ / orthogonal model with a new filter based on	α_i	1.14	2.27	4.68	7.64
bilinear transformation ($a = 0.26, b = 0.62, c = 3.38$)	b_i	1.28	0.57	3.90	7.64
$J = 0.6465 * 10^{-3}$ / orthogonal model with generalized Malmquist	α_i	1.78	1.18	4.63	6.42
filter ($b = 0.84$)	b_i	2.16	−0.19	3.27	3.64
$J = 0.8132 * 10^{-3}$ / orthogonal model with generalized Legendre	α_i	0.38	0.76	0.98	1.95
filter ($\lambda = 0.58$)	b_i	0.88	1.02	0.67	0.47
$J = 2.5071 * 10^{-3}$ / orthogonal model with Laguerre filter	α_i	2.19	2.19	2.19	2.19
	b_i	3.17	−0.60	2.09	2.03

Table 1: Obtained parameters of the orthogonal models based on a new and existing types of filters

The model of the first cascade has the following form

$$W_1(s) = b_0\frac{1}{s+\alpha_0} + \sum_{k=1}^{3} b_l \frac{1}{s+\alpha_0} \prod_{k=1}^{l} \frac{s + \dfrac{a\alpha_{k-1}+b}{c\alpha_{k-1}-a}}{s+\alpha_k}, \tag{28}$$

where obtained values for parameters of bilinear transformation, poles, and summation coefficients are given in Table 1.

From Table 1 we can see that the mean squared error is much bigger for the classic orthogonal filters (Laguerre and Legendre filters), and also for generalized Malmquist filter [9], than for the filter presented in this paper. The excellent matching between the system output and the output of the adjustable model based on the proposed filter (Fig. 5) has shown the quality and need for new filters described in this paper. Finally, a transfer function of the model based on new orthogonal filters by using (28) and obtained values in Table 1 can be written as

$$W_1(s) = 1.44\left(\frac{0.067s^3 + 0.496s^2 + 1.131s + 1}{0.011^4 + 0.17s^3 + 0.970s^2 + 1.662s + 1}\right). \tag{29}$$

Modeling of other cascades of the protector cooling system is performed in the same manner. Models of other cascades are very similar to (29). Namely, gains of other cascades have very close values because contractions of the rubber strip which is cooled become smaller and smaller. Finally, the gains of the second to the last cascade and the last cascade are equal.

5. Conclusion

This paper presents a new class of cascade orthogonal filters based on use of bilinear transformation for mapping poles to zeroes and vice versa. By using this transformation we obtained a sequence of generalized Malmquist functions which were applied in designing of new filters. New filters are orthogonal in the complex domain on the contour which surrounds all the poles of the filters, while zeroes lie outside the contour. From generalized Malmquist functions a new class of Müntz polynomials which are orthogonal with respect to the new inner product was obtained. Outputs of the filter are determined using these polynomials. The outputs are also orthogonal in the time domain with respect to the special inner product. The proposed filter is adjustable because parameters of bilinear transformation can be set. The filters obtained in this way present a generalization of the existing classes of orthogonal filters: Legendre, Laguerre, Takenaka-Malmquist, generalized Malmquist filters, etc. Each of these classes can be generated by choosing the specific values of parameters of bilinear transformation. Having in mind that proposed orthogonal filters provide adjustments of parameters of bilinear transformation, beside the adjustment of the values of the poles, we expect better results in modeling of dynamic systems regarding to the mentioned classical orthogonal filters. These filters can be applied in determining mathematical models using different criterions for optimizations of the process model (*min*/*max* criterion, or criterions used in adaptive control). The quality of the new class of cascade orthogonal filters is demonstrated in the case of determining a model of the complex technological process in tire industry. This model can be used for adaptive control design of this process. Design of more general cascade orthogonal filters, based on mapping poles to zeroes using more general symmetric transformation, could be of interest in some future works.

References

[1] D. Antić, B. Danković, S. Nikolić, M. Milojković, Z. Jovanović, Approximation based on orthogonal and almost orthogonal functions, Journal of the Franklin Institute 349(1) (2012) 323–336.

[2] D. Antić, Z. Jovanović, V. Nikolić, M. Milojković, S. Nikolić, N. Danković, Modelling of Cascade-Connected Systems using Quasi-Orthogonal Functions, Elektronika ir Elektrotechnika 18(10) (2012), DOI: http://dx.doi.org/10.5755/j01.eee.18.10.3051.

[3] D. Antić, Z. Jovanović, V. Nikolić, M. Milojković, S. Nikolić, N. Danković, Sensitivity analysis of imperfect systems using almost orthogonal filters, Acta Polytechnica Hungarica 8(6) (2011) 79–94.

[4] G. Badalyan, Generalisation of Legendre polynomials and some of their applications, Akad. Nauk. Armyan. SSR Izv. Ser. Fiz.-Mat. Estest. Tekhn. Nauk 8(5) (1955), 1–28.

[5] P. Bodin, L. F. Villemoes, B. Wahlberg, Selection of Best Orthonormal Rational Basis, SIAM Journal on Control and Optimization 38(4) (2000) 995–1032.

[6] P. Borwein, T. Erdlyi, J. Zhang, Müntz Systems and Orthogonal Müntz-Legendre Polynomials, Transactions of the American Mathematical Society 342(2) (1994) 523–542.

[7] B. Danković, G. Milovanović, S. Rančić, Malmquist and Müntz orthogonal systems and applications, Inner product spaces and applications, Harlow: Addison-Wesley Longman (1997) 22–41.

[8] B. Danković, S. Nikolić, M. Milojković, Z. Jovanović, A class of almost orthogonal filters, Journal of Circuits, Systems and Computers 18(5) (2009) 923–931.

[9] N. Danković, D. Antić, S. Nikolić, S. Perić, M. Milojković, A new class of cascade orthogonal filters based on a special inner product with application in modeling of dynamical systems, Acta Polytechnica Hungarica 13(7) (2016) 63–82.

[10] M. Djrbashian, Orthogonal systems of rational functions on the circle with given set of poles, Doklady Akademii Nauk SSSR 147 (1962) 1278–1281.

[11] M. Djrbashian, A Survey on the Theory of Orthogonal System and Some Open Problems, Orthogonal Polynomials: Theory and Practice, Springer Netherlands (1990) 135–146.

[12] P. Heuberger, P. M. J. Van den Hof, B. Wahlberg, Modelling and identification with rational orthogonal basis functions, Springer-Verlag (2005).

[13] S. Marinković, B. Danković, M. Stanković, P. Rajković, Orthogonality of some sequences of the rational functions and the Müntz polynomials, Journal of Computational and Applied Mathematics 163(2) (2004) 419–427.

[14] P. Mccarthy, J. Sayre, B. Shawyer, Generalized Legendre Polynomials, Journal of Mathematical Analysis and Applications 177(2) (1993) 530–537.

[15] W. Mi, T. Qian, F. Wan, A fast adaptive model reduction method based on Takenaka-Malmquist systems, Systems & Control Letters 61(1) (2012) 223–230.

[16] M. Milojković, D. Antić, S. Nikolić, Z. Jovanović, S. Perić, On a new class of quasi-orthogonal filters, International Journal of Electronics 100(10) (2013) 1361–1372.

[17] M. Milojković, S. Nikolić, B. Danković, D. Antić, Z. Jovanović, Modelling of dynamical systems based on almost orthogonal polynomials, Mathematical and Computer Modelling of Dynamical Systems 16(2) (2010) 133–144.

[18] G. Milovanović, B. Danković, S. Rančić, Some Müntz orthogonal systems, Journal of Computational and Applied Mathematics 99(1) (1998) 299–310.

[19] S. Nikolić, D. Antić, S. Perić, N. Danković, M. Milojković, Design of generalised orthogonal filters: application to the modelling of dynamical systems, International Journal of Electronics 103(2) (2016) 269–280.

[20] , B. Ninness, F. Gustafsson, A unifying construction of orthonormal bases for system identification, IEEE Transactions on Automatic Control 42(4) (1997) 515–521.

[21] G. Szegö, Orthogonal polynomials, Providence, RI: American Mathematical Society, Colloquium (1975).

[22] A. Taslakyan, Some properties of Legendre quasi-polynomials with respect to a Müntz system, Mathematics 2 (1984) 179–189.

[23] D. Trajković, V. Nikolić, D. Antić, B. Danković, Analyzing, modeling and simulation of the cascade connected transporters in tire industry using signal and bond graphs, Machine Dynamics Problems 29(3) (2005) 91–106.