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Partial Soft Separation Axioms and Soft Compact Spaces

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Abstract. The main aim of the present paper is to define new soft separation axioms which lead us, first, to generalize existing comparable properties via general topology, second, to eliminate restrictions on the shape of soft open sets on soft regular spaces which given in [22], and third, to obtain a relationship between soft Hausdorff and new soft regular spaces similar to those exists via general topology. To this end, we define partial belong and total non belong relations, and investigate many properties related to these two relations. We then introduce new soft separation axioms, namely p-soft T_i-spaces ($i = 0, 1, 2, 3, 4$), depending on a total non belong relation, and study their features in detail. With the help of examples, we illustrate the relationships among these soft separation axioms and point out that p-soft T_i-spaces are stronger than soft T_i-spaces, for $i = 0, 1, 4$. Also, we define a p-soft regular space, which is weaker than a soft regular space and verify that a p-soft regular condition is sufficient for the equivalent among p-soft T_i-spaces, for $i = 0, 1, 2$. Furthermore, we prove the equivalent among finite p-soft T_i-spaces, for $i = 1, 2, 3$ and derive that a finite product of p-soft T_i-spaces is p-soft T_i, for $i = 0, 1, 2, 3, 4$. In the last section, we show the relationships which associate some p-soft T_i-spaces with soft compactness, and in particular, we conclude under what conditions a soft subset of a p-soft T_2-space is soft compact and prove that every soft compact p-soft T_2-space is soft T_3-space. Finally, we illuminate that some findings obtained in general topology are not true concerning soft topological spaces which among of them a finite soft topological space need not be soft compact.

1. Introduction

Molodtsov [16] initiated the concept of soft sets in 1999 as a mathematical tool to copy with uncertainties, and he pointed out the merits of soft set theory to solve complicated problems compared with probability theory and fuzzy sets theory. In 2002, Maji et al. [13] presented an application of soft sets in a decision making problem, and in 2005, Chen et al. [8] pointed out some incorrect statements in [13] and proposed a new definition of parameter reduction for soft sets to improve their applications. Usage of algebraic concepts of soft set theory was first studied in [4]. Later on, many researchers investigated the applications of soft sets in algebraic structures (see, for example, [3], [10], [21]). In 2009, Ali et al. [5] defined new operators between two soft sets and illustrated with the help of examples that several assertions which

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were presented in [14] are incorrect.

In 2011, Shabir and Naz [22] employed the notion of soft sets to initiate the concept of soft topological spaces. They defined and studied elementary soft topological notions such as soft closure and soft interior operators, soft subspace and soft separation axioms. Min [15] did corrections for some mistakes in [22]. Hussain and Ahmad [11] studied the properties of soft interior, soft closure and soft boundary operators and investigated some findings that connected among them. The soft product spaces and soft compact spaces were introduced and discussed by Aygünoğlu and Aygün [7]. In 2012, Zorlutuna et al. [25] came up with an idea of soft point and employed it to study some properties of soft interior points and soft neighborhood systems. Also, they introduced a concept of soft continuous maps and discussed its characterizations. In 2013, the authors of [9] and [17] simultaneously modified a concept of soft point in order to study soft metric spaces and keep classical limit points laws true for soft sets. Tantawy et al. [23] utilized a soft points notion which introduced in [9] and [17] to give and study new soft axioms, namely soft T_i-spaces, for $i = 0, 1, 2, 3, 4, 5$. The authors of [1] and [2] proposed some new operators on soft sets by relaxing conditions on a parameters set and investigated their basic properties. Recently, Al-shami [6] pointed out that some results obtained in [23] are not true and corrected them with the help of illustrative examples.

The present paper is organized as follows: Section 2 is the preliminary part where some definitions and properties of soft sets and soft topologies are given. In Section 3, we introduce the notions of partial belong and total non belong relations, which are more effective to theoretical and application studies on soft topological spaces, and derive some results related to them. Then, in section 4, we employ these two new relations to introduce new soft separation axioms, namely p-soft T_i-spaces ($i = 0, 1, 2, 3, 4$). We point out the relationships among them and present several of their properties. One of the most important point in this section is defining a p-soft regular spaces concept, which is weaker than soft regular space [22], and deeply studying its properties. Also, we demonstrate the equivalent between soft T_1 and p-soft T_1-spaces under a condition of soft regular spaces. In the last section, we investigate some properties of soft Lindelöf (soft compact) spaces and prove that every soft compact p-soft T_2-space is p-soft regular and every uncountable (infinite) soft subset of a soft Lindelöf (soft compact) space has a soft limit point. Also, we illuminate an important role of enriched soft topological spaces to preserve a compactness property between soft topological spaces and topological spaces. By constructing some soft topological spaces, we show that some results obtained in general topology need not be true concerning soft topological spaces such as that a soft compact subset of a soft T_2-space need not be soft closed and a finite soft topological spaces need not be soft compact.

2. Preliminaries

We present in this section some definitions and results which will be needed in the sequels. Throughout this work, A, B and E denote to the sets of parameters.

2.1. Soft sets

Definition 2.1. *[16] A pair (G, E) is said to be a soft set over X provided that G is a map of a set of parameters E into 2^X.*
In this work, a soft set is denoted by G_E instead of (G, E) and it is identified with the set $G_E = \{(e, G(e)) : e \in E$ and $G(e) \in 2^X\}$. The set of all soft sets, over X under a parameter set E, is denoted by $S(X_E)$.

Definition 2.2. *[16] For a soft set G_E over X and $x \in X$, we say that $x \in G_E$ if $x \in G(e)$, for each $e \in E$ and $x \notin G_E$ if $x \notin G(e)$, for some $e \in E$.*

Definition 2.3. *[14] A soft set G_E over X is said to be:*

(i) *A null soft set, denoted by $\widetilde{\Phi}$, if $G(e) = \emptyset$, for each $e \in E$.*

(ii) *An absolute soft set, denoted by $\widetilde{X}$, if $G(e) = X$, for each $e \in E$.*

Definition 2.4. *[22] A soft set x_E over X is defined by $x(e) = \{x\}$, for each $e \in E$.*

Definition 2.5. *[18] A soft set G_A is a soft subset of a soft set F_B, denoted by $G_A \widetilde{\subseteq} F_B$, if $A \subseteq B$ and for all $a \in A$, $G(a) \subseteq F(a)$. The soft sets G_A and G_B are soft equal if each one of them is a soft subset of the other.*

Definition 2.6. *[14] The union of two soft sets G_A and F_B over X, denoted by $G_A \widetilde{\bigcup} F_B$, is the soft set H_D, where $D = A \bigcup B$ and a map $H : D \to 2^X$ is given as follows:*

$$V(d) = \begin{cases} G(d) & : \ d \in A - B \\ F(d) & : \ d \in B - A \\ G(d) \bigcup F(d) & : \ d \in A \bigcap B \end{cases}$$

Definition 2.7. *[18] The intersection of two soft sets G_A and F_B over X, denoted by $G_A \widetilde{\bigcap} F_B$, is the soft set H_D, where $D = A \bigcap B$, and a map $H : D \to 2^X$ is given by $V(d) = G(d) \bigcap F(d)$.*

It is noteworthy that many types of soft subset, soft union and soft intersection between two soft sets were given in literature. Also, the soft union and soft intersection operators were generalized for arbitrary number of soft sets. For more details in these topics, we refer the reader to ([1], [2], [5], [20]) and references mentioned therein.

Definition 2.8. *[5] The relative complement of a soft set G_E, denoted by G^c_E, is given by $G^c_E = (G^c)_E$, where $G^c : E \to 2^X$ is a mapping defined by $G^c(e) = X \setminus G(e)$, for each $e \in E$.*

Definition 2.9. *([9], [17]) A soft subset P_E of $\widetilde{X}$ is called soft point if there exists $e \in E$ and there exists $x \in X$ such that $P(e) = \{x\}$ and $P(\alpha) = \emptyset$, for each $\alpha \in E \setminus \{e\}$. A soft point will be shortly denoted by P^x_e.*

Definition 2.10. *[17] A soft set F_E over X under a parameters set E is said to be pseudo constant soft set if $F(e) = X$ or $\emptyset$, for each $e \in E$. A set of all pseudo constant soft sets is denoted by $CS(X_E)$.*

Definition 2.11. *[9] A soft set H_E over X is called a countable (resp. finite) soft set if $H(e)$ is countable (resp. finite) for each $e \in E$.*

Definition 2.12. *[19] Let G_A and H_B be soft sets over X and Y, respectively. Then the cartesian product of G_A and H_B, denoted by $(G \times H)_{A \times B}$, is defined as $(G \times H)(a, b) = G(a) \times H(b)$, for each $(a, b) \in A \times B$.*

Definition 2.13. *[25] A soft mapping between $S(X_A)$ and $S(Y_B)$ is a pair (f, ϕ), denoted also by f_ϕ, of mappings such that $f : X \to Y$, $\phi : A \to B$. Let G_A and H_B be soft subsets of $S(X_A)$ and $S(Y_B)$, respectively. Then the image of G_A and pre-image of H_B are defined by:*

(i) *$f_\phi(G_A) = (f_\phi(G))_B$ is a soft subset of $S(Y_B)$ such that*

$$f_\phi(G)(b) = \begin{cases} \bigcup_{a \in \phi^{-1}(b)} f(G(a)) & : \ \phi^{-1}(b) \neq \emptyset \\ \emptyset & : \ \phi^{-1}(b) = \emptyset \end{cases}$$

for each $b \in B$.

(ii) *$f^{-1}_\phi(H_B) = (f^{-1}_\phi(H))_A$ is a soft subset of $S(X_A)$ such that $f^{-1}_\phi(H)(a) = f^{-1}(H(\phi(a)))$, for each $a \in A$.*

Definition 2.14. *[25] A soft map $f_\phi : S(X_A) \to S(Y_B)$ is said to be injective (resp. surjective, bijective) if ϕ and f are injective (resp. surjective, bijective).*

2.2. Soft topology

Definition 2.15. *[22] A collection τ of soft sets over X under a parameters set E is said to be a soft topology on X if the following three axioms hold:*

(i) *$\widetilde{X}$ and $\widetilde{\Phi}$ belong to τ.*

(ii) *The intersection of a finite family of soft sets in τ belongs to τ.*

(iii) *The union of an arbitrary family of soft sets in τ belongs to τ.*

The triple (X, τ, E) is called a soft topological space (briefly, STS). Every member of τ is called a soft open set and its relative complement is called a soft closed set. An STS (X, τ, E) is called finite (resp. countable) provided that X is finite (resp. countable).

Proposition 2.16. *[22] If (X, τ, E) is an STS, then for each $e \in E$, a family $\tau_e = \{G(e) : G_E \in \tau\}$ forms a topology on X.*

Definition 2.17. *[22] An STS (X, τ, E) is said to be:*

(i) *Soft T_0-space if for every pair of distinct points $x, y \in X$, there is a soft open set G_E such that $x \in G_E$ and $y \notin G_E$ or $y \in G_E$ and $x \notin G_E$.*

(ii) *Soft T_1-space if for every pair of distinct points $x, y \in X$, there are soft open sets G_E and F_E such that $x \in G_E$, $y \notin G_E$ and $y \in F_E$, $x \notin F_E$.*

(iii) *Soft T_2-space if for every pair of distinct points $x, y \in X$, there are disjoint soft open sets G_E and F_E such that $x \in G_E$ and $y \in F_E$.*

(iv) *Soft regular if for every soft closed set H_E and $x \in X$ such that $x \notin H_E$, there are disjoint soft open sets G_E and F_E such that $H_E \widetilde{\subseteq} G_E$ and $x \in F_E$.*

(v) *Soft normal if for every two disjoint soft closed sets H_{1_E} and H_{2_E}, there exist two disjoint soft open sets G_E and F_E such that $H_{1_E} \widetilde{\subseteq} G_E$ and $H_{2_E} \widetilde{\subseteq} F_E$.*

(vi) *Soft T_3 (resp. Soft T_4) -space if it is both soft regular (resp. soft normal) and soft T_1-space.*

Definition 2.18. *[22] Let Y be a non-empty subset of an STS (X, τ, E). Then $\tau_Y = \{\widetilde{Y} \widetilde{\bigcap} G_E : G_E \in \tau\}$ is said to be a soft relative topology on Y and the triple (Y, τ_Y, E) is said to be a soft subspace of (X, τ, E).*

Definition 2.19. *[22] The closure of a soft subset H_E of an STS (X, τ, E), denoted by $\overline{H_E}$, is the intersection of all soft closed sets containing H_E.*

Definition 2.20. *[17] Let G_E be a soft subset of an STS (X, τ, E). Then P_e^x is called a soft limit point of G_E if $[F_E \backslash P_e^x] \widetilde{\bigcap} G_E \neq \widetilde{\Phi}$, for each soft open set F_E containing P_e^x.*

Definition 2.21. *[7] A soft topology τ on X is said to be an enriched soft topology if axiom **(i)** of Definition(2.15) is replaced by the following condition: $G_E \in \tau$, for all $G_E \in CS(X_E)$. The triple (X, τ, E) is called an enriched soft topological space.*

Theorem 2.22. *[19] Let (X, τ, A) and (Y, θ, B) be two STSs. Let $\Omega = \{G_A \times F_B : G_A \in \tau$ and $F_B \in \theta\}$. Then the family of all arbitrary union of elements of Ω is a soft topology on $X \times Y$.*

Theorem 2.23. *[19] An STS (X, τ, E) is soft disconnected if and only if it contains a soft set that are both soft open and soft closed.*

Definition 2.24. *[7]*

(i) *A family* $\{G_{i_E} : i \in I\}$ *of soft open subsets of an STS* (X, τ, E) *is called soft open cover of* $\widetilde{X}$ *provided that* $\widetilde{X} = \widetilde{\bigcup}_{i\in I} G_{i_E}$.

(ii) *An STS* (X, τ, E) *is called soft compact (resp. soft Lindelöf) provided that every soft open cover of* $\widetilde{X}$ *has a finite (resp. countable) subcover.*

Theorem 2.25. *[7] The product of soft compact spaces is soft compact.*

Definition 2.26. *[25] A soft map* $f_\phi : (X, \tau, A) \to (Y, \theta, B)$ *is called:*

(i) *Soft continuous if the inverse image of each soft open subset of* (Y, θ, B) *is a soft open subset of* (X, τ, A).

(ii) *Soft open (resp. soft closed) if the image of each soft open (resp. soft closed) subset of* (X, τ, A) *is a soft open (resp. soft closed) subset of* (Y, θ, B).

(iii) *Soft homeomorphism if it is bijective, soft continuous and soft open.*

3. Partial belong and total non belong relations

In this section, the notions of partial belong and total non belong relations are introduced and their relationships with belong and non belong relations are illustrated. Their behaviour with some maps are investigated and some of their properties with cartesian product of soft sets are studied.

Definition 3.1. *Let* G_E *be a soft set over* X *and* $x \in X$. *We say that:*

(i) $x \Subset G_E$, *reading as x partially belongs to a soft set* G_E, *if* $x \in G(e)$, *for some* $e \in E$.

(ii) $x \not\Subset G_E$, *reading as x does not totally belong to a soft set* G_E, *if* $x \notin G(e)$, *for each* $e \in E$.

For the sake of economy, we omit the proof of the next proposition.

Proposition 3.2. *For two soft sets* G_E *and* H_E *in* $S(X_E)$ *and* $x \in X$, *we have the following results.*

(i) *If* $x \in G_E$, *then* $x \Subset G_E$.

(ii) $x \not\Subset G_E$ *if and only if* $x \in G^c_E$.

(iii) $x \Subset G_E \widetilde{\bigcup} H_E$ *if and only if* $x \Subset G_E$ *or* $x \Subset H_E$.

(iv) *If* $x \Subset G_E \widetilde{\bigcap} H_E$, *then* $x \Subset G_E$ *and* $x \Subset H_E$.

(v) *If* $x \in G_E$ *or* $x \in H_E$, *then* $x \in G_E \widetilde{\bigcup} H_E$.

(vi) $x \in G_E \widetilde{\bigcap} H_E$ *if and only if* $x \in G_E$ *and* $x \in H_E$.

The converse of items **(i)**, **(iv)** and **(v)** of the above proposition need not be true in general as the next example shows.

Example 3.3. *Let the two soft sets* G_E *and* H_E *over* $X = \{x_1, x_2\}$ *and a parameters set* $E = \{e_1, e_2\}$ *be defined as follows:*

$$G_E = \{(e_1, \{x_1\}), (e_2, \{x_2\})\}$$
$$H_E = \{(e_1, \emptyset), (e_2, X)\}$$

It can be noted the following:

(i) $x_2 \Subset G_E$, *but* $x_2 \notin G_E$.

(ii) *$x_1 \Subset G_E$ and $x_1 \Subset H_E$, but $x_1 \not\Subset G_E \widetilde{\bigcap} H_E$.*

(iii) *$x_1 \in G_E \widetilde{\bigcup} H_E$, whereas neither $x_1 \in G_E$ nor $x_1 \in H_E$.*

Remark 3.4. *If for each $x \Subset G_E$ implies that $x \Subset H_E$, then $G_E \widetilde{\subseteq} H_E$ need not be true in general. It can be noted this fact in the above example.*

Definition 3.5. *A soft set G_E over X is said to be stable if there exists a subset S of X such that $G(e) = S$, for each $e \in E$ and it is denoted by $\widetilde{S}$.*

Proposition 3.6. *Let G_E be a stable soft set. Then $x \in G_E$ iff $x \Subset G_E$.*

Proof. Straightforward. □

Proposition 3.7. *The following two properties are satisfied for any two soft sets G_A and H_B over X.*

(i) *$(x, y) \Subset G_A \times H_B$ if and only if $x \Subset G_A$ and $y \Subset H_B$.*

(ii) *$(x, y) \in G_A \times H_B$ if and only if $x \in G_A$ and $y \in H_B$.*

Proof. We only prove **(i)** and the second one follows similar lines.
(i):Necessity: Let $(x, y) \Subset G_A \times H_B = F_{A\times B}$. Then there exists $(a, b) \in A\times B$ such that $(x, y) \in F(a, b) = G(a)\times H(b)$. So $x \in G(a)$ and $y \in H(b)$. Consequently, $x \Subset G_A$ and $y \Subset H_B$.
Sufficiency: It is obvious. □

Proposition 3.8. *Let G_{A_1}, H_{A_2}, F_{A_3} and W_{A_4} be soft sets. Then*

(i) $G_{A_1} \times [H_{A_2} \widetilde{\bigcap} F_{A_3}] = [G_{A_1} \times H_{A_2}] \widetilde{\bigcap} [G_{A_1} \times F_{A_3}]$.

(ii) $G_{A_1} \times [H_{A_2} \widetilde{\bigcup} F_{A_3}] = [G_{A_1} \times H_{A_2}] \widetilde{\bigcup} [G_{A_1} \times F_{A_3}]$.

(iii) $[G_{A_1} \times H_{A_2}] \widetilde{\bigcap} [F_{A_3} \times W_{A_4}] = [G_{A_1} \widetilde{\bigcap} F_{A_3}] \times [H_{A_2} \widetilde{\bigcap} W_{A_4}]$.

(iv) $[G_{A_1} \times H_{A_2}] \widetilde{\bigcup} [F_{A_3} \times W_{A_4}] \widetilde{\subseteq} [G_{A_1} \widetilde{\bigcup} F_{A_3}] \times [H_{A_2} \widetilde{\bigcup} W_{A_4}]$.

Proof. Let us prove the second and third one, the other can be made similarly.

(ii): It is well known from set theory that $A_1 \times (A_2 \bigcup A_3) = (A_1 \times A_2) \bigcup (A_1 \times A_3)$. So The sets of parameters of both sides are equal.
Now, we prove the equality of the approximate elements of both sides as follows:

$$\begin{aligned} G_{A_1} \times [H_{A_2} \widetilde{\bigcup} F_{A_3}] &= \{(x, y) : x \Subset G_{A_1} \text{ and } y \Subset [H_{A_2} \text{ or } F_{A_3}]\} \\ &= \{(x, y) : [x \Subset G_{A_1} \text{ and } y \Subset H_{A_2}] \text{ or } [x \Subset G_{A_1} \text{ and } y \Subset F_{A_3}]\} \\ &= \{(x, y) : (x, y) \Subset G_{A_1} \times H_{A_2} \text{ or } (x, y) \Subset G_{A_1} \times F_{A_3}\} \\ &= [G_{A_1} \times H_{A_2}] \widetilde{\bigcup} [G_{A_1} \times F_{A_3}]. \end{aligned}$$

(iii): It is well known from set theory that $(A_1 \times A_2) \bigcap (A_3 \times A_4) = (A_1 \bigcap A_2) \times (A_3 \bigcap A_4)$. So The sets of parameters of both sides are equal.
Now, we prove the equality of the approximate elements of both sides as follows:
Necessity: Let $(x, y) \Subset [G_{A_1} \times H_{A_2}] \widetilde{\bigcap} [F_{A_3} \times W_{A_4}]$. Then, by Proposition(3.2)**(iv)**, there exist $(r, s) \in [(A_1 \times A_2) \bigcap (A_3 \times A_4)]$ such that $(x, y) \in G(r) \times H(s)$ and $(x, y) \in F(r) \times W(s)$. Now, we have $r \in (A_1 \bigcap A_3)$ such that $x \in G(r) \bigcap F(r)$ and $s \in (A_2 \bigcap A_4)$ such that $y \in H(s) \bigcap W(s)$. This implies that $(x, y) \in [G(r) \bigcap F(r)] \times [H(s) \bigcap W(s)]$. Thus $[G_{A_1} \times H_{A_2}] \widetilde{\bigcap} [F_{A_3} \times W_{A_4}] \widetilde{\subseteq} [G_{A_1} \widetilde{\bigcap} F_{A_3}] \times [H_{A_2} \widetilde{\bigcap} W_{A_4}]$.
Sufficiency: Let Let $(x, y) \Subset [G_{A_1} \widetilde{\bigcap} F_{A_3}] \times [H_{A_2} \widetilde{\bigcap} W_{A_4}]$. Then there exist $r \in (A_1 \bigcap A_3)$ and $s \in (A_2 \bigcap A_4)$ such that $x \in G(r) \bigcap F(r)$ and $y \in H(s) \bigcap W(s)$. So $(x, y) \in G(r) \times H(s)$ and $(x, y) \in F(r) \times W(s)$. This means that there exists $(r, s) \in (A_1 \bigcap A_2) \times (A_3 \bigcap A_4)$ satisfies $(x, y) \in [(G \times H)(r, s)] \bigcap [(F \times W)(r, s)]$. Thus $[G_{A_1} \widetilde{\bigcap} F_{A_3}] \times [H_{A_2} \widetilde{\bigcap} W_{A_4}] \widetilde{\subseteq} [G_{A_1} \times H_{A_2}] \widetilde{\bigcap} [F_{A_3} \times W_{A_4}]$. Hence the desired result is proved. □

Example 3.9. *To illustrate that the converse of item **(iv)** of the above proposition fails, it is sufficient to show that a set of parameters $(A_1 \times A_2) \bigcup (A_3 \times A_4)$ in the left side is a proper subset of a set of parameters $(A_1 \bigcup A_3) \times (A_2 \bigcup A_4)$ in the right side. Suppose that $A_1 = A_2 = \{e_1\}$ and $A_3 = A_4 = \{e_2, e_3\}$. Then we find the following:*

(i) $(A_1 \times A_2) \bigcup (A_3 \times A_4) = \{(e_1, e_1), (e_2, e_2), (e_2, e_3), (e_3, e_2), (e_3, e_3)\}$.

(ii) $(A_1 \bigcup A_3) \times (A_2 \bigcup A_4) = \{(e_1, e_1), (e_1, e_2), (e_1, e_3), (e_2, e_1), (e_2, e_2), (e_2, e_3), (e_3, e_1), (e_3, e_2), (e_3, e_3)\}$.

Hence $(A_1 \bigcup A_3) \times (A_2 \bigcup A_4) \nsubseteq (A_1 \times A_2) \bigcup (A_3 \times A_4)$.

Proposition 3.10. *Consider $f_\phi : S(X_A) \to S(Y_B)$ is a soft map and let G_A be a soft set in $S(X_A)$. Then the following statements hold.*

(i) *If $x \Subset G_A$, then $f(x) \Subset f_\phi(G_A)$.*

(ii) *If f is injective and $x \not\Subset G_A$, then $f(x) \not\Subset f_\phi(G_A)$.*

(iii) *If ϕ is surjective and $x \in G_A$, then $f(x) \in f_\phi(G_A)$.*

(iv) *If f_ϕ is injective and $x \notin G_A$, then $f(x) \notin f_\phi(G_A)$.*

Proof. **(i)** Let $x \Subset G_A$. Then there exists a parameter $a \in A$ such that $x \in G(a)$. Now, there exists a parameter $b \in B$ such that $a \in \phi^{-1}(b)$. Therefore $f(x) \in f(G(a)) \subseteq \bigcup_{\alpha \in \phi^{-1}(b)} f(G(\alpha)) = f(G)(b)$. Thus $f(x) \Subset f_\phi(G_A)$.

(ii) Let $x \not\Subset G_A$. Then $x \notin G(a)$, for each $a \in A$. Since f is injective, then $f(x) \notin f(G(a)) \subseteq f_\phi(G)(b)$, for each $b \in B$. Thus $f(x) \not\Subset f_\phi(G_A)$.

(iii) Let $x \in G_A$. Then $x \in G(a)$, for each $a \in A$ and this implies that $f(x) \in f(G(a))$, for each $a \in A$. Since ϕ is surjective, then from Definition(2.13), we have $f_\phi(G)(b) = \bigcup_{a \in \phi^{-1}(b)} f(G(a))$, for each $b \in B$. Therefore $f(x) \in f_\phi(G)(b)$, for each $b \in B$. Thus $f(x) \in f_\phi(G_A)$.

(iv) Let $x \notin G_A$. Then there exists at least a parameter $a \in A$ such that $x \notin G(a)$. Since f is injective, then $f(x) \notin f(G(a))$ and since ϕ is injective, then there exists a parameter $b \in B$ such that $a = \phi^{-1}(b)$. So $f(x) \notin f_\phi(G)(b)$. Hence $f(x) \notin f_\phi(G_A)$.

□

Proposition 3.11. *Consider $f_\phi : S(X_A) \to S(Y_B)$ is a soft map and et H_B be a soft set in $S(Y_B)$. Then the following statements hold.*

(i) *If ϕ is surjective and $y \Subset H_B$, then $x \Subset f_\phi^{-1}(H_B)$, for each $x \in f^{-1}(y)$.*

(ii) *If $y \not\Subset H_B$, then $x \not\Subset f_\phi^{-1}(H_B)$, for each $x \in f^{-1}(y)$.*

(iii) *If $y \in H_B$, then $x \in f_\phi^{-1}(H_B)$, for each $x \in f^{-1}(y)$.*

(iv) *If ϕ is surjective and $y \notin H_B$, then $x \notin f_\phi^{-1}(H_B)$, for each $x \in f^{-1}(y)$.*

Proof. **(i)** Let $x \in f^{-1}(y)$. Since $y \Subset H_B$, then there exists a parameter $b \in B$ such that $y \in H(b)$ and since ϕ is surjective, then there exists a parameter $a \in A$ such that $\phi(a) = b$. Therefore $y \in H(b) = H(\phi(a))$. Thus $f^{-1}(y) \subseteq f^{-1}(H(\phi(a))) = f_\phi^{-1}(H)(a)$. Hence $x \Subset f_\phi^{-1}(H_B)$.

(ii) Let $x \in f^{-1}(y)$. Since $y \not\Subset H_B$, then $y \notin H(b)$, for each $b \in B$. This implies that $y \notin H(\phi(a))$, for each $a \in A$. Therefore $f^{-1}(y) \bigcap f^{-1}(H(\phi(a))) = f^{-1}(\{y\} \bigcap H(\phi(a))) = \emptyset$. Thus $x \not\Subset f_\phi^{-1}(H_B)$.

(iii) Let $x \in f^{-1}(y)$. Since $y \in H_B$, then $y \in H(b)$, for each $b \in B$. This implies that $y \in H(\phi(a))$, for each $a \in A$. Therefore $f^{-1}(y) \subseteq f^{-1}(H(\phi(a)))$. Thus $x \in f_\phi^{-1}(H_B)$.

(iv) Let $x \in f^{-1}(y)$. Since $y \notin (H_B)$, then there exists a parameter $b \in B$ such that $y \notin H(b)$ and since ϕ is surjective, then there exists a parameter $a \in A$ such that $\phi(a) = b$. Therefore $y \notin H(b) = H(\phi(a))$. Thus $f^{-1}(y) \bigcap f^{-1}(H(\phi(a))) = \emptyset$. Hence $x \notin f_{\phi}^{-1}(H_B)$.

□

Proposition 3.12. *Let $f_{\phi} : S(X_A) \to S(Y_B)$ be a bijective soft map. Then $(f_{\phi}(G_A))^c = f_{\phi}(G_A^c)$, for each soft subset G_A of $\widetilde{X}$.*

Proof. Suppose that G_A is a soft subset of $\widetilde{X}$. Then $f_{\phi}((G_A)^c) = (f_{\phi}(G^c))_B$, where

$$f_{\phi}(G^c)(b) = \begin{cases} \widetilde{\bigcup}_{a \in \phi^{-1}(b)} f(G^c(a)) & : \quad \phi^{-1}(b) \neq \emptyset \\ \emptyset & : \quad \phi^{-1}(b) = \emptyset \end{cases}$$

for each $b \in B$.
Since f_{ϕ} is bijective, then $f(G^c(a)) = [f(G(a))]^c$, for each $a \in A$. Therefore

$$f_{\phi}(G^c)(b) = \begin{cases} \widetilde{\bigcup}_{a \in \phi^{-1}(b)} [f(G(a))]^c & : \quad \phi^{-1}(b) \neq \emptyset \\ \widetilde{(X)}^c & : \quad \phi^{-1}(b) = \emptyset \end{cases}$$

Thus $(f_{\phi}(G_A))^c = f_{\phi}(G_A^c)$. □

Corollary 3.13. *Let $f_{\phi} : (X, \tau, A) \to (Y, \theta, B)$ be a bijective soft map. Then f_{ϕ} is soft open if and only if it is soft closed.*

4. Partial soft separation axioms

We initiate in this section the concepts of p-soft T_i-spaces ($i = 0, 1, 2, 3, 4$) and p-soft regular spaces utilizing belong and total non belong relations. Several properties of them are studied and the relationships among them are illustrated with the help of examples. Also, we point out that a finite soft T_2-spaces is a p-soft T_3-space. Finally, we investigate under what maps some p-soft separation axioms are preserved.

Definition 4.1. *An STS (X, τ, E) is said to be:*

(i) *p-soft T_0-space if for every two distinct points $x, y \in X$, there exists a soft open set G_E such that $x \in G_E$ and $y \not\Subset G_E$ or $y \in G_E$ and $x \not\Subset G_E$.*

(ii) *p-soft T_1-space if for every two distinct points $x, y \in X$, there exist soft open sets G_E and F_E such that $x \in G_E$, $y \not\Subset G_E$, $y \in F_E$ and $x \not\Subset F_E$.*

(iii) *p-soft T_2-space if for every two distinct points $x, y \in X$, there exist two disjoint soft open sets G_E and F_E such that $x \in G_E$, $y \not\Subset G_E$, $y \in F_E$ and $x \not\Subset F_E$.*

Remark 4.2. *On the one hand, $x \not\Subset G_E$ implies that $x \notin G_E$, then a p-soft T_2-space is a soft T_2-space which defined in [22]. On the other hand, the definition of soft T_2-space in [22] implies that for every two distinct points $x, y \in X$, there exist two disjoint soft open sets G_E and F_E containing x and y, respectively, such that $y \notin G_E$ and $x \notin F_E$. Since G_E and F_E are disjoint, then $y \not\Subset G_E$ and $x \not\Subset F_E$. Hence the definitions of p-soft T_2-space and soft T_2-space are equivalent.*

Proposition 4.3. *Every p-soft T_i-space is a soft T_i-space, for $i = 0, 1$.*

Proof. It follows from the fact that a total non belong relation $\not\Subset$ implies a non belong relation $\notin$. □

The next example points out that the vice-versa of the above proposition is not true.

Example 4.4. *Let $E = \{e_1, e_2\}$ and $\tau = \{\widetilde{\Phi}, \widetilde{X}, K_{i_E} : i = 1, 2, ..., 6\}$ be a soft topology on $X = \{x_1, x_2\}$, where the six soft sets are defined as follows:*

$K_{1_E} = \{(e_1, X), (e_2, \{x_1\})\}$;
$K_{2_E} = \{(e_1, X), (e_2, \{x_2\})\}$;
$K_{3_E} = \{(e_1, \emptyset), (e_2, \{x_2\})\}$;
$K_{4_E} = \{(e_1, \emptyset), (e_2, \{x_1\})\}$;
$K_{5_E} = \{(e_1, \emptyset), (e_2, X)\}$ *and*
$K_{6_E} = \{(e_1, X), (e_2, \emptyset)\}$.

Then (X, τ, E) is a soft T_1-space. On the other hand, there does not exist a soft open set containing x_1 such that x_2 does not totally belong to it. Thus (X, τ, E) is not a p-soft T_0-space.

In what follows, we investigate some results related to a p-soft T_0-space.

Lemma 4.5. *Let G_E be a soft subset of an STS (X, τ, E) and $x \in X$. Then $x \notin \overline{G_E}$ iff there exists a soft open set F_E containing x such that $G_E \widetilde{\bigcap} F_E = \widetilde{\Phi}$.*

Proof. Let $x \notin \overline{G_E}$. By Proportion (3.2)**(ii)**, $x \in (\overline{G_E})^c = F_E$. So $G_E \widetilde{\bigcap} F_E = \widetilde{\Phi}$. Conversely, if there exists a soft open set F_E containing x such that $G_E \widetilde{\bigcap} F_E = \widetilde{\Phi}$, then $G_E \subseteq F^c_E$. Therefore $\overline{G_E} \subseteq F^c_E$. Since $x \notin F^c_E$, then $x \notin \overline{G_E}$. □

Theorem 4.6. *If (X, τ, E) is a p-soft T_0-space, then $\overline{x_E} \neq \overline{y_E}$, for every $x \neq y \in X$.*

Proof. Let $x \neq y$ in a p-soft T_0-space. Then there is a soft open set G_E such that $x \in G_E$ and $y \notin G_E$ or $y \in G_E$ and $x \notin G_E$. Say, $x \in G_E$ and $y \notin G_E$. Now, $y_E \widetilde{\bigcap} G_E = \widetilde{\Phi}$. So, by the above lemma, $x \notin \overline{y_E}$. But $x \in \overline{x_E}$. Hence the proof is complete. □

Corollary 4.7. *If (X, τ, E) is a p-soft T_0-space, then $\overline{P^x_\alpha} \neq \overline{P^y_\beta}$, for all $x \neq y$ and $\alpha, \beta \in E$.*

It can be seen from the next example that the converse of the above theorem fails.

Example 4.8. *Assume that (X, τ, E) is the same as in Example (4.4). Then $\overline{x_{1_E}} \neq \overline{x_{2_E}}$. On the other hand, $x_1 \neq x_2$ and there do not exist a soft open set satisfies a condition of a p-soft T_0-space. Hence (X, τ, E) is not a p-soft T_0-space.*

We give a complete description for a p-soft T_1-space in the following result and then we establish some properties of this soft space.

Theorem 4.9. *An STS (X, τ, E) is a p-soft T_1-space if and only if x_E is soft closed, for all $x \in X$.*

Proof. Necessity: For each $y_i \in X\backslash\{x\}$, there is a soft open set G_{i_E} such that $y_i \in G_{i_E}$ and $x \notin G_{i_E}$. Therefore $X\backslash\{x\} = \bigcup_{i\in I} G_i(e)$ and $x \notin \bigcup_{i\in I} G_i(e)$, for each $e \in E$. Thus $\widetilde{\bigcup}_{i\in I} G_{i_E} = \widetilde{X\backslash\{x\}}$ is soft open. Hence, x_E is soft closed.
Sufficiency: For each $x \neq y$, we have x_E and y_E are soft closed sets. Now, $y \in (x_E)^c = (X\backslash\{x\})_E$ and $x \in (y_E)^c = (X\backslash\{y\})_E$. Since $x \notin (X\backslash\{x\})_E$ and $y \notin (X\backslash\{y\})_E$, then (X, τ, E) is a p-soft T_1-space. □

Theorem 4.10. *Let E be a finite set. Then (X, τ, E) is a p-soft T_1-space if and only if $x_E = \widetilde{\bigcap}\{G_E : x_E \widetilde{\subseteq} G_E \in \tau\}$, for all $x \in X$.*

Proof. To prove the "If" part, let $y \in X$. Then for each $x \in X \setminus \{y\}$, we have a soft open set G_E such that $x \in G_E$ and $y \notin G_E$. Then $y \notin \bigcap G(e)$, for each $e \in E$. Therefore $y \notin \widetilde{\bigcap}\{G_E : x_E \widetilde{\subseteq} G_E \in \tau\}$. Since y is chosen arbitrary, then the proof of this part is complete.
To prove the "only if" part, let the given conditions be satisfied and let $x \neq y$. As $y \notin x_E$ and E is finite, say $| E |= m$, then we can choose at most m soft open sets G_{i_E} such that $y \notin G_i(e_j)$ and $x \in G_{i_E} : j = 1, 2, ..., m$. Therefore $\widetilde{\bigcap}_{i=1}^{i=m} G_{i_E}$ is a soft open set such that $y \notin \widetilde{\bigcap}_{i=1}^{i=m} G_{i_E}$ and $x \in \widetilde{\bigcap}_{i=1}^{i=m} G_{i_E}$. Similarly, we can get a soft open set W_E such that $y \in W_E$ and $x \notin W_E$. Thus (X, τ, E) is a p-soft T_1-space. □

Corollary 4.11. *If (X, τ, E) is a p-soft T_1-space, then $x_E = \widetilde{\bigcap}\{G_E : x \in G_E\}$, for all $x \in X$.*

Theorem 4.12. *If (X, τ, E) is an enriched p-soft T_1-space, then P^x_e is soft closed for all $P^x_e \in \widetilde{X}$.*

Proof. From Theorem (4.9), we get $\widetilde{X \setminus \{x\}}$ is soft open. As (X, τ, E) is enriched, then a soft set H_E, defining as $H(e) = \emptyset$ and $H(\alpha) = X$ for each $\alpha \neq e$, is soft open. Therefore $\widetilde{X \setminus \{x\}} \widetilde{\bigcup} H_E$ is soft open. Thus $(\widetilde{X \setminus \{x\}} \widetilde{\bigcup} H_E)^c = P^x_e$ is soft closed. □

Corollary 4.13. *If (X, τ, E) is an enriched p-soft T_1-space, then the intersection of all soft open sets containing U_E is exactly U_E, for each $U_E \widetilde{\subseteq} \widetilde{X}$.*

Proof. Let U_E be a soft subset of $\widetilde{X}$ and $P^x_e \in U^c_E$. As P^x_e is soft closed, then $\widetilde{X} \setminus P^x_e$ is a soft open set containing U_E. We do similarly, for each $P^x_e \in U^c_E$. Hence the proof is complete. □

Theorem 4.14. *A finite STS (X, τ, E) is p-soft T_1 if and only if it is p-soft T_2.*

Proof. Necessity: For each $y \in X \setminus \{x\}$ and $x \in X \setminus \{y\}$, we have y_E and x_E are soft closed. Since X is finite, then $\widetilde{\bigcup}_{y \in X \setminus \{x\}} y_E$ and $\widetilde{\bigcup}_{x \in X \setminus \{y\}} x_E$ are soft closed. Therefore $(\widetilde{\bigcup}_{y \in X \setminus \{x\}} y_E)^c = x_E$ and $(\widetilde{\bigcup}_{x \in X \setminus \{y\}} x_E)^c = y_E$ are soft open. Thus (X, τ, E) is a p-soft T_2-space.
Sufficiency: It follows immediately from Definition (4.1). □

Corollary 4.15. *A finite p-soft T_1-space is soft disconnected.*

Remark 4.16. *If X is infinite, then a soft set x_E in a p-soft T_1-space (X, τ, E) need not be soft open as illustrated in the following example.*

Example 4.17. *Let E be the set of natural numbers $\mathbf{N}$ and $\tau = \{\widetilde{\Phi}, G_E : G^c_E$ is finite$\}$ be a soft topology on the set of real numbers $\boldsymbol{R}$. Then x_E is not soft open, for each $x \in \boldsymbol{R}$.*

Remark 4.18. *In the definition of soft regular space in [22], if we say that $x \notin H_E$, where H_E is a soft closed set, then we have two cases:*

(i) *Either there exists at least $e \in E$ such that $x \notin H(e)$ and $x \in H(\alpha)$, for each $e \neq \alpha$. Then we can not find any two disjoint soft sets G_E and F_E containing x and H_E, respectively.*

(ii) *Or $x \notin H(e)$, for each $e \in E$. Then it is possible to find two disjoint soft open sets G_E and F_E containing x and H_E, respectively.*

*Now, **(ii)** is the only possible case and so we can conclude that any soft open (soft closed) subset of a soft regular space must be stable. An unawareness of the authors in [22] about a strict condition on the shape of soft open subsets of a soft regular space caused some errors in their work which investigated and corrected by [15]. To avoid this strict condition, we introduce a concept of a p-soft regular space by using a total non belong relation instead of a non belong relation.*

Definition 4.19. *An STS (X, τ, E) is said to be p-soft regular if for every soft closed set H_E and $x \in X$ such that $x \notin\!\!\!\notin H_E$, there exist disjoint soft open sets G_E and F_E such that $H_E \widetilde{\subseteq} G_E$ and $x \in F_E$.*

Proposition 4.20. *Every soft regular space is p-soft regular.*

Proof. The proof follows easily from Definition(4.19) and Proposition(3.2). □

We show in the next example that the converse of the above proposition fails.

Example 4.21. *Let $E = \{e_1, e_2, e_3\}$ be a set of parameters and $\tau = \{\widetilde{\Phi}, \widetilde{X}, G_{i_E} : i = 1, 2, ..., 7\}$ be a soft topology on $X = \{x_1, x_2\}$, where*

$G_{1_E} = \{(e_1, \{x_1\}), (e_2, \{x_1\}), (e_3, \{x_1\})\}$;
$G_{2_E} = \{(e_1, \{x_2\}), (e_2, \{x_2\}), (e_3, \{x_2\})\}$;
$G_{3_E} = \{(e_1, \emptyset), (e_2, \{x_1\}), (e_3, \{x_1\})\}$;
$G_{4_E} = \{(e_1, \emptyset), (e_2, \{x_2\}), (e_3, \{x_2\})\}$;
$G_{5_E} = \{(e_1, \{x_1\}), (e_2, X), (e_3, X)\}$;
$G_{6_E} = \{(e_1, \{x_2\}), (e_2, X), (e_3, X)\}$ *and*
$G_{7_E} = \{(e_1, \emptyset), (e_2, X), (e_3, X)\}$.

Then (X, τ, E) is p-soft regular. On the other hand, a soft open set G_{3_E} is not stable, hence (X, τ, E) is not soft regular.

We characterize a p-soft regular space in the following result.

Theorem 4.22. *An STS (X, τ, E) is p-soft regular iff for each $x \in X$ and soft open subset F_E of (X, τ, E) containing x, there exists a soft open set G_E such that $x \in G_E \widetilde{\subseteq} \overline{G_E} \widetilde{\subseteq} F_E$.*

Proof. Let $x \in X$ and F_E be a soft open set containing x. Then F_E^c is soft closed and $x_E \widetilde{\bigcap} F_E^c = \widetilde{\Phi}$. So there are disjoint soft open sets W_E and G_E such that $F_E^c \widetilde{\subseteq} W_E$ and $x \in G_E$. Therefore $G_E \widetilde{\subseteq} W_E^c \widetilde{\subseteq} F_E$. Thus $\overline{G_E} \widetilde{\subseteq} W_E^c \widetilde{\subseteq} F_E$. Conversely, let F_E^c be a soft closed set. Then for each $x \notin F_E^c$, we have $x \in F_E$. By hypothesis, there is a soft open set G_E containing x such that $\overline{G_E} \widetilde{\subseteq} F_E$. Therefore $F_E^c \widetilde{\subseteq} (\overline{G_E})^c$ and $G_E \widetilde{\bigcap} (\overline{G_E})^c = \widetilde{\Phi}$. Thus (X, τ, E) is p-soft regular, as required. □

Theorem 4.23. *For any p-soft regular space, the following three statements are equivalent:*

(i) *(X, τ, E) is a p-soft T_2-space.*

(ii) *(X, τ, E) is a p-soft T_1-space.*

(iii) *(X, τ, E) is a p-soft T_0-space.*

Proof. Obviously, **(i)** → **(ii)** → **(iii)**.
(iii) → **(i)**: Let x, y be two distinct point in X and (X, τ, E) be a p-soft T_0-space. Then there exists a soft open set G_E such that $x \in G_E$ and $y \notin G_E$ or $y \in G_E$ and $x \notin G_E$. Say, $x \in G_E$ and $y \notin G_E$. By Proposition (3.2), we obtain that $x \notin G_E^c$ and $y \in G_E^c$. Since (X, τ, E) is p-soft regular, then there exist two disjoint soft open sets W_{1_E} and W_{2_E} such that $x \in W_{1_E}$ and $y \in G_E^c \widetilde{\subseteq} W_{2_E}$. This completes the proof. □

Theorem 4.24. *Let an STS (X, τ, E) be soft regular. Then (X, τ, E) is p-soft T_1-space iff it is soft T_1-space.*

Proof. The proof of the "if" part follows directly from Proposition(4.3).
To prove the "only if" part, suppose x, y are two distinct points in X. Then there exist soft open sets G_E and F_E such that $x \in G_E$, $y \notin G_E$, $y \in F_E$ and $x \notin F_E$. Since G_E and F_E are soft open subsets of a soft regular space, then they are stable. So $y \notin G_E$ and $x \notin F_E$. Thus (X, τ, E) is a p-soft T_1-space. □

Theorem 4.25. *A finite p-soft T_2-space is p-soft regular.*

Proof. Let H_E be a soft closed set and $x \in X$ such that $x \notin H_E$. Then for each $y \Subset H_E$, we have $x \neq y$. Therefore there exist disjoint soft open sets G_{i_E} and F_{i_E} such that $x \in G_{i_E}$, $y \in F_{i_E}$. Since a set $\{y : y \in X\}$ is finite, then we can take a finite number of soft open sets F_{i_E} such that $H_E \widetilde{\subseteq} \widetilde{\bigcup}_{i=1}^{i=m} F_{i_E}$. Now, $\widetilde{\bigcap}_{i=1}^{i=m} G_{i_E}$ is a soft open set containing x and $[\widetilde{\bigcup}_{i=1}^{i=m} F_{i_E}] \widetilde{\bigcap} [\widetilde{\bigcap}_{i=1}^{i=m} G_{i_E}] = \widetilde{\Phi}$. Hence (X, τ, E) is p-soft regular. □

Definition 4.26. *An STS (X, τ, E) is called:*

(i) *p-soft T_3-space if it is both p-soft regular and p-soft T_1-space.*

(ii) *p-soft T_4-space if it is both soft normal and p-soft T_1-space.*

Proposition 4.27. *Every soft T_3-space is a p-soft T_3-space.*

Proof. One can obtain the proof easily from Proposition(4.20) and Theorem(4.24). □

To see that the converse of the above proposition is not true in general, one can observe that a given soft topological space in Example (4.21) is p-soft T_3, but it is not soft T_3.

Proposition 4.28. *Every p-soft T_4-space is a soft T_4-space.*

Proof. Straightforward. □

To show that the converse of the above proposition fails, we give the next example.

Example 4.29. *Let $E = \{e_1, e_2, e_3\}$ and $\tau = \{\widetilde{\Phi}, \widetilde{X}, G_{i_E} : i = 1, 2, ..., 6\}$ be a soft topology on $X = \{x_1, x_2\}$, where*
$G_{1_E} = \{(e_1, X), (e_2, \{x_1\}), (e_3, X)\}$;
$G_{2_E} = \{(e_1, X), (e_2, \{x_2\}), (e_3, X)\}$;
$G_{3_E} = \{(e_1, X), (e_2, \emptyset), (e_3, X)\}$;
$G_{4_E} = \{(e_1, \emptyset), (e_2, \{x_1\}), (e_3, \emptyset)\}$;
$G_{5_E} = \{(e_1, \emptyset), (e_2, \{x_2\}), (e_3, \emptyset)\}$ *and*
$G_{6_E} = \{(e_1, \emptyset), (e_2, X), (e_3, \emptyset)\}$.

Then (X, τ, E) is a soft T_4-space. On the other hand, there does not exist a soft open set containing x_2 such that x_1 does not totally belong to it. So (X, τ, E) is not a p-soft T_1-space, hence it is not a p-soft T_4-space.

Now, we elucidate a relationship among p-soft T_i-spaces and deduce some results which associate them with some soft topological notions such as soft subspace and soft product space.

Proposition 4.30. *Every p-soft T_i-space is a p-soft T_{i-1}-space, for $i = 1, 2, 3, 4$.*

Proof. We prove the proposition in case of $i = 3, 4$. The other proofs follow similar lines.
For $i = 3$, let $x \neq y$ and (X, τ, E) be p-soft T_1. Then x_E is soft closed. Since $y \notin x_E$ and (X, τ, E) is p-soft regular, then there are disjoint soft open sets G_E and F_E such that $x_E \widetilde{\subseteq} G_E$ and $y \in F_E$. Therefore (X, τ, E) is a p-soft T_2-space.
For $i = 4$, let $x \in X$ and H_E be a soft closed set such that $x \notin H_E$. Since (X, τ, E) is p-soft T_1, then x_E is soft closed. Since $x_E \widetilde{\bigcap} H_E = \widetilde{\Phi}$ and (X, τ, E) is soft normal, then there are disjoint soft open sets G_E and F_E such that $H_E \widetilde{\subseteq} G_E$ and $x_E \widetilde{\subseteq} F_E$. Hence (X, τ, E) is a p-soft T_3-space. □

Theorem 4.31. *The following three properties are equivalent if X is finite.*

(i) *(X, τ, E) is a p-soft T_3-space.*

(ii) *(X, τ, E) is a p-soft T_2-space.*

(iii) *(X, τ, E) is a p-soft T_1-space.*

Proof. **(i)** → **(ii)** → **(iii)**: This is obtained from Proposition(4.30).
(iii) → **(ii)**: This is obtained from Theorem(4.14).
(ii) → **(i)**: This is obtained from Theorem(4.25) and Proposition(4.30). □

Lemma 4.32. *If $H_{A_1 \times A_2}$ is a soft closed subset of a soft product space $(X \times Y, \tau_1 \times \tau_2, A_1 \times A_2)$, then $H_{A_1 \times A_2} = [(G_{A_1})^c \times \widetilde{Y}] \widetilde{\bigcup} [\widetilde{X} \times (U_{A_2})^c]$, for some $G_{A_1} \in \tau_1$ and $U_{A_2} \in \tau_2$.*

Theorem 4.33. *A finite product of p-soft T_i-spaces (X_r, τ_r, A_r) is a p-soft T_i-space, for $i = 0, 1, 2, 3$.*

Proof. We prove the theorem for two soft topological spaces in case of $i = 0, 3$. The other proofs follow similar lines.

(i) Consider (X_1, τ_1, A_1) and (X_2, τ_2, A_2) are two p-soft T_0-spaces and let $(x_1, y_1) \neq (x_2, y_2)$ in $(X_1 \times X_2, \tau_1 \times \tau_2, A_1 \times A_2)$. Without lose of generality, let $x_1 \neq x_2$. Then there exists a soft open subset G_{A_1} of (X_1, τ_1, A_1) such that $x_1 \in G_{A_1}$ and $x_2 \notin G_{A_1}$ or $x_2 \in G_{A_1}$ and $x_1 \notin G_{A_1}$. Say, $x_1 \in G_{A_1}$ and $x_2 \notin G_{A_1}$. Therefore $(x_1, y_1) \in G_{A_1} \times \widetilde{X_2}$ and $(x_2, y_2) \notin G_{A_1} \times \widetilde{X_2}$. Thus $(X_1 \times X_2, \tau_1 \times \tau_2, A_1 \times A_2)$ is a p-soft T_0-space.

(ii) Let $H_{A_1 \times A_2}$ be a soft closed subset of a soft space $(X_1 \times X_2, \tau_1 \times \tau_2, A_1 \times A_2)$. Then $H_{A_1 \times A_2} = [(G_{A_1})^c \times \widetilde{X_2}]\widetilde{\bigcup}[\widetilde{X_1} \times (U_{A_2})^c]$, for some $G_{A_1} \in \tau_1$ and $U_{A_2} \in \tau_2$. For every $(x, y) \notin H_{A_1 \times A_2}$, we have $(x, y) \notin (G_{A_1})^c \times \widetilde{X_2}$ and $(x, y) \notin \widetilde{X_1} \times (U_{A_2})^c$. From Proposition(3.7), we obtain that $x \notin (G_{A_1})^c$ and $y \notin (U_{A_2})^c$. Since (X_1, τ_1, A_1) and (X_2, τ_2, A_2) are p-soft regular, then there exist disjoint soft open sets $F_{1_{A_1}}$ and $F_{2_{A_1}}$ containing x and $(G_{A_1})^c$, respectively, and disjoint soft open sets $F_{3_{A_2}}$ and $F_{4_{A_2}}$ containing y and $(U_{A_2})^c$, respectively. Thus $H_{A_1 \times A_2} \widetilde{\subseteq} [F_{2_{A_1}} \times \widetilde{X_2}]\widetilde{\bigcup}[\widetilde{X_1} \times F_{4_{A_2}}]$ and $(x, y) \in [F_{1_{A_1}} \times F_{3_{A_2}}]$. From Proposition(3.8), we obtain that $[F_{1_{A_1}} \times F_{3_{A_2}}]\widetilde{\bigcap}\{[F_{2_{A_1}} \times \widetilde{X_2}]\widetilde{\bigcup}[\widetilde{X_1} \times F_{4_{A_2}}]\} = \widetilde{\Phi_{A_1 \times A_2}}$. Hence the proof is complete.

□

Theorem 4.34. *Every soft subspace (Y, τ_Y, E) of a p-soft T_i-space (X, τ, E) is a p-soft T_i-space, for $i = 0, 1, 2, 3$.*

Proof. we prove the theorem in case of $i = 3$ and the other proofs follow similar lines.
To prove (Y, τ_Y, E) is p-soft T_1, let $x \neq y \in Y$. Since (X, τ, E) is a p-soft T_1-space, then there exist soft open sets G_E and F_E such that $x \in G_E, y \notin G_E$, $y \in F_E$, and $x \notin F_E$. Therefore $x \in H_{1_E} = \widetilde{Y}\widetilde{\bigcap}G_E$ and $y \in H_{2_E} = \widetilde{Y}\widetilde{\bigcap}F_E$. Since $y \notin G_E$ and $x \notin F_E$, then $y \notin H_{1_E}$ and $x \notin H_{2_E}$. Thus (Y, τ_Y, E) is p-soft T_1.
To prove (Y, τ_Y, E) is p-soft regular, let $y \in Y$ and L_E be a soft closed subset of (Y, τ_Y, E) such that $y \notin L_E$. Then there exists a soft closed subset H_E of (X, τ, E) such that $L_E = \widetilde{Y}\widetilde{\bigcap}H_E$ and $y \notin H_E$. Therefore there exist disjoint soft open sets G_E and F_E such that $H_E \widetilde{\subseteq} G_E$ and $y \in F_E$. Now, we find that $L_E \widetilde{\subseteq} W_{1_E} = \widetilde{Y}\widetilde{\bigcap}G_E$, $y \in W_{2_E} = \widetilde{Y}\widetilde{\bigcap}F_E$ and $W_{1_E}\widetilde{\bigcap}W_{2_E} = \widetilde{\Phi}$. Thus (Y, τ_Y, E) is p-soft regular.
Hence (Y, τ_Y, E) is p-soft T_3. □

Proposition 4.35. *Let $f_\phi : (X, \tau, A) \to (Y, \theta, B)$ be a soft continuous map. Then if (Y, θ, B) is a p-soft T_i-space and f is injective, then (X, τ, A) is a p-soft T_i-space, for $i = 0, 1, 2$.*

Proof. We only prove the proposition for $i = 2$.
Consider a and b are two distinct points in X. By injective of f, there are two distinct points x and y in Y such that $f(a) = x$ and $f(b) = y$. Since (Y, θ, B) is a p-soft T_2-space, then there are two soft open sets G_B and F_B such that $x \in G_B$, $y \in F_B$ and $G_B\widetilde{\bigcap}F_B = \widetilde{\Phi_B}$. From Proposition (3.11), we obtain that $a \in f_\phi^{-1}(G_B)$, $b \in f_\phi^{-1}(F_B)$ and $f_\phi^{-1}(G_B)\widetilde{\bigcap}f_\phi^{-1}(F_B) = \widetilde{\Phi_A}$. Thus (X, τ, A) is a p-soft T_2-space. □

For the sake of brevity, we omit the proofs of the next three results.

Proposition 4.36. *Let $f_\phi : (X, \tau, A) \to (Y, \theta, B)$ be a bijective soft open map. Then if (X, τ, A) is a p-soft T_i-space, then (Y, θ, B) is a p-soft T_i-space, for $i = 0, 1, 2, 3, 4$.*

Proposition 4.37. *Let $f_\phi : (X, \tau, A) \to (Y, \theta, B)$ be a bijective soft continuous map. Then if (Y, θ, B) is a p-soft T_i-space, then (X, τ, A) is a p-soft T_i-space, for $i = 0, 1, 2, 3, 4$.*

Proposition 4.38. *The property of being p-soft T_i is a topological property, for $i = 0, 1, 2, 3, 4$.*

5. Some results related to soft compact and soft Lindelöf spaces

Some properties of soft compact (soft Lindelöf) spaces are presented and investigated in this section. The relationships among soft compact p-soft T_2-spaces, p-soft regular spaces and p-soft T_3-spaces are illustrated and some properties of enriched soft compact spaces with compact spaces are studied. A formula for computing the number of all soft subsets of an STS is given and then it is used to point out that a finite STS need not be soft compact.

Proposition 5.1. *Every soft closed subset H_E of a soft compact (resp. soft Lindelöf) space is soft compact (resp. soft Lindelöf).*

Proof. Straightforward. □

Corollary 5.2. *If H_E is a soft closed set and F_E is a soft compact (resp. soft Lindelöf) set in (X, τ, E), then $H_E \widetilde{\bigcap} F_E$ is soft compact (resp. soft Lindelöf).*

Remark 5.3. *The result which state that: every compact subset of a T_2-space is closed, is one of the results in general topology which is not true concerning soft topology. it worthily noting that some authors did mistake when they generalized this result without imposing conditions on a soft compact set, see for example Theorem 3.34 in [24]. The following example illuminates this error.*

Example 5.4. *Assume that (X, τ, E) is the same as in Example (4.21). Obviously, (X, τ, E) is a soft T_2-space and a soft set $H_E = \{(e_1, \{x_1\}), (e_2, X), (e_3, \{x_2\})\}$ is a soft compact subset of $\widetilde{X}$. However, H_E is not soft closed.*

In the next proposition, we give a sufficient condition for a soft subset of a p-soft T_2-space to be soft closed.

Proposition 5.5. *Every stable soft compact subset $\widetilde{S}$ of a p-soft T_2-space is soft closed.*

Proof. Let the given conditions be satisfied and let $P_e^x \Subset \widetilde{S}^c$. Since $\widetilde{S}$ is stable, then for each $P_e^y \Subset \widetilde{S}$, we get $x \neq y$. Therefore there are two disjoint soft open sets G_{i_E} and W_{i_E} such that $x \in G_{i_E}$ and $y \in W_{i_E}$. It follows that $\{W_{i_E} : i \in I\}$ forms a soft open cover of $\widetilde{S}$. Consequently, $\widetilde{S} \widetilde{\subseteq} \widetilde{\bigcup}_{i=1}^{i=n} W_{i_E}$. Putting $\widetilde{\bigcap}_{i=1}^{i=n} G_{i_E} = H_E$ and $\widetilde{\bigcup}_{i=1}^{i=n} W_{i_E} = V_E$. Now, H_E and V_E are soft open sets such that $H_E \widetilde{\bigcap} V_E = \widetilde{\Phi}$. Therefore $H_E \widetilde{\bigcap} \widetilde{S} = \widetilde{\Phi}$ and this implies that $H_E \widetilde{\subseteq} \widetilde{S}^c$. Since P_e^x is chosen arbitrary, then $\widetilde{S}^c$ is soft open. Hence $\widetilde{S}$ is soft closed. □

According to the definition of soft regular spaces [22], the result, on general topology, reported that every compact T_2-space is regular is not valid on soft topology as it can be seen from Example (4.21). As a matter of fact, this result can be generalized on soft topology with respect to a p-soft regular space. To this end, we shall utilize the following auxiliary result.

Theorem 5.6. *Let F_E be a soft compact subset of a p-soft T_2-space. If $x \notin F_E$, then there are disjoint soft open sets G_E and V_E such that $x \in G_E$ and $F_E \subseteq V_E$.*

Proof. Let $x \notin F_E$. Then for each $y \Subset F_E$, we get that. $x \neq y$. Since (X, τ, E) is a p-soft T_2-space, then there exist soft open sets G_{i_E} and V_{i_E} such that $x \in G_{i_E}$, $y \in V_{i_E}$ and $G_{i_E} \widetilde{\bigcap} V_{i_E} = \widetilde{\Phi}$. Therefore $\{V_{i_E}\}$ forms a soft open cover of F_E. As F_E is soft compact, then $F_E \subseteq \widetilde{\bigcup}_{i=1}^{i=n} V_{i_E}$. By putting $\widetilde{\bigcup}_{i=1}^{i=n} V_{i_E} = V_E$ and $\widetilde{\bigcap}_{i=1}^{i=n} G_{i_E} = G_E$, it follows that V_E and G_E are disjoint soft open sets. This completes the proof. □

Theorem 5.7. *Every soft compact p-soft T_2-space is p-soft regular.*

Proof. Let H_E be a soft closed subset of a soft compact p-soft T_2-space (X, τ, E) and let $x \notin H_E$. By Proposition(5.1), we get H_E is soft compact. By Theorem(5.6), there exist soft open sets G_E and V_E such that $x \in G_E$, $H_E \subseteq V_E$ and $G_E \widetilde{\bigcap} V_E = \widetilde{\Phi}$. Thus (X, τ, E) is p-soft regular. □

Corollary 5.8. *Every soft compact p-soft T_2-space is a p-soft T_3-space.*

Lemma 5.9. *Let F_E be a soft open subset of a soft regular space. Then for each $P_e^x \in F_E$, there exists a soft open set G_E such that $P_e^x \in \overline{G_E} \widetilde{\subseteq} F_E$.*

Proof. Let F_E be a soft open set such that $P_e^x \in F_E$. Then $x \notin F_E^c$. Since (X, τ, E) is soft regular, then there exist two disjoint soft open sets G_E and W_E containing x and F_E^c, respectively. Thus $x \in G_E \widetilde{\subseteq} W_E^c \widetilde{\subseteq} F_E$. Hence $P_e^x \in G_E \widetilde{\subseteq} \overline{G_E} \widetilde{\subseteq} W_E^c \widetilde{\subseteq} F_E$. □

Theorem 5.10. *Let H_E be a soft compact subset of a soft regular space and F_E be a soft open set containing H_E. Then there exists a soft open set G_E such that $H_E \widetilde{\subseteq} G_E \widetilde{\subseteq} \overline{G_E} \widetilde{\subseteq} F_E$.*

Proof. Let the given conditions be satisfied. Then for each $P_e^x \in H_E$, we have $P_e^x \in F_E$. Therefore there is a soft open set W_{xe_E} such that $P_e^x \in W_{xe_E} \widetilde{\subseteq} \overline{W_{xe_E}} \widetilde{\subseteq} F_E$. Now, a collection of soft open sets $\{W_{xe_E} : P_e^x \in F_E\}$ forms an open cover of H_E. Since H_E is soft compact, then $H_E \widetilde{\subseteq} \widetilde{\bigcup}_{i=1}^{i=n} W_{xe_E}$. Putting $G_E = \widetilde{\bigcup}_{i=1}^{i=n} W_{xe_E}$. Thus $H_E \widetilde{\subseteq} G_E \widetilde{\subseteq} \overline{G_E} \widetilde{\subseteq} F_E$. □

Corollary 5.11. *Every soft compact soft T_3-space (X, τ, E) is a p-soft T_4-space.*

Proof. Suppose that F_{1_E} and F_{2_E} are two disjoint soft closed sets. Then $F_{2_E} \widetilde{\subseteq} F_{1_E}^c$. Since (X, τ, E) is soft compact, then F_{2_E} is soft compact and since (X, τ, E) is soft regular, then there is a soft open set G_E such that $F_{2_E} \widetilde{\subseteq} G_E \widetilde{\subseteq} \overline{G_E} \widetilde{\subseteq} F_{1_E}^c$. Obviously, $F_{2_E} \widetilde{\subseteq} G_E$, $F_{1_E} \widetilde{\subseteq} (\overline{G_E})^c$ and $G_E \widetilde{\bigcap} (\overline{G_E})^c = \widetilde{\Phi}$. Thus (X, τ, E) is soft normal. Since (X, τ, E) is soft T_3, then it follows from Theorem(4.24) that (X, τ, E) is a p-soft T_1-space. Hence (X, τ, E) is a p-soft T_4-space □

It is well known in general topology that if X is finite, then X is compact. This fact stimulate us to ask the following question: Is a finite soft topological space is soft compact?. The following theorem and example point out that the answer is "No".

Theorem 5.12. *The number of all soft subsets of an absolute soft set $\widetilde{X}$ is $2^{|E||X|}$.*

Proof. For any soft subset G_{i_E} of an absolute soft set $\widetilde{X}$, we have a map: $G_i : E \to X$. It will be known that the number of all subsets of X is $2^{|X|}$. Then for $e_1 \in E$, we can define $G_i(e_1)$ by $2^{|X|}$ distinct ways. Similarly, for $e_j \in E$, we can define $G_i(e_j)$ by $2^{|X|}$ distinct ways. By using the counting principle, the number of all soft subsets of $\widetilde{X}$ is $\underbrace{2^{|X|} \times 2^{|X|} \times ...2^{|X|}}_{|E| \ times}$. This completes the proof. □

Corollary 5.13. *An STS (X, τ, E) is discrete if and only if τ consists of $2^{|E||X|}$ distinct soft open sets.*

Example 5.14. *Let $E = \{e_n : n \in \mathbf{N}\}$ be a set of parameters and τ be a soft discrete topology on $X = \{1, 2, 3\}$. A collection Λ which consists of all soft points of $\widetilde{X}$ forms a soft open cover of $\widetilde{X}$. Obviously, Λ has not a finite subcover. So $\widetilde{X}$ is not soft compact in spite of X is finite.*

Theorem 5.15. *If (X, τ, E) is an enriched soft Lindelöf (resp. enriched soft compact) space, then a topological space (X, τ_e) is Lindelöf (resp. compact), for each $e \in E$.*

Proof. Suppose that $\{H_j(e) : j \in J\}$ is an open cover of a topological space (X, τ_e). Now, we construct a soft open cover of (X, τ, E) as follows:

(i) All soft open sets F_{j_E} in which $F_j(e) = H_j(e)$, for each $j \in J$.

(ii) Since (X, τ, E) is enriched, then we take a soft open set G_E such that $G(e) = \emptyset$ and $G(\alpha) = X$, for all $e \neq \alpha$.

Obviously, $\{F_{j_E} : j \in J\}\widetilde{\bigcup}G_E$ is a soft open cover of (X, τ, E). As (X, τ, E) is soft Lindelöf, then there exists a countable set S such that $\widetilde{X} = \bigcup_{j\in S} F_{j_E}\widetilde{\bigcup}G_E$. Therefore $X = \bigcup_{j\in S} F_j(e) = \bigcup_{j\in S} H_j(e)$. Thus (X, τ_e) is Lindelöf.
One can similarly prove a case between parenthesise. □

In case of a compact space (X, τ_e), it can be concluded from Example (5.14), that the converse of the above theorem fails. So in the following result, we show under what condition the converse of the above theorem is true.

Theorem 5.16. *If a topological space (X, τ_e) is Lindelöf (resp. compact) for each $e \in E$ and E is countable (resp. finite), then an STS (X, τ, E) is soft Lindelöf (resp. soft compact).*

Proof. Let $\{G_{j_E} : j \in J\}$ be a soft open cover of (X, τ, E). Then $X = \bigcup_{j\in J} G_j(e)$, for each $e \in E$. Since E is countable, then $| E |= \aleph$, where $\aleph$ is the cardinal number of the natural numbers set, and since (X, τ_e) is Lindelöf for each $e \in E$, then there exists a family of countable sets S_m such that $X = \bigcup_{j\in S_m} G_j(e)$, for each $e \in E$. Therefore $\widetilde{X} = \widetilde{\bigcup}_{j\in \bigcup_{m\in\aleph} S_m} G_{j_E}$. Since a countable union of countable sets is countable, then (X, τ, E) is soft Lindelöf.
One can similarly prove a case between parenthesise. □

Theorem 5.17. *Every uncountable (resp. infinite) soft subset of a soft Lindelöf (resp. soft compact) space has a soft limit point.*

Proof. We prove the theorem for an uncountable soft set and the other proof follows similar lines.
Let H_E be an uncountable soft subset of a soft Lindelöf space (X, τ, E). Suppose that H_E has not a soft limit point. Then for each $P^x_e \in \widetilde{X}$, there exists a soft open set $G_{x_{i_E}}$ containing P^x_e such that $G_{x_{i_E}}\widetilde{\bigcap}[H_E \setminus P^x_e] = \widetilde{\Phi}$. Now, the collection $\Lambda = \{G_{x_{i_E}}\}$ forms a soft open cover of $\widetilde{X}$. As $\widetilde{X}$ is soft Lindelöf, then there exists a countable set S such that $\widetilde{X} = \widetilde{\bigcup}_{i\in S} G_{x_{i_E}}$. Therefore X has at most countable soft points of H_E. This implies that H_E is countable. But this contradicts that H_E is uncountable. Hence H_E has a soft limit point. □

6. Conclusion

We present in this paper many effective notions to study soft set theory and soft topological spaces. First, we introduce and study some properties of the notions of partial belong and total non belong relations. We then use it to define p-soft T_i-spaces which are stronger than soft T_i-spaces [22], for $(i = 0, 1, 4)$, and to define p-soft T_3-spaces which are weaker than soft T_3-spaces [22]. One of the most significant idea is defining a p-soft regular space based on a total non belong relation and clarify that p-soft regular space is weaker than a soft regular space. In the investigation, we elucidate the relationships between compactness and some p-soft separation axioms. The motivation of giving these soft separation axioms is, first, to generalize existing comparable topological properties via soft topology (see, for example, Theorem (4.9) and Proposition (4.30)), second, to eliminate restrictions on the shape of soft open sets on soft regular spaces (see, Remark (4.18)), and third, to obtain a relationship between soft Hausdorff and p-soft regular spaces similar to those exists on general topology (see, for example, Theorem (5.6) and Theorem (5.7)). In the end, we hope that the concepts initiated herein will find their applications in many fields soon.

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