Filomat 32:18 (2018), 6219–6220
https://doi.org/10.2298/FIL1818219O

Published by Faculty of Sciences and Mathematics,
University of Niš, Serbia
Available at: http://www.pmf.ni.ac.rs/filomat

Addendum to: Permutability Degrees of Finite Groups

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Abstract. The present note fixes some problems of computational nature in [D.E. Otera and F.G. Russo, Permutability degrees of finite groups, Filomat 30 (2016), 2165–2175].

The *permutability degree* of a finite group G is introduced in [2] and defined by

$$pd(G) = \frac{1}{|G|\,|\mathcal{L}(G)|} \sum_{X \in \mathcal{L}(G)} |P_G(X)|,$$

where $P_G(X) = \langle g \in G \mid \langle g \rangle X = X \langle g \rangle \rangle$ is called *permutizer* of the subgroup X in G and $\mathcal{L}(G)$ denotes the lattice of subgroups of G. Note that [1] also investigates permutizers in addition to the references in [2].

After the publication of [2], we noted some errors which we want to fix here. The main results of [2] are not directly affected, but we cannot avoid to give more details on some points of our investigations.

(1) Definition of $P(G)$ in [2, Introduction]. Two lines below. Replace the sentence "Note that for any $X \in \mathcal{L}(P(G))$ one has $P_G(X) = G$" by "Note that when $X = P(G)$ one has $P_G(X) = P(G)$".

(2) The value of $pd(D_8)$ in [2, Example 3.2] is wrong. In fact $pd(D_8) = 1$, in agreement with [2, Proposition 6.1] when we show that $pd(D_{2p}) = 1$ for dihedral groups (of order $2p$ with p odd). The error of [2, Example 3.2] is here:

$$4 = |M_1| = |P_{D_8}(H)| = |P_{D_8}(K)| = |P_{D_8}(V)| = |P_{D_8}(U)|,$$

in fact we must replace this computation with

$$8 = |P_{D_8}(H)| = |P_{D_8}(K)| = |P_{D_8}(V)| = |P_{D_8}(U)|.$$

Consequently, $P(D_8) = D_8$ and $P(D_8) \neq Q(D_8)$. In particular, the final two sentences from "This example shows ..." until the end of [2, Example 3.2] must be removed. In addition,

2010 *Mathematics Subject Classification*. Primary 20D15; 20P02; Secondary 06B23, 20D60.
Keywords. Permutability degree; dihedral groups.
Received: 30 November 2017; Accepted: 22 December 2018
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(a) the final sentence of [2, Remark 2.3] must be removed;

(b) the sentence just before [2, Theorem 5.1] must be removed;

(c) the sentences before [2, Theorem 5.2] "Of course, D_8 ... $P(D_8) \simeq C_2$ " must be removed.

(d) the sentences "On the other hand... further Theorem 5.2" before [2, Theorem 5.3] must be removed.

(3) The lower bound of [2, Theorem 4.1] is correct, while the upper bound of [2, Theorem 4.1] is true only in special cases (via computational evidence), so it is better to amend it in the following form:

Upper bound in [2, Theorem 4.1]. *Let H be a subgroup of a group G. If $|P_G(X) : P_H(X)| \leq |G : H|$ for all $X \in \mathcal{L}(G)$, then*

$$pd(G) \leq \frac{|G : H|}{|G|\,|\mathcal{L}(G)|} \sum_{X \in \mathcal{L}(G)} |P_H(X)|.$$

Proof. We have indeed

$$|G|\,|\mathcal{L}(G)|\,pd(G) = \sum_{X \in \mathcal{L}(G)} |P_G(X)| \leq |G : H| \sum_{X \in \mathcal{L}(G)} |P_H(X)|.$$

(4) In [2, Theorem 4.3] we use an assumption which is again motivated by evidences of computational nature. Therefore [2, Theorem 4.3] is better if reformulated in the following way:

New version of [2, Theorem 4.3]. *Let G be a noncyclic group and p the smallest prime divisor of $|G|$. If $P_G(X)$ is a proper subgroup of G for all $X \in \mathcal{L}(G)$, $|P(G)| = p$ and $|\mathcal{L}(G)| = m + \frac{p^{m+1}+p-2}{p-1}$ for some $m \geq 0$, then*

$$pd(G) \leq \frac{p^{m+1} + 2p^2 + (m-3)p - m}{p^{m+2} + (m+1)p^2 - (m+2)p}.$$

Proof. The same of [2, Theorem 4.3] without the first two sentences.

References

[1] A. Ballester–Bolinches and R. Esteban-Romero, On a Question of Beidleman and Robinson, *Comm. Algebra* **30** (2002), 5757–5770.

[2] D.E. Otera and F.G. Russo, Permutability degrees of finite groups, *Filomat* **30** (2016), 2165–2175.