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A Novel Ant Colony Optimization Algorithm For The Shortest-path Problem In Traffic Networks

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Abstract. The Ant Colony Optimization (ACO) algorithm is a metaheuristic nature-inspired technique for solving various combinatorial optimization problems. The shortest-path problem is an important combinatorial optimization problem in network optimization. In this paper, a novel algorithm based on ACO to solve the single-pair shortest-path problem in traffic networks is introduced. In this algorithm, a new strategy is developed to find the best solution in a local search, by which the ants seek the shortest path using both a pheromone-trail-following mechanism and an orientation-guidance mechanism. A new method is designed to update the pheromone trail. To demonstrate the good performance of the algorithm, an experiment is conducted on a traffic network. The experimental results show that the proposed algorithm produces good-quality solutions and has high efficiency in finding the shortest path between two nodes; it proves to be a vast improvement in solving shortest-path problems in traffic networks. The algorithm can be used for vehicle navigation in intelligent transportation systems.
Keywords: Shortest-path problem, ACO algorithm, Orientation-guidance, Traffic network.

1. INTRODUCTION

The adaptive behavior of ants has aroused people's great interest. Taking inspiration from the ants' foraging behavior, Dorigo et al. first proposed an ACO algorithm called the ant system (AS) [1]. The ACO algorithm was originally used to solve the well-known Travelling Salesman Problem (TSP) [2]. In recent years, ACO algorithms have become an important branch in the field of intelligent computing. Many ACO algorithms have been developed and widely applied to various combinatorial optimization problems, such as the Job-Shop scheduling problem [3], multidimensional knapsack problem [4], robot path-planning problem [5], and the vehicle-routing problem [6]. It has achieved encouraging results in solving combinatorial optimization problems that are difficult to solve by traditional optimization methods.

The shortest-path problem in traffic networks is a key element in intelligent traffic systems. There are many classical algorithms for solving the shortest-path problem [7]. Now, the scale of traffic networks is becoming larger, and as a result, the shortest-path problem is facing new challenges. In large traffic networks, it is not wise for the shortest-path algorithm to aggressively pursue the accuracy of its solution because the higher the accuracy of an algorithm, the greater time it takes to reach a conclusion, and therefore it cannot meet real-time requirements. Intelligent algorithm models are simple with few constraints in the

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objective function. Practice has proven that intelligence algorithms have excellent performance in some complex optimization problems. Various ACO algorithms have been used for shortest-path problems. Glabowski et al. introduced an AS called the ShortestPathACO algorithm based on ACO for solving the Shortest-Path problem [8]. In that study, several problems about using ACO to find the shortest path were discussed. Attiratanasunthron and Fakcharoenphol studied time complexity for ACO applied to the single destination shortest-path problem [9]. They proved a bound of $O\left(\frac{1}{\rho}n^2 m \log n\right)$ for the expected number of iterations required for an ACO algorithm for solving the single destination shortest-path problem on a directed acyclic network with n nodes and m edges, where ρ is an evaporation rate. Angus improved the ability of the ACO algorithm for shortest-path problems [10]. The author proved the ability of the ACO algorithm to balance multiple factors such as cost and length in the solution construction process. Ok, Seo, and Ahn proposed a preference-based shortest-path ACO algorithm [11]. In this algorithm, the shortest path was obtained taking into consideration the properties of the links. Faisal et al. introduced an algorithm based on the ant system called AntStar for solving single-pair shortest-path problems. In this case, AS and A* algorithm were integrated to enhance the optimization performance [12].

The purpose of this paper is to study further the shortest-path problem in traffic networks and to develop a novel ACO algorithm to solve the problem efficiently. In this paper, we propose a novel shortest-path algorithm based on ACO. The novel algorithm simulates the foraging behavior of natural ants, which seek the shortest path from their nest to a food source by not only the pheromones on the path but also the orientation between the original node and the destination.

2. SHORTEST-PATH PROBLEM IN A TRAFFIC NETWORK

A traffic network can be modeled as a directed and connected weighted network $G : \{V, E, C, W_{(i,j)}(t)\}$, where V is a set of the nodes of the traffic network [13]. $V = \{(1, 2, \cdots, n) \mid n$ is the node number$\}$, where a node represents an intersection of the traffic network, as shown in Figure 1; E is a set of road arcs within the traffic network, $E \in V \times V = \{(i, j) \mid i \neq j; i, j \in V\}$, where a road arc (i, j) represents a link between nodes i and j; C is a set of the coordinates of the nodes of the traffic network, $C = (x_i, y_i) \mid x, y \in R^+; i, j \in V$; $W_{(i,j)}(t)$ denotes a set of weights of the road arcs at time t, where the values are measured using the travel time of the road arcs, and each road arc $(i, j) \in E$ has an associated travel time at t.

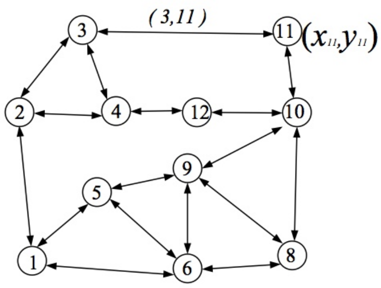

Figure 1: An example of a Traffic Network (the numbers in the circles are the node numbers).

In this paper, we focus on a single-pair shortest-path problem, which is to find the shortest path from a given original node to a destination. A path in a traffic network from an original node to a destination is defined as an alternating sequence of different nodes and road arcs $(o, (o, k), k, (k, u), u, \cdots v, (v, d), d)$, where the node o is the original node, node d is the destination, and there are no circuits in the path.

In a traffic network, assuming $R^t_{o,d}(n)$ is the set of all paths between node o and node d at time t, the number of all paths is $N = \mid R^t_{o,d}(n) \mid$, $R^t_{o,d}(l)$ denotes one of the paths in $R^t_{o,d}(n)$, if there is one path $R^t_{o,d}(k) \in R^t_{o,d}(n)$, and the following formula is workable for all $l \in \{1, 2, \cdots, N\}$:

$$F\left(R^t_{o,d}(k)\right) \leq F(R^t_{o,d}(l)), \tag{1}$$

where $F(\cdot)$ is the sum of travel times of all road arcs in path $R^t_{o,d}(k)$, k denotes one path from node o to node d, then $R^t_{o,d}(k)$mis defined as the shortest path between node o and node d at time t. The formal mathematic model that describes the shortest-path problem is shown as follows:

$$\min T = \sum_{(i,j)\in E} W_{(i,j)}(t) X_{i,j} \tag{2}$$

s.t.

$$X_{ij} \in \{0, 1\}; i, j = 1, 2, \cdots, n; i \neq j \tag{3}$$

$$\left(\sum_{i,j\in B} X_{i,j} \right) < |B|; 2 \leq |B| \leq n - 2, B \subset \{1, 2, \cdots, n\}, \tag{4}$$

where formula (2) is the object function, T is the sum of the travel times of all road arcs in one path from the original node to the destination; $W_{(i,j)}(t)$ is the travel time of road arc (i, j); X_{ij} denotes whether or not the path passes the road arc (i, j); $X_{ij} = 0$ means the path passes road arc (i, j) at a point in time, $X_{ij} = 0$ means the path does not pass road arc (i, j). The constraint condition formula (4) can avoid a loop in the path; set B is the subset of the set of all nodes that the path passes, where B is the number of inequality nodes in set B.

3. ACO

To find the shortest path in traffic networks, the artificial ants construct solutions by successively choosing nodes to visit, until the destination node has been visited. In the process, ants choose their next node according to two aspects: the information they sense in the pheromones left on the road arc and the local heuristic information, which is called visibility. Namely, ant k that is in location i at time t moves to j with a particular transition probability. In ACO the transition probability is calculated using formula (5) as follows:

$$P^k_{ij}(t) = \begin{cases} \dfrac{\tau^{\alpha}_{ij}(t)\eta^{\beta}_{ij}(t)}{\Sigma_{S\in\Omega_k} \tau^{\alpha}_{is}(t)\eta^{\beta}_{is}(t)}, & j \in \Omega_k \\ 0, & \text{otherwise} \end{cases}, \tag{5}$$

where $P^k_{ij}(t)$ is the probability of ant k choosing node j to move to in location i at time t, $\pi_{ij}(t)$ is the amount of pheromone on road arc (i, j) at time t, η_{ij} is the heuristic value from location i to j; parameters α and β denote the significance of pheromone and heuristic information, which control the influence of pheromone versus heuristic information on ants selecting traveling direction; the set $\Omega_k = 0, 1, \cdots, n$ represents the next position that ant k is allowed to reach from position i, it is a subset of the nodes connected with node i. It can be seen from formula (5) that the transition probability $P^k_{ij}(t)$ is directly proportional to $\tau^{\alpha}_{ij}\eta^{\beta}_{ij}$; when the amount of pheromone on the road arc (i, j) is more and the heuristic information is more important, then the ant is more likely to choose node j to move to.

From the above, we can see that the operation mechanism of ACO is as follows:

(1) Ants select the travel direction according to a transition probability, which depends on the amount of pheromone on the road arc and the heuristic information;

(2) After each iteration, the ants deposit pheromone on each visited road arc.

4. NOVEL SHORTEST PATH ACO ALGORITHM

In fact, the ants often use the path orientation information to guide their foraging process [14]. The ants that have arrived at the food source can remember the orientation of the paths from the ant nest to the food source [15, 16]. The orientation of the paths that the ants remembered can guide ants to find the shortest path to the food source more rapidly than pheromone trails [17]. The ant colony can make composite collective decisions more efficiently using both path orientation information and pheromone trails [18]. Therefore, it is necessary to consider the path orientation information in ACO. In this paper, we propose a novel ACO algorithm for finding the shortest path in a traffic network; the artificial ants simulate the phenomenon that the real ants will use the path orientation information in addition to the pheromone information to find the shortest path from the ant nest to the food source in the foraging behavior.

A traffic network is a geographical network with a certain spatial distribution of each of its intersections and road arcs. The shortest path between two nodes always distributes along the straight line between the two nodes. As shown in Figure 2, given that an ant intends to find a shortest path from node 3 to node 6, the ant starts from node 3, assuming it moves along the path 3–4–12–10 and arrives at node 10, if the ant selects the next node to move to according to the pheromone on the road arc and the distance of the road arc, then the ant is more likely to select node 11 as the next node to move to, so it will enter an invalid path [1]. Given the ant has obtained the orientation information from node 3 to node 6, when the orientation information is applied to guide the ant to select the forward direction, the ant will select node 9 or node 8 with large probability because the orientations of edges 10–9 and 10–8 are more inclined to node 6. As can be seen, orientation guidance can effectively guide the ant to find the shortest path.

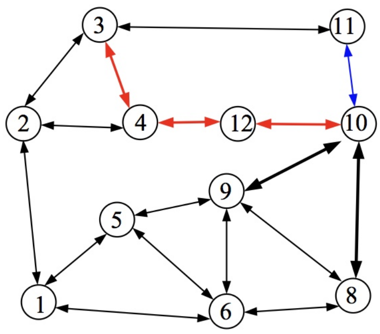

Figure 2: An example of path orientation guidance.

4.1. METAHEURISTIC METHOD

We use the following approach to express the orientation guidance. As shown in Figure 3, given A is the original node, B is the destination, the line $\overline{AB}$ can be used to represent the orientation of the shortest path from A to B. Given an ant has reached node i from A, and given that there are nodes j and z to choose to move to, it can be seen from the figure that the value of the included angle ω between lines $\overline{ij}$ and $\overline{iB}$ is less than the included angle φ between lines $\overline{iz}$ and $\overline{iB}$; so, the line $\overline{ij}$ inclines to the line $\overline{AB}$ more. Assuming the ant remembered the orientation of the path from A to B (the orientation of line $\overline{AB}$), if the amount of pheromone on road arc (i, j) is approximately equal to that of road arc (i, z) and the road conditions of the two road arcs are roughly similar, then the ant will select the node j to move to with large probability, which is closer to the orientation.

To express the orientation of road arcs, one coefficient μ_{ij} is defined as follows:

$$\mu_{ij} = \frac{1}{1+\theta} \tag{6}$$

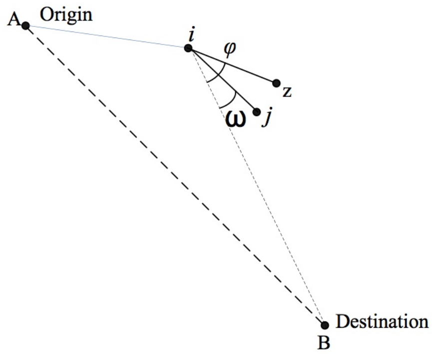

Figure 3: A diagrammatic drawing of path orientation.

where θ is the included angle described above.

In this paper, the visibility η_{ij} is defined as follows:

$$\eta_{ij} = \frac{d_{ij}}{w_{ij}} \tag{7}$$

where d_{ij} is the distance of the road arc (i, j), w_{ij} is the travel time of the road arc (i, j), $\frac{d_{ij}}{w_{ij}}$ is the average speed of the vehicles traveling on the road arc.

Therefore, formula (5) is modified to formula (8) when calculating the transition probability of ants selecting their next position to move to in the process of searching the path.

$$P_{ij}^{k}(t) = \begin{cases} \dfrac{\tau_{ij}^{\alpha}(t)\eta_{ij}^{\beta}(t)u_{ij}^{\gamma}}{\sum_{S\in\Omega_k}\tau_{is}^{\alpha}(t)\eta_{is}^{\beta}(t)u_{ij}^{\gamma}}, & j \in \Omega_k \\ 0, & \text{otherwise} \end{cases} \tag{8}$$

where the parameter γ denotes the influence of the orientation information on the ant selecting the next node. If $\gamma > 0$, the orientation information is taken into consideration when calculating the transition probability, if $\gamma = 0$, the orientation information is not considered.

As shown in Figure 4, assuming that the ants start from node 10 to find the shortest path to node 20, it can be seen from the figure that the ants have three nodes, nodes 1, 9, and 11, to choose as the next node to move forward from node 10. Assuming that the initial pheromone of each road arc is 10, the parameter $\alpha = 1, \beta = 2$, and $\gamma = 2$, the length of each road arc is 300 meters, the travel time of road arcs 10–1, 10–9, 10–11 are 0.36 minutes, 0.6 minutes, and 0.45 minutes, respectively, then the average travel speeds of road arcs 10–1, 10–9, 10–11 are 13.9 m/s, 8.3 m/s, 11.1 m/s, respectively. In the figure, the dashed line represents the orientation from the origin node 10 to the destination node 20. The included angles between the road arcs 10–1, 10–9, and 10–11, and the line 10–20 are 2.03 radians, 0.46 radians, and 1.11 radians, respectively. The transition probability is calculated according to equation 8, and then the results of the transition probability of the ants choosing node 1, node 9, and node 11 as the next node to move to are 26.5%, 39.6%, and 33.9%, respectively.

4.2. PHEROMONE UPDATE

In an iteration, after all ants find one path from the ant nest to a food source, at that point the pheromone on the road arcs will be updated according to formula (9). In this paper, only the ants with good solutions contribute to the pheromone update. The approach is as follows:

(1) First, rank the ants according to the length of the paths from short to long, the paths with short length are good solutions;

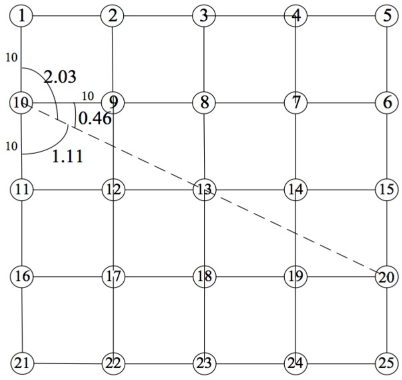

Figure 4: An example of transition probability calculation.

(2) Second, select the former h ants, called elitist ants;

(3) Third, use the h ants to update the pheromone, the ants deposit the pheromone on the road arcs according to solution quality, the pheromone is calculated with formula (11).

$$\tau_{ij}(t+1) = \rho \cdot \tau_{ij}(t) + \Delta\tau_{ij}(t, t+1), \tag{9}$$

$$\Delta\tau_{ij}(t+1) = \sum_{k=1}^{h} \Delta\tau_{ij}^{k}(t, t+1), \tag{10}$$

$$\Delta\tau_{ij}^{k} = \begin{cases} \dfrac{Q}{k \times L_k}, & \text{if } (i, j) \text{ is passed by the } k\text{th ant} \\ 0 & \text{otherwise} \end{cases}, \tag{11}$$

where ρ is the persistence coefficient ($0 < \rho < 1$), the pheromones on a road arc volatilize at a certain rate $1 - \rho$; $\Delta\tau_{ij}^{k}(t, t+1)$ is the pheromone that the kth ant deposits on that road arc, L_k is the length of the kth path, Q is a constant; $\Delta\tau_{ij}(t, t+1)$ is the total amount of pheromone that the h elitist ants deposit on road arc (i, j) within one iteration.

4.3. ALGORITHM PROCEDURE

As mentioned above, the algorithm works as follows. The artificial ants search the shortest paths from the original node to the destination. During the search, each ant constructs its solution by repeatedly using the transition probability rule to select the nodes to move to until reaching the destination. The transition probability is obtained by formula (8). The artificial ants tend to choose the nodes that are connected by an edge with many pheromones and fast travel speed, and whose orientation inclines to the destination.

After initializing the parameters of the algorithm, the two basic steps, construction of the shortest path and pheromone update, are repeated for a given number of iterations. When the algorithm satisfies one of the following conditions, the algorithm terminates: (1) the number of iterations reaches a given maximum number; it indicates that the ants have already done enough work; (2) the same optimal solution is repeated multiple times, it means that the algorithm has converged and no longer needs to continue. The following is a formal description of the algorithm represented as pseudocode.

```
1  Begin
2  Initialize the parameters of the algorithm and the value of pheromones on the road arcs according to
   the traffic conditions.
3  For i = 1 to Max_Iteration
4      Place the m ants on the original node.
5      Ants start moving from original node.
6      For k = 1 to m_ants
7          kth ant searches the path to the destination using the transition rule.
8      End For
9      Evaluate each solution, if a solution meets the termination conditions then output the shortest
       path, or go to next step.
10     Rank the solutions.
11     Select the elitist ants.
12     Update the pheromone.
13     Go to step 4.
14 End For
15 End
```

The flowchart of the novel algorithm is shown in Figure 5.

5. CASE STUDY

In this section, an experiment is conducted to evaluate the novel ACO algorithm proposed above. A traffic network that is composed of 528 nodes and 906 road arcs (Figure 6) was taken as the experimental object. The experiment is implemented using the programming language C#, based on a digital map that is constructed on the platform of the geographic information system software ArcGIS10.2. In the experiment, the performances of the novel algorithm and the traditional ACO algorithm are compared using the same configuration except for the parameter γ. The algorithm parameters are set as follows: the population size of the ant colony is $m = 30$; the initial pheromone is $\tau_c = 10$; the pheromones that the ants deposit on the path after each iteration is $Q = 50$; the persistence coefficient of the pheromone is $\rho = 0.8$; where parameter $\alpha = 1$, $\beta = 2$, and $\gamma = 2$ in the novel ACO algorithm, $\gamma = 0$ in the traditional ACO algorithm.

The experiment is tested successfully several times. Figure 7 illustrates the experimental result of one time. The experimental results showed that the algorithm can find optimal solutions, and the efficiency of the algorithm has been greatly improved. As shown in Figure 7, at the beginning, there are only initial pheromones available on the road arcs; each ant selects the moving direction mainly based on orientation guidance, so, in the first iteration, the length of the path of the novel ACO algorithm was shorter. There is a clear improvement in the performance of the novel algorithm compared with the basic ACO algorithm. This excellent enhancement was due to the ant path selection mechanism being integrated with orientation guidance. Figure 6 shows the traffic network adopted in the experiment, where the figure on the right side is an enlarged drawing of part of the network represented in the figure on the left side and the numbers are the serial numbers of the nodes.

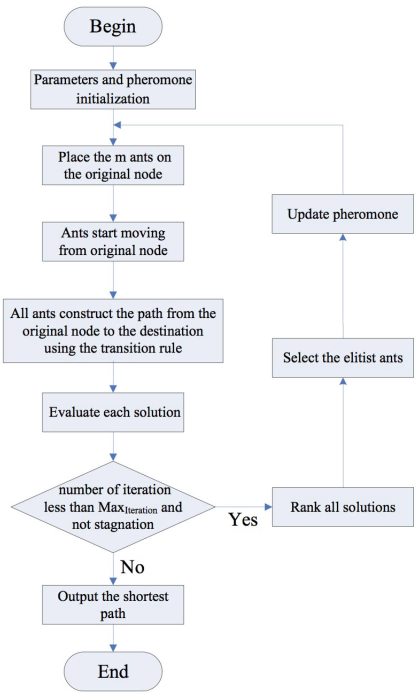

Figure 5: Flowchart of the novel algorithm.

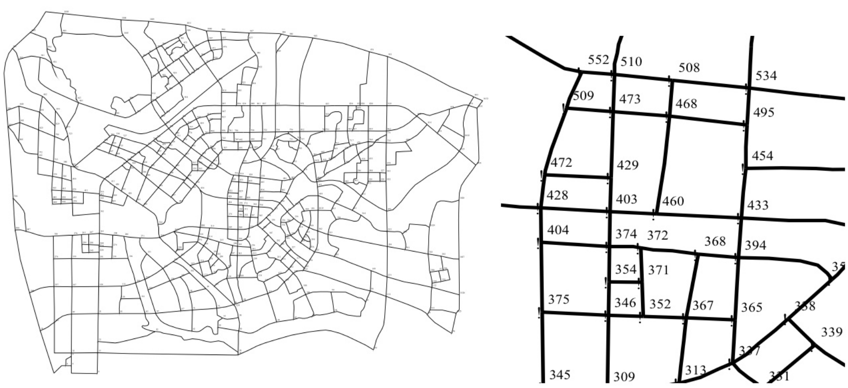

Figure 6: Traffic network used in the experiment.

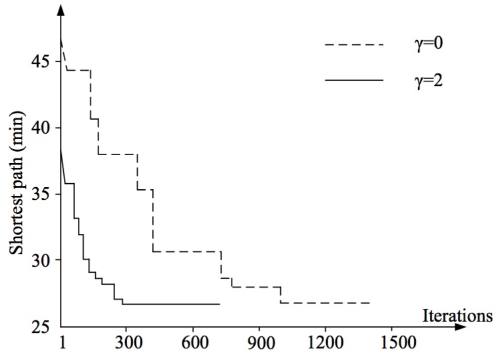

Figure 7: Explicitly state which results show the original ACO and which the proposed improved ACO.

6. DISCUSSION AND CONCLUSION

The vehicle speed on the road arcs reflects the traffic conditions of the roads of the traffic network. In the process of selecting the travel path from the origin to the destination, people always move along the orientation that leads to the destination, not selecting the forward direction completely randomly, and often choose main roads or wide roads with fast speed to identify the path with the shortest travel time. In this novel algorithm, the speed factor is added to the calculation of the transition probability. The mode of searching for the shortest path in the algorithm is in accord with the psychology of people in the selection of their travel path.

Under conditions when congestion in traffic networks is a common occurrence, there is significant practical meaning in studying the shortest-path problem. The algorithm proposed in this paper can be applied to guide travelers along the optimum path to travel; this will help travelers arrive at their destination more quickly and conveniently, which promotes a reduction of the time vehicles spend in traffic networks. On a macrolevel, this can reduce the overall amount of travel time in a whole traffic network, adjust the traffic flow, and reduce traffic jams, thus the efficiency of the whole transportation system is improved.

The proposed algorithm enhances the optimization and performance of the ACO by integrating the orientation information to guide the movements of ants to search for the shortest path. In a future study, the complexity of the algorithm will be further analyzed theoretically, such as the coefficients' impact on the performance of the algorithm. In addition, a study on how to apply it to guide travelers along the optimum path to travel will be carried out.

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