Filomat 33:13 (2019), 4085–4093
https://doi.org/10.2298/FIL1913085K

Published by Faculty of Sciences and Mathematics,
University of Niš, Serbia
Available at: http://www.pmf.ni.ac.rs/filomat

k-Metric Antidimension of Some Generalized Petersen Graphs

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Abstract. Resistance of social graphs to active attacks is a very important feature which must be maintained in the modern networks. Recently introduced k-metric antidimension graph invariant is used to define a new measure for resistance of social graphs. In this paper we have found and proved the k-metric antidimension for generalized Petersen graphs $GP(n,1)$ and $GP(n,2)$. It is proven that $GP(2m+1,1)$ and $GP(8,2)$ are 2-metric antidimensional, while all other $GP(n,1)$ and $GP(n,2)$ graphs are 3-metric antidimensional.

1. Introduction

The notion of (k,l)-anonymity was introduced by Trujillo-Rasua and Yero (2016) in [8]. As explained in that paper the motivation was to establish a new measure for evaluating the resistance of social graphs against active attacks. This measure uses a new graph invariant: k-metric antidimension.

Let $G=(V,E)$ be a simple connected graph and $d(u,v)$ is the length of the shortest path between the vertices u and v. The metric representation $r(v|S)$ of vertex v with respect to an ordered set of vertices $S=\{u_1,...,u_t\}$ is defined as $r(v|S)=(d(v,u_1),...,d(v,u_t))$. Values $d(v,u_i)$ are considered as metric coordinates of v with respect to vertices u_i.

Definition 1.1. ([8]) *Let k be the largest positive integer with the property that for every vertex $v \in V(G)\setminus S$ there exist at least $k-1$ different vertices $v_1,...,v_{k-1} \in V(G)\setminus S$ with $r(v|S)=r(v_1|S)=...=r(v_{k-1}|S)$. In other words, v and $v_1,...,v_{k-1}$ have the same metric representation with respect to S. Then, set S is called a k-antiresolving set for G.*

Definition 1.2. ([8]) *For fixed k, the minimum cardinality amongst all k-antiresolving sets in G is called the k-metric antidimension of graph G, and it is denoted by $adim_k(G)$. A k-antiresolving set of that minimum cardinality $adim_k(G)$ is called a k-antiresolving basis of G.*

Definition 1.3. ([8]) *If $k = max\{t \,|\, adim_t(G) \text{ exists}\}$ then graph G is called k-metric antidimensional.*

Observation 1.4. ([8]) *If G has maximum degree Δ and G is k-metric antidimensional then $1 \le k \le \Delta$ holds.*

2010 *Mathematics Subject Classification*. 05C12; 05C07
Keywords. Generalized Petersen graphs, k-metric antidimension, Graph theory
Received: 17 January 2019; Revised: 30 May 2019; Accepted: 14 June 2019
Communicated by Paola Bonacini
This research was partially supported by Serbian Ministry of Education, Science and Technological Development under the grants no. 174010 and 174033.
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In the sequel we shall use the equivalence relation defined in [1, 2]. Let $S \subseteq V(G)$ be a subset of vertices of a connected graph G and let ρ_S be equivalence relation on $V(G) \setminus S$ defined by

$$(\forall a, b \in V(G) \setminus S)\ (a\rho_S b \iff r(a|S) = r(b|S))$$

and let $S_1, ..., S_m$ be the equivalence classes of ρ_S. Then the following property can be proved.

Proposition 1.5. ([1, 2]) *Let k be a fixed integer, $k \geq 1$. Then S is a k-antiresolving set in G if and only if $\min\limits_{1\leq i\leq m} |S_i| = k$.*

In [2, 10] it has been proved that the problem of determining the k-metric antidimension of a graph for a fixed k is NP-complete in general case.

For some graphs with special structures it would be interesting to investigate the privacy measure based on the k-metric antidimension. Such investigations are considered in the literature:

- In [9] are considered 1-metric antidimensional trees and unicyclic graphs;
- Privacy violation properties of eight real social networks and large number of synthetic networks generated by both the classical Erdös-Rényi model and the Barbábasi-Albert preferential-attachment model were analyzed in [4];
- First privacy-preserving graph transformation improving privacy is presented in [6]. Experiments on random graphs show that the proposed method effectively counteracts active attacks;
- k-metric antidimensions of wheels and grid graphs are given in [1].

In this paper we study the k-metric antidimension of generalized Petersen graphs introduced by Coxeter [3]. The generalized Petersen graph $GP(n, k)$ ($n \geq 3$; $1 \leq k < n/2$) has $2n$ vertices and $3n$ edges, where vertex set V and edge set E are defined as follows: $V = \{u_i, v_i \mid 0 \leq i \leq n-1\}$, $E = \{\{u_i, u_{i+1}\}, \{u_i, v_i\}, \{v_i, v_{i+k}\} \mid 0 \leq i \leq n-1\}$, with vertex indices taken modulo n. In this notation the well-known Petersen graph presented on Figure 1 is $GP(5, 2)$.

There are a lot of papers devoted to generalized Petersen graphs and their invariants. Some recent results include: metric dimension [7], strong metric dimension [5], and power domination [11].

Example 1.6. *Consider the Petersen graph G given on Figure 1. By total enumeration it is easy to see that G is 3-antidimensional: 1-antiresolving basis is $\{u_0, u_2\}$, 2-antiresolving basis is $\{u_0, v_0\}$, while 3-antiresolving basis is $\{v_0\}$. Therefore, $adim_k(G) = \begin{cases} 2, & k = 1,2 \\ 1, & k = 3 \end{cases}$.*

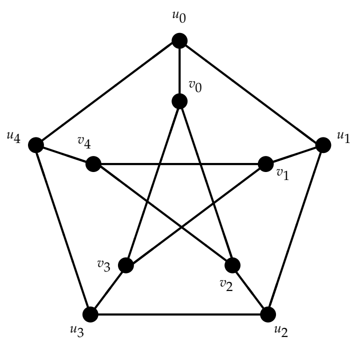

Figure 1: Petersen graph G

It should be noted that, according to Definition 1.3, if a graph is k-metric antidimensional, it does not mean that there exists an l-antiresolving set for each $l \in \{2, ..., k-1\}$. For example, wheel graphs studied in [1] are n-metric antidimensional, but for $4 \le l \le n-1$ there are no l-antiresolving sets in wheel graphs. Therefore, as mentioned and presented in [2, 4], it is an interesting problem to find families of graphs for which there exist l-antiresolving sets for all values of l, such that $2 \le l \le k-1$. In the next two sections we show that $GP(n, 1)$ and $GP(n, 2)$ satisfy the previous property.

In Section 2 we prove that $GP(2m, 1)$ is 3-metric antidimensional, while $GP(2m+1, 1)$ is 2-metric antidimensional. In Section 3 it is shown that $GP(n, 2)$ is 3-metric antidimensional, except for $n = 8$, when it is 2-metric antidimensional.

2. k-metric antidimension of GP(n,1)

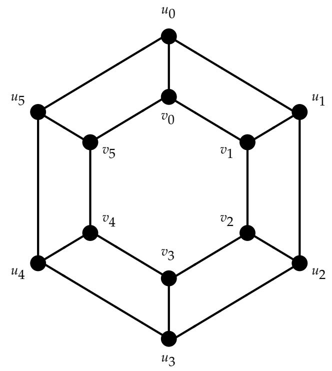

Figure 2: Graph GP(6,1)

Theorem 2.1. *Graph $GP(2m, 1)$ is 3-metric antidimensional and*

(i) $adim_1(GP(2m, 1)) = 1$

(ii) $adim_2(GP(2m, 1)) = 4$

(iii) $adim_3(GP(2m, 1)) = 2$

Proof. (i) Let us consider set $S = \{u_0\}$. The equivalence classes of ρ_S are given in Table 1. More precisely, the first column of Table 1 contains set S, while in the second one the equivalence classes of relation ρ_S are given, and in the third column the metric representations with respect to S are shown for all their vertices. Since the minimal cardinality of equivalence classes is one, according to Property 1.5, it follows that $S = \{u_0\}$ is 1-antiresolving set. Since $|S| = 1$, $S = \{u_0\}$ is a 1-antiresolving basis of $GP(2m, 1)$, so $adim_1(GP(2m, 1)) = 1$.
(ii) Due to symmetry of $GP(2m, 1)$ and the fact that set $\{u_0\}$ is 1-antiresolving, it follows that every set S consisting of only one vertex of $GP(2m, 1)$ is 1-antiresolving. Let us consider sets S of cardinality two. From symmetry properties of $GP(2m, 1)$, without loss of generality we can assume $u_0 \in S$. We have two cases.
Case 1. $v_m \notin S$. Then from Table 1 it follows that v_m is the only vertex with the metric coordinate with respect to vertex u_0 which is equal to $m+1$ and, consequently, S is 1-antiresolving.
Case 2. If $v_m \in S$ then $S = \{u_0, v_m\}$ and the corresponding equivalence classes are given in Table 1. From Table 1 and Property 1.5 it follows that set $\{u_0, v_m\}$ is 3-antiresolving.
Cases 1 and 2 demonstrate that there does not exist set S of cardinality 2 which is 2-antiresolving for $GP(2m, 1)$.
Next we consider sets S with cardinality three. Again, we can suppose that $u_0 \in S$. If we $v_m \notin S$, as in Case 1, we can conclude that S is 1-antiresolving. Suppose that $v_m \in S$ and consider cases $v_0 \in S$ or $u_m \in S$. If $v_0 \in S$, i.e. $S = \{u_0, v_0, v_m\}$, then equivalence class $\{u_m, v_{m-1}, v_{m+1}\}$ from Table 1 is partitioned into 2 classes: $\{u_m\}$ with metric representation equal to $(m, 1, m+1)$ and $\{v_{m-1}, v_{m+1}\}$ with metric representation equal to

$(m, 1, m-1)$. Similarly, if $u_m \in S$, i.e. $S = \{u_0, u_m, v_m\}$, then class $\{u_1, u_{m-1}, v_0\}$ from Table 1 is partitioned into $\{u_1, u_{m-1}\}$ with metric representation equal to $(1, m, m-1)$ and $\{v_0\}$ with metric representation equal to $(1, m, m+1)$. Hence, if $u_0, v_m \in S$ and $v_0 \in S$ or $u_m \in S$ set S is 1-antiresolving. Finally, if $u_0, v_m \in S$ and $v_0 \notin S$ and $u_m \notin S$ we consider equivalence class $\{u_m, v_{m-1}, v_{m+1}\}$ from Table 1. Table 2 contains distances of u_m, v_{m-1}, v_{m+1} from all possible third elements of S. From Table 2 it follows that in all cases equivalence class $\{u_m, v_{m-1}, v_{m+1}\}$ is partitioned with respect to the third coordinate into two classes, one of cardinality 2 and the other of cardinality 1. Consequently, set S is again 1-antiresolving. Therefore, there does not exist set S of cardinality 3 which is 2-antiresolving for $GP(2m, 1)$.

Consider now set $S = \{u_0, v_0, u_m, v_m\}$ of cardinality 4 and the corresponding classes in Table 1. Since all classes have cardinality 2, it follows that S is 2-antiresolving for $GP(2m, 1)$. Since $adim_2(GP(2m, 1)) > 3$, we conclude $adim_2(GP(2m, 1)) = 4$.

(iii) Let $S = \{u_0, v_m\}$. As we have already concluded in (ii), from Table 1 it follows that S is 3-antiresolving set for $GP(2m, 1)$ and consequently $adim_3(GP(2m, 1)) \leq 2$. Let us prove that there does not exist a 3-antiresolving set S' of cardinality one. By symmetry, we can suppose that $S' = \{u_0\}$. As proved in (i), S' is 1-antiresolving set.

Since $GP(2m, 1)$ is 3-regular, according to Observation 1.4, it follows that $GP(2m, 1)$ is k-metric antidimensional for some $k \leq 3$. From (i)-(iii) it follows that $GP(2m, 1)$ is 3-metric antidimensional. □

Table 1: Equivalence classes of ρ_S on $GP(2m, 1)$

S	*Equivalence class*	*Metric representation*
$\{u_0\}$	$\{u_1, u_{n-1}, v_0\}$	(1)
	$\{u_i, u_{n-i}, v_{i-1}, v_{n-i+1}\}$	$(i), 2 \leq i \leq m-1$
	$\{u_m, v_{m-1}, v_{m+1}\}$	(m)
	$\{v_m\}$	$(m+1)$
$\{u_0, v_m\}$	$\{u_1, u_{n-1}, v_0\}$	$(1, m)$
	$\{u_i, u_{n-i}, v_{i-1}, v_{n-i+1}\}$	$(i, m-i+1), 2 \leq i \leq m-1$
	$\{u_m, v_{m-1}, v_{m+1}\}$	$(m, 1)$
$\{u_0, v_0, u_m, v_m\}$	$\{u_1, u_{n-1}\}$	$(1, 2, m-1, m)$
	$\{u_i, u_{n-i}\}$	$(i, i+1, m-i, m-i+1)$
	$\{v_{i-1}, v_{n-i+1}\}$	$(i, i-1, m-i+2, m-i+1)$
	$\{u_{m-1}, v_{m+1}\}$	$(m, m-1, 2, 1)$

Table 2: Distances of u_m, v_{m-1}, v_{m+1} from the third element of S

Third element	u_m	v_{m-1}	v_{m+1}
$u_i, 1 \leq i \leq m-1$	$m-i$	$m-i$	$m-i+2$
$u_{n-i}, 1 \leq i \leq m-1$	$m-i$	$m-i+2$	$m-i$
$v_i, 1 \leq i \leq m-1$	$m-i+1$	$m-i-1$	$m-i+1$
$v_{n-i}, 1 \leq i \leq m-1$	$m-i+1$	$m-i+1$	$m-i-1$

Theorem 2.2. *Graph $GP(2m+1, 1)$ is 2-metric antidimensional and*

(i) $adim_1(GP(2m+1, 1)) = 2$

(ii) $adim_2(GP(2m+1, 1)) = 1$

Proof. (i) Let $S = \{u_0, v_1\}$. It is easy to see that vertex v_2 has unique metric representation with respect to S equal to $(3, 1)$. According to Property 1.5, S is 1-antiresolving set of $GP(2m+1, 1)$.

Let us prove that S is 1-antiresolving basis of $GP(2m+1, 1)$. Suppose contrary, that there exists 1-antiresolving

set S' of cardinality 1. Without loss of generality, due to the symmetry of $GP(2m+1,1)$, we can assume that $S' = \{u_0\}$. The equivalence classes of $\rho_{S'}$ are given in Table 3. From Table 3 it follows that set S' is 2-antiresolving, which is a contradiction. Therefore, $S = \{u_0, v_1\}$ is an 1-antiresolving basis of $GP(2m+1,1)$, i.e. $adim_1(GP(2m+1,1)) = 2$.
(ii) Let $S = \{u_0\}$. From Table 3 it is evident that set $S = \{u_0\}$ is 2-antiresolving set of $GP(2m+1,1)$. Since $|S| = 1$, S is a 2-antiresolving basis of $GP(2m+1,1)$ and hence $adim_2(GP(2m+1,1)) = 1$.
From (i) and (ii) it follows that $GP(2m+1,1)$ is k-metric antidimensional for $k \geq 2$. On the other side, according to Observation 1.4, $k \leq 3$. Let us prove that $GP(2m+1,1)$ is not 3-metric antidimensional, i.e. that in this graph there does not exist a 3-antiresolving set.
Let S be a set of vertices from V. Without loss of generality, we can assume $u_0 \in S$. Consider the following two cases:
Case 1. $v_m \notin S$ or $v_{m+1} \notin S$. According to Table 3, the equivalence class with respect to $S' = \{u_0\}$ with metric coordinate $m+1$ is $\{v_m, v_{m+1}\}$. Therefore, the equivalence class with respect to S, $S \supseteq S'$, whose members have distance from u_0 equal to $m+1$ has cardinality less or equal to 2. It follows that S is not a 3-metric antidimensional set.
Case 2. Suppose that $v_m \in S$ and $v_{m+1} \in S$. Then each vertex $u_i, i = 1, ..., n-1$, $v_j, j = 0, ..., n-1, j \neq m, m+1$ has unique metric representation with respect to $\{u_0, v_m, v_{m+1}\} \subseteq S$ and therefore S is 1-antiresolving set.
Cases 1 and 2 demonstrate that in $GP(2m+1,1)$ there does not exist a 3-antiresolving set. Therefore, $GP(2m+1,1)$ is 2-metric antidimensional. □

Table 3: Equivalence classes of $\rho_{S'}$ on $GP(2m,1)$

S'	*Equivalence class*	*Metric representation*
$\{u_0\}$	$\{u_1, u_{n-1}, v_0\}$	(1)
	$\{u_i, u_{n-i}, v_{i-1}, v_{n-i+1}\}$	$(i), 2 \leq i \leq m$
	$\{v_m, v_{m+1}\}$	$(m+1)$

3. k-metric antidimension of GP(n,2)

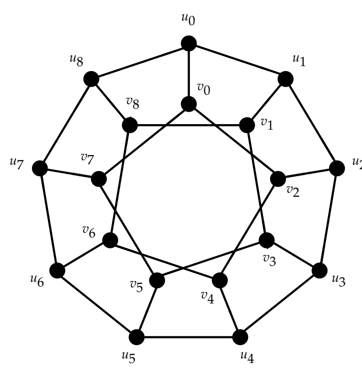

Figure 3: Graph GP(9,2)

Theorem 3.1. *For $m \neq 2$ graph $GP(4m,2)$ is 3-metric antidimensional and*

(i) $adim_1(GP(4m,2)) = 2$

(ii) $adim_2(GP(4m, 2)) = 1$

(iii) $adim_3(GP(4m, 2)) = 1$

Proof. (i) Let $S = \{u_0, u_{2m}\}$. It is easy to see that v_0 has unique metric representation $(1, m + 1)$ with respect to S. Therefore, S is 1-antiresolving set. Suppose that there exists 1-antiresolving set S' of cardinality 1. Due to the symmetry of $GP(4m, 2)$, we can assume that $S' = \{u_0\}$ or $S' = \{v_0\}$. From Table 4 it can be seen that the equivalence classes in both cases have cardinality at least 2, which is a contradiction. Hence, $adim_1(GP(4m, 2)) = 2$.
(ii) Let $S = \{v_0\}$. According to Table 4, S is a 2-antiresolving basis of cardinality 1, so $adim_2(GP(4m, 2)) = 1$.
(iii) Let $S = \{u_0\}$. From Table 4 we conclude that S is a 3-antiresolving basis of $GP(4m, 2)$, i.e. $adim_3(GP(4m, 2)) = 1$.
From (i)-(iii) it follows that $GP(4m, 2)$ is k-metric antidimensional for $k \geq 3$. Since $GP(4m, 2)$ is 3-regular, according to Observation 1.4, it follows that $k = 3$, i.e. $GP(4m, 2)$ is 3-metric antidimensional. □

Table 4: Equivalence classes of ρ_S on $GP(4m, 2)$

S	*Equivalence class*	*Metric representation*
$\{u_0\}$	$\{u_1, u_{4m-1}, v_0\}$	(1)
	$\{u_i, u_{4m-i}, v_{2i-3}, v_{2i-2}, v_{4m-2i+2}, v_{4m-2i+3}\}$	$(i), i = 2, 3, 4$
	$\{u_{2i-5}, u_{2i-4}, u_{4m-2i+4}, u_{4m-2i+5}, v_{2i-3}, v_{2i-2}, v_{4m-2i+2}, v_{4m-2i+3}\}$	$(i), i = 5, ..., m$
	$\{u_{2m-3}, u_{2m-2}, u_{2m+2}, u_{2m+3}, v_{2m-1}, v_{2m}, v_{2m+1}\}$	$(m + 1)$
	$\{u_{2m-1}, u_{2m}, u_{2m+1}\}$	$(m + 2)$
$\{v_0\}$	$\{u_0, v_2, v_{4m-2}\}$	(1)
	$\{u_1, u_2, u_{4m-2}, u_{4m-1}, v_4, v_{4m-4}\}$	(2)
	$\{u_{2i-3}, u_{2i-2}, u_{4m-2i+2}, u_{4m-2i+3}, v_{2i-5}, v_{2i}, v_{4m-2i}, v_{4m-2i+5}\}$	$(i), i = 3, ..., m - 1$
	$\{u_{2m-3}, u_{2m-2}, u_{2m+2}, u_{2m+3}, v_{2m-5}, v_{2m}, v_{2m+5}\}$	(m)
	$\{u_{2m-1}, u_{2m}, u_{2m+1}, v_{2m-3}, v_{2m+3}\}$	$(m + 1)$
	$\{v_{2m-1}, v_{2m+1}\}$	$(m + 2)$

Theorem 3.2. *Graph $GP(4m + 1, 2)$ is 3-metric antidimensional and*

(i) $adim_1(GP(4m + 1, 2)) = 2$

(ii) $adim_2(GP(4m + 1, 2)) = 2$

(iii) $adim_3(GP(4m + 1, 2)) = 1$

Proof. (i) The proof is similar to the proof of (i) in Theorem 3.1. Let $S = \{u_0, u_{2m}\}$. Then vertex v_0 has unique metric representation $(1, m + 1)$, which implies that S is an 1-antiresolving set. Using Table 5 and the same argument as in (i) of Theorem 3.1 we conclude that $\{u_0\}$ and $\{v_0\}$ are not 1-antiresolving sets, and due to the symmetry of $GP(4m+1, 2)$ the same holds for all singleton subsets of V. Therefore, $adim_1(GP(4m+1, 2)) = 2$.
(ii) Let $S = \{u_0, v_0\}$. According to Table 5, S is a 2-antiresolving set since all equivalence classes are of cardinality at least 2. Since by Table 5 equivalence classes for sets $\{u_0\}$ and $\{v_0\}$ are of cardinality at least 3, similarly as in (i) we conclude $adim_2(GP(4m + 1, 2)) = 2$.
(iii) For $S = \{v_0\}$, directly from Table 5 it follows that $adim_3(GP(4m + 1, 2)) = 1$.
From (i)-(iii) it follows that $GP(4m+1, 2)$ is k-metric antidimensional for $k \geq 3$. By Observation 1.4 it follows that $k = 3$, i.e. $GP(4m + 1, 2)$ is 3-metric antidimensional. □

Table 5: Equivalence classes of ρ_S on $GP(4m+1,2)$

S	*Equivalence class*	*Metric representation*
$\{u_0\}$	$\{u_1, u_{4m}, v_0\}$	(1)
	$\{u_i, u_{4m-i+1}, v_{2i-3}, v_{2i-2}, v_{4m-2i+3}, v_{4m-2i+4}\}$	$(i), i = 2, 3, 4$
	$\{u_{2i-5}, u_{2i-4}, u_{4m-2i+5}, u_{4m-2i+6}, v_{2i-3}, v_{2i-2}, v_{4m-2i+3}, v_{4m-2i+4}\}$	$(i), i = 5, ..., m+1$
	$\{u_{2m-1}, u_{2m}, u_{2m+1}, u_{2m+2}\}$	$(m+2)$
$\{v_0\}$	$\{u_0, v_2, v_{4m-1}\}$	(1)
	$\{u_1, u_2, u_{4m-1}, u_{4m}, v_4, v_{4m-3}\}$	(2)
	$\{u_{2i-3}, u_{2i-2}, u_{4m-2i+3}, u_{4m-2i+4}, v_{2i-5}, v_{2i}, v_{4m-2i+1}, v_{4m-2i+6}\}$	$(i), i = 3, ..., m$
	$\{u_{2m-1}, u_{2m}, u_{2m+1}, u_{2m+2}, v_{2m-3}, v_{2m-1}, v_{2m+2}, v_{2m+4}\}$	$(m+1)$
$\{u_0, v_0\}$	$\{u_1, u_{4m}\}$	$(1, 2)$
	$\{v_2, v_{4m-1}\}$	$(2, 1)$
	$\{u_2, u_{4m-1}\}$	$(2, 2)$
	$\{v_1, v_{4m}\}$	$(2, 3)$
	$\{v_4, v_{4m-3}\}$	$(3, 2)$
	$\{u_3, u_{4m-2}\}$	$(3, 3)$
	$\{v_3, v_{4m-2}\}$	$(3, 4)$
	$\{u_4, u_{4m-3}, v_6, v_{4m-5}\}$	$(4, 3)$
	$\{u_{2i-5}, u_{2i-4}, u_{4m-2i+5}, u_{4m-2i+6}, v_{2i-2}, v_{4m-2i+3}\}$	$(i, i-1), i = 5, ..., m+1$
	$\{v_{2i-3}, v_{4m-2i+4}\}$	$(i, i+1), i = 4, ..., m$
	$\{v_{2m-1}, v_{2m+2}\}$	$(m+1, m+1)$
	$\{u_{2m-1}, u_{2m}, u_{2m+1}, u_{2m+2}\}$	$(m+2, m+1)$

Theorem 3.3. *For $m \geq 3$ graph $GP(4m+2,2)$ is 3-metric antidimensional and*

(i) $adim_1(GP(4m+2,2)) = 1$

(ii) $adim_2(GP(4m+2,2)) = 2$

(iii) $adim_3(GP(4m+2,2)) = 2$

Proof. (i) Let $S = \{u_0\}$. Then vertex u_{2m+1} has the unique metric representation $(m+3)$ and therefore, $adim_1(GP(4m+2,2)) = 1$.
(ii) $S = \{u_0, u_{2m+1}\}$. From Table 6, S is a 2-antiresolving set. If we consider singleton subsets of V, due to symmetry it is sufficient to analyze cases $\{u_0\}$ and $\{v_0\}$. By (i), $\{u_0\}$ is 1-antiresolving and since v_{2m+1} has unique metric representation $(m+3)$ with respect to $\{v_0\}$, set $\{v_0\}$ is also 1-antiresolving. It means that all singleton subsets of V are not 2-antiresolving. This implies that $adim_2(GP(4m+2,2)) = 2$.
(iii) For $S = \{v_0, v_{2m+1}\}$ from Table 6 it follows that S is a 3-antiresolving set. Since all singleton vertices are 1-antiresolving sets it follows that $adim_3(GP(4m+2,2)) = 2$.
From (i)-(iii) it follows that $GP(4m+2,2)$ is k-metric antidimensional for $k \geq 3$. According to Observation 1.4 it follows that $k = 3$, i.e. $GP(4m+2,2)$ is 3-metric antidimensional. □

Table 6: Equivalence classes of ρ_S on $GP(4m+2,2)$

S	*Equivalence class*	*Metric representation*
$\{u_0, u_{2m+1}\}$	$\{u_1, u_{4m+1}, v_0\}$	$(1, m+2)$
	$\{u_i, u_{4m-i+1}\}$	$(i, m-i+2),\ i = 2,3,4$
	$\{v_{2i-3}, v_{2i-2}, v_{4m-2i+4}, v_{4m-2i+5}\}$	$(i, m-i+3),\ i = 2,...,m$
	$\{u_{2i-5}, u_{2i-4}, u_{4m-2i+6}, u_{4m-2i+7}\}$	$(i, m-i+5),\ i = 5,...,m+1$
	$\{u_{2m-2}, u_{2m+4}\}$	$(m+1, 3)$
	$\{u_{2m-3}, u_{2m+5}\}$	$(m+1, 4)$
	$\{u_{2m}, u_{2m+2}, v_{2m+1}\}$	$(m+2, 1)$
	$\{u_{2m-1}, u_{2m+3}\}$	$(m+2, 2)$
$\{v_0, v_{2m+1}\}$	$\{u_0, v_2, v_{4m}\}$	$(1, m+2)$
	$\{u_1, u_2, u_{4m}, u_{4m+1}, v_4, v_{4m-2}\}$	$(2, m+1)$
	$\{u_{2i-3}, u_{2i-2}, u_{4m-2i+4}, u_{4m-2i+5}, v_{2i-5}, v_{2i}, v_{4m-2i+2}, v_{4m-2i+7}\}$	$(i, m-i+3),\ i = 3,...,m$
	$\{u_{2m-1}, u_{2m}, u_{2m+2}, u_{2m+3}, v_{2m-3}, v_{2m+5}\}$	$(m+1, 2)$
	$\{u_{2m+1}, v_{2m-1}, v_{2m+3}\}$	$(m+2, 1)$

Theorem 3.4. *For $m \geq 2$ graph $GP(4m+3,2)$ is 3-metric antidimensional and*

(i) $adim_1(GP(4m+3,2)) = 2$

(ii) $adim_2(GP(4m+3,2)) = 1$

(iii) $adim_3(GP(4m+3,2)) = 1$

Proof. (i) Let $S = \{u_0, u_2\}$. Then vertex u_1 has unique metric representation $(1,1)$ and consequently, S is 1-antiresolving set. Since by Table 7 sets $\{u_0\}$ and $\{v_0\}$ are 2-antiresolving and 3-antiresolving, respectively, then $adim_1(GP(4m+3,2)) = 2$.
(ii) and (iii) follow directly from Table 7.
Since $GP(4m+3,2)$ is 3-regular, according to Observation 1.4, it follows that $GP(4m+3,2)$ is k-metric antidimensional for some $k \leq 3$. From (i)-(iii) it follows that $GP(4m+3,2)$ is 3-metric antidimensional. □

Table 7: Equivalence classes of ρ_S on $GP(4m+3,2)$

S	*Equivalence class*	*Metric representation*
$\{u_0\}$	$\{u_1, u_{4m+2}, v_0\}$	(1)
	$\{u_i, u_{4m-i+3}, v_{2i-3}, v_{2i-2}, v_{4m-2i+5}, v_{4m-2i+6}\}$	$(i), i = 2,3,4$
	$\{u_{2i-5}, u_{2i-4}, u_{4m-2i+7}, u_{4m-2i+8}, v_{2i-3}, v_{2i-2}, v_{4m-2i+5}, v_{4m-2i+6}\}$	$(i),\ i = 5,...,m+1$
	$\{u_{2m-1}, u_{2m}, u_{2m+3}, u_{2m+4}, v_{2m+1}, v_{2m+2}\}$	$(m+2)$
	$\{u_{2m+1}, u_{2m+2}\}$	$(m+3)$
$\{v_0\}$	$\{u_0, v_2, v_{4m+1}\}$	(1)
	$\{u_1, u_2, u_{4m+1}, u_{4m+2}, v_4, v_{4m-1}\}$	(2)
	$\{u_{2i-3}, u_{2i-2}, u_{4m-2i+5}, u_{4m-2i+6}, v_{2i-5}, v_{2i}, v_{4m-2i+3}, v_{4m-2i+8}\}$	$(i),\ i = 3,...,m$
	$\{u_{2m-1}, u_{2m}, u_{2m+3}, u_{2m+4}, v_{2m-3}, v_{2m+1}, v_{2m+2}, v_{2m+6}\}$	$(m+1)$
	$\{u_{2m+1}, u_{2m+2}, v_{2m-1}, v_{2m+4}\}$	$(m+2)$

The values for the metric antidimension of the cases which are not covered by Theorems 3.1 - 3.4 are obtained by total enumeration and given in the next two observations.

Observation 3.5. *Graph $GP(8,2)$ is 2-metric antidimensional and $adim_1(GP(8,2)) = 1$ and $adim_2(GP(8,2)) = 1$.*

Observation 3.6. *Graphs* $GP(6,2)$, $GP(7,2)$ *and* $GP(10,2)$ *are 3-metric antidimensional and*

$$adim_k(GP(6,2)) = \begin{cases} 1, & k = 1,2 \\ 2, & k = 3 \end{cases}$$

$$adim_k(GP(7,2)) = \begin{cases} 2, & k = 1,2 \\ 1, & k = 3 \end{cases}$$

$$adim_k(GP(10,2)) = \begin{cases} 1, & k = 1 \\ 4, & k = 2 \\ 2, & k = 3 \end{cases} .$$

4. Conclusions

In this article the recently introduced k-metric antidimension problem is considered. We have studied mathematical properties of the k-antiresolving sets and the k-metric antidimension of some generalized Petersen graphs. Exact formulas for the k-metric antidimension of $GP(n,1)$ and $GP(n,2)$ are obtained.

A possible direction of future research could be considering the k-metric antidimension of some other challenging classes of graphs.

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