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A Remark on Asymptotic Regularity and Fixed Point Property

Ravindra Kishor Bisht[a]

[a]*Department of Mathematics, National Defence Academy, Khadakwasla, Pune, India*

Abstract. In this paper, we show that orbital continuity of a pair of non-commuting mappings of a complete metric space is equivalent to fixed point property under the Proinov type condition. Furthermore, we establish a situation in which orbital continuity turns out to be a necessary and sufficient condition for the existence of a common fixed point of a pair of mappings yet the mappings are not necessarily continuous at the common fixed point.

1. Introduction

For a self-mapping f of a metric space (X, d), the quasi-contraction due to Ćirić [9] is as follows:

$$d(fx, fy) \leq M \cdot P(x, y), \tag{1}$$

where $P(x, y) = \max\Big\{d(x, y), d(x, Tx), d(y, Ty), d(x, Ty), d(y, Tx)\Big\}, x, y \in X$ and $0 \leq M < 1$.

In a comparative study of contractive definitions, Rhoades [30] partially ordered 250 contractive definitions and pointed out that Ćirić's quasi-contraction as the most general contraction condition (see condition (24) in Rhoades [30]). The following contractive condition (condition (25) in Rhoades [30])) is more general than (1):

$$d(fx, fy) < P(x, y), x \neq y. \tag{2}$$

In 1995, Osilike [21] introduced the following class of mappings and utilized it to establish some stability results for various iterative procedures:

$$d(fx, fy) \leq Md(x, y) + Kd(x, fx), 0 \leq M < 1, K \geq 0. \tag{3}$$

In 2006, Proinov [28] established equivalence between the Meir-Keeler type contractive conditions [20] and the contractive definitions equipped with gauge functions. The following fixed point theorem is due to Proinov [28]:

Theorem 1.1. *Let (X, d) be a complete metric space and f be a continuous and asymptotically regular self-mapping on X satisfying the following:*

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Email address: ravindra.bisht@yahoo.com (Ravindra Kishor Bisht)

(i) *$d(fx, fy) \leq \psi(L(x, y))$ for all $x, y \in X$, where $L(x, y) = d(x, y) + K[d(x, fx) + d(y, fy)]$, $K \geq 0$ and $\psi : \mathbb{R}_+ \to \mathbb{R}_+$ satisfies the following; for any $\epsilon > 0$ there exists $\delta > \epsilon$ such that $\epsilon < t < \delta$ implies $\psi(t) \leq \epsilon$;*

(ii) *$d(fx, fy) < L(x, y)$ for all $x, y \in X$ with $x \neq y$.*

Then f has a unique fixed point, say z and all of the Picard iterates of f converge to z. Moreover, if $L(x, y) = d(x, y) + d(x, fx) + d(y, fy)$ and ψ is continuous and satisfies $\psi(t) < t$ for all $t > 0$, then the continuity of f can be dropped.

It may be observed that the class of mappings (condition (i) of Theorem 1.1) considered by Proinov [28] subsumes condition given in (2). In ([2], see also [3]), the author has shown that Theorem 1.1 still holds true if continuity of f is replaced by orbital continuity or $k-$ continuity.

In 2019, Górnicki [13] proved the following fixed point theorem:

Theorem 1.2. *Let (X, d) be a complete metric space and $f : X \to X$ be an asymptotically regular continuous mapping. Suppose there exist $0 \leq M < 1$ and $0 \leq K < +\infty$ satisfying*

$$d(fx, fy) \leq Md(x, y) + K\{d(x, fx) + d(y, fy)\} \tag{4}$$

for all $x, y \in X$. Then f has a unique fixed point $z \in X$ and $f^n x \to z$ for each $x \in X$.

In [1], the author has shown that Theorem 1.2 pertains to both continuous and discontinuous mappings.

Theorem 1.3. *Let (X, d) be a complete metric space and $f : X \to X$ be an asymptotically regular mapping. Suppose there exist $0 \leq M < 1$ and $0 \leq K < +\infty$ satisfying (4) for all $x, y \in X$. Then f has a unique fixed point $z \in X$ provided f is either $k-$continuous for some $k \geqslant 1$ or orbitally continuous. Moreover, $f^n x \to z$ for each $x \in X$.*

Remark 1.4. *Theorems 1.2 and 1.3 generalize some of the well-known fixed point theorems in metric fixed point theory. In the fixed point theorems of Kannan[18] and Reich [29], the constant K lies in between $[0, 1/2)$, whereas in Theorems 1.2 and 1.3, K ranges in $[0, \infty)$. In addition to it, Kannan's or Ćirić's fixed point theorem forces the mapping to be continuous at the fixed point, whereas in Theorem 1.3, the mapping need not be continuous at the fixed point (see [1]).*

If $fx = gx = y$ for some x, y in X, then x and y are called coincidence point and point of coincidence of f and g, respectively. The set of coincidence points and point of coincidences of f and g are denoted by $C(f, g)$ and $PC(f, g)$ respectively. If $y = x$ then x is a common fixed point of f and g. In 1967, Machuca [19] proved a coincidence theorem as an abstraction of the Banach contraction principle but under heavy topological conditions. Subsequently, Goebel [10] improved Machuca's result under much weaker assumption. More specifically, given a non empty set Y, a metric space (X, d) and two mappings $f, g : Y \to X$, he gave sufficient conditions for the existence of a point $x \in Y$ such that $fx = gx$.

Theorem 1.5. *Let $g(Y)$ be a complete subspace of a metric space (X, d) and $f(Y) \subseteq g(Y)$ satisfying*

$$d(fx, fy) \leq Md(gx, gy), 0 \leq M < 1, \tag{5}$$

for all $x, y \in Y$. Then f and g have a coincidence point.

Jungck [16] observed the interdependence of common fixed points and commuting mappings, and proved a common fixed point theorem for a pair of mappings besides providing partial answer to the historical open question (see [4, 15]): For a pair of commuting self mappings on $[0, 1]$, what additional conditions guarantee that f and g have a common fixed point?

Following Jungck, several authors obtained common fixed point theorems for both commuting and non-commuting pairs of mappings satisfying contractive or noncontractive type conditions. An updated survey and comparison of various generalized non-commuting mappings and their applications has been given in [11].

Definition 1.6. *Let f and g be two self-mappings of a metric space (X, d). Then*

(i). f is asymptotically regular with respect to g at $x_0 \in X$ [6, 23, 32] if there exists a sequence $\{x_n\}$ in X such that $gx_{n+1} = fx_n, n = 0, 1, 2, ...,$ and $\lim_{n\to\infty} d(gx_{n+1}, gx_{n+2}) = 0$.

(ii). f and g are called R-weakly commuting mappings[24] if there exists some real number $R > 0$ such that $d(fgx, gfx) \leq Rd(fx, gx)$ for all $x \in X$.

(iii). f and g are called R-weakly commuting mappings of type A_g (of type A_f) [22] if there exists some real number $R > 0$ such that $d(ffx, gfx) \leq Rd(fx, gx)$ ($d(fgx, ggx) \leq Rd(fx, gx)$) for all $x \in X$.

(iv) f and g are called nontrivially weakly compatibile ([17] if f and g commute on the set of coincidence points whenever the set of their coincidences is non-empty.

(v) f and g are called noncompatible [25] if there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = t$ for some t in X but $\lim_{n\to\infty} d(fgx_n, gfx_n)$ is either non-zero or non-existent.

(vi) f and g are called $g-$ reciprocally continuous [26] if $\lim_{n\to\infty} ffx_n = ft$ and $\lim_{n\to\infty} gfx_n = gt$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = t$ for some t in X.

Definition 1.7. *Let f and g be two self-mappings of a metric space (X, d) and let $\{x_n\}$ be a sequence in X such that $fx_n = gx_{n+1}$. Then the set $O(x_0, f, g) = \{fx_n : n = 0, 1, 2, ...\}$ is called the $(f, g)-$orbit at x_0 and g (or f) is called (f, g)-orbitally continuous [8, 23] if $\lim_{n\to\infty} fx_n = z$ implies $\lim_{n\to\infty} gfx_n = gz$ or ($\lim_{n\to\infty} fx_n = z$ implies $\lim_{n\to\infty} ffx_n = fz$). We say f and g are orbitally continuous if f is (f, g)-orbitally continuous and g is (f, g)-orbitally continuous.*

In 2010, using axiom of choice Haghi et al. [14] established a lemma and claimed that some coincidence point or common fixed point abstractions in metric fixed point theory are not real abstractions.

Lemma 1.8. *Let X be a nonempty set and $f : X \to X$ a function. Then there exists a subset $Y \subset X$ such that $f(Y) = f(X)$ and $f : Y \to X$ is one-to-one.*

We point out that in the proof of Theorem 1.5, Geobel [10]defined the mapping $h = fg^{-1} : g(Y) \to g(Y)$ and with the help of Banach's contraction principle, he showed that both the mappings f and g have a coincidence point. On the other hand, Haghi et al. [14] considered the same mapping h and using some fixed point results they concluded that coincidence point or common fixed point theorems are not real generalizations of their corresponding fixed point theorems. The unique difference in Geobel's proof is the argument used to say that h is well defined. Moreover, the proofs given in [14] are essentially Goebel's proof. Interestingly, the idea given by Haghi et al. [14] covers only a handful results in metric fixed point theory and coincidence point or common fixed point theorems are indeed real generalizations of their corresponding fixed point theorems (see Example 2.3).

In this paper, we show that orbital continuity of a pair of R-weakly commuting mappings of type A_g or A_f [22] of a complete metric space is equivalent to fixed point property under the Proinov type condition. We also establish a situation in which orbital continuity turns out to be a necessary and sufficient condition for the existence of a common fixed point of a pair of mappings yet the mappings are not necessarily continuous at the common fixed point.

2. Main results

Our main result of this section is the following:

Theorem 2.1. *Let (X, d) be a complete metric space and $f, g : X \to X$ be R-weakly commuting mappings of type A_g or of type A_f. Suppose that f is asymptotically regular with respect to g and there exist $0 \le M < 1$ and $0 \le K < +\infty$ satisfying*

$$d(fx, fy) \le Md(gx, gy) + K\{d(fx, gx) + d(fy, gy)\} \tag{6}$$

for all $x, y \in X$. Then f and g have a unique common fixed point iff f and g are (f, g)-orbitally continuous.

Proof. Since f is asymptotically regular with respect to g at $x_0 \in X$, there exists a sequence $\{y_n\}$ in X such that $y_n = fx_n = gx_{n+1}$ for all $n \in \mathbb{N}\bigcup\{0\}$ and $\lim_{n\to\infty} d(gx_{n+1}, gx_{n+2}) = \lim_{n\to\infty} d(y_n, y_{n+1}) = 0$.

We claim that $\{y_n\}$ is a Cauchy sequence. In light of triangle inequality, (6) and for any n and $p \ge 1$, we have

$$\begin{aligned} d(y_{n+p}, y_n) &\le d(y_{n+p}, y_{n+p+1}) + d(y_{n+p+1}, y_{n+1}) + d(y_{n+1}, y_n) \\ &\le d(y_{n+p}, y_{n+p+1}) + Md(y_{n+p}, y_n) + K\{d(y_{n+p+1}, y_{n+p}) + d(y_{n+1}, y_n)\} + d(y_{n+1}, y_n), \end{aligned}$$

which implies

$$(1 - M)d(y_{n+p}, y_n) \le (1 + K)\{d(y_{n+p}, y_{n+p+1}) + d(y_{n+1}, y_n)\}. \tag{7}$$

Asymptotic regularity of f with respect to g implies that $d(y_{n+p}, y_n) \to 0$ as $n \to \infty$. Therefore $\{y_n\}$ is a Cauchy sequence. Since X is complete, there exists a point t in X such that $y_n \to t$ as $n \to \infty$. Moreover, $y_n = fx_n = gx_{n+1} \to t$.

Suppose that f and g are R-weakly commuting mappings of type A_g. Orbital continuity of f and g implies that

$$\lim_{n\to\infty} ffx_n = \lim_{n\to\infty} fgx_n = ft,$$

and

$$\lim_{n\to\infty} gfx_n = \lim_{n\to\infty} ggx_n = gt.$$

Then R-weakly commuting mappings of type A_g of f and g yields $d(ffx_n, gfx_n) \le d(fx_n, gx_n)$. On letting $n \to \infty$, we get $ft = gt$. Again R-weakly commuting mappings of type A_g of f and g implies commutativity at t, i.e., $fgt = gft$. Hence $gft = fgt = fft = ggt$. Now if $ft \ne fft$, using (6) we obtain $d(ft, fft) \le Md(gt, gft) + K\{d(ft, gt) + d(fft, gft)\} = Md(ft, fft)$, that is, $ft = fft$. Hence $ft = fft = gft$ and ft is a common fixed point of f and g. The proof is similar if f and g are assumed R-weakly commuting of type A_f . Moreover, (6) implies uniqueness of the common fixed point.

Conversely, let us assume that the mappings f and g satisfy (6) and possess a common fixed point, say t. Then $t = ft = gt$. Also, the (f, g)-orbit of any point x_0 defined by asymptotic regularity of f with respect to g converges to t, i.e., $fx_n = gx_{n+1} \to t$. Suppose that f and g are R-weakly commuting of type A_g. Then we have $d(ffx_n, gfx_n) \le Rd(fx_n, gx_n)$. This implies $\lim_{n\to\infty} d(ffx_n, gfx_n) = 0$.
Now by virtue of (6) we have

$$d(ffx_n, ft) \le Md(gfx_n, gt) + K\{d(ffx_n, gfx_n) + d(ft, gt)\},$$

i.e.,

$$d(ffx_n, ft) \le M\{d(gfx_n, ffx_n) + d(ffx_n, gt)\}.$$

In view of $\lim_{n\to\infty} d(ffx_n, gfx_n) = 0$ and $t = ft = gt$ above inequality yields $\lim_{n\to\infty} ffx_n = \lim_{n\to\infty} gfx_n = ft = gt$. Hence f is (f, g)-orbitally continuous and g is (f, g)-orbitally continuous, i.e., f and g are orbitally continuous.

Similarly, f and g are orbitally continuous if f and g are assumed to be R-weakly commuting of type A_f. This completes the theorem. □

Corollary 2.2. *Let (X, d) be a complete metric space and $f, g : X \to X$ be R-weakly commuting mappings of type A_f or of type A_g. Suppose that f is asymptotically regular with respect to g and there exists $0 \leq K < +\infty$ satisfying*

$$d(fx, fy) \leq K\{d(fx, gx) + d(fy, gy)\} \tag{8}$$

for all $x, y \in X$. Then f and g have a unique common fixed point iff f and g are (f, g)-orbitally continuous.

The following example illustrates the above theorem.

Example 2.3. *Let $X = [2, 20]$ and let d be the usual metric on X. Define self mappings f and g on X as follows*

$$fx = 2 \text{ if } x = 2 \text{ or } x > 5,\ fx = 6 \text{ if } 2 < x \leq 5,$$
$$g2 = 2,\ gx = 11 \text{ if } 2 < x \leq 5,\ gx = \tfrac{x+1}{3} \text{ if } x > 5.$$

Then f and g satisfy the following conditions of Theorem 2.1 and have a fixed point $x = 2$ at which f and g are discontinuous.

(i). *f and g satisfy the condition $d(fx, fy) \leq \frac{4}{5}\{d(fx, gx) + d(fy, gy)\}$ for $M = 0$ and $x, y \in X$;*
(ii). *f and g are R-weakly commuting mappings of type A_g, i.e., $d(gfx, ffx) \leq d(fx, gx)$ for $R = 1$.*
(iii). *f and g are orbitally continuous.*

Remark 2.4. *In view of the above example one can check that in Theorem 2.1 f and g are discontinuous at the fixed point $x = 2$ but Theorem 1.2 demands continuity of the mapping f. Hence, Theorem 2.1 is not a consequence of Theorem 1.2. Therefore, coincidence point or common fixed point theorems are indeed real generalizations of their corresponding fixed point theorems.*

In the next theorem, we replace condition (6) in Theorem 2.1 by a $\psi-$ type condition, where the contractive function ψ is required to satisfy only $\psi(t) < t$ for each $t > 0$. Analogous results using the contractive function ψ require to be either upper semicontinuous [5] or nondecreasing with $g(t) = t/(t - \psi(t))$ nonincreasing [7] or nondecreasing and continuous from right [27].

Theorem 2.5. *Let (X, d) be a metric space and $f, g : X \to X$ be noncompatible R-weakly commuting mappings of type A_g such that for all $x, y \in X$*

$$d(fx, fy) \leq \psi(d(gx, gy) + K\{d(fx, gx) + d(fy, gy)\}), \tag{9}$$

where $\psi : \mathbb{R}_+ \to \mathbb{R}_+$ is such that $\psi(t) < t$ for each $t > 0$ and $0 \leq K < \infty$. Suppose that f and g are g-reciprocally continuous. Then f and g have a unique common fixed point.

Proof. Non-compatibility of f and g implies that there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = t$ for some t in X but $\lim_{n\to\infty} d(fgx_n, gfx_n)$ is either nonzero or nonexistent. g-reciprocal continuity of f and g implies $\lim_{n\to\infty} ffx_n = ft$ and $\lim_{n\to\infty} gfx_n = gt$. Further, R-weak commutativity of type A_g yields $d(ffx_n, gfx_n) \leq Rd(fx_n, gx_n)$. Making $n \to \infty$ we get $ft = gt$. R-weakly commuting mappings of type A_g of f and g implies that $fgt = gft$. This further implies that $gft = fgt = fft = ggt$. Using (9), we obtain $d(ft, fft) \leq \psi(d(gt, gft) + K[d(ft, gt) + d(fft, gft)]) < d(ft, fft)$, a contradiction. Hence $ft = fft = gft$ and ft is a common fixed point of f and g. Uniqueness of the common fixed point follows easily. □

Using the minimal commutativity condition, i.e., non-trivial weak compatibility we now prove a common fixed point theorem for two mappings satisfying Proinov type condition.

Theorem 2.6. *Let f and g be self-mappings on an arbitrary non-empty set Y with values in a metric space (X, d). Suppose $\psi : \mathbb{R}_+ \to \mathbb{R}_+$ is a continuous gauge function in the sense that for any $\epsilon > 0$ there exists $\delta > \epsilon$ such that $\epsilon < t < \delta$ implies $\psi(t) \leq \epsilon$, and suppose that f is asymptotically regular with respect to g at $x_0 \in Y$ satisfying*

$$d(fx, fy) \leq \psi(d(gx, gy) + K[d(fx, gx) + d(fy, gy)]) \tag{10}$$

for all $x, y \in Y$ and $0 \leq K < 1$. If gY is a complete subset of X, then f and g have a coincidence point. Moreover, if $Y = X$, then f and g have a unique common fixed point provided f and g are non-trivially weakly compatible.

Proof. Since f is asymptotically regular with respect to g at $x_0 \in Y$, there exist two sequences $\{x_n\}$ and $\{y_n\}$ in Y such that $y_n = gx_{n+1} = fx_n, n = 0, 1, 2, ...$ and $\lim_{n\to\infty} d(gx_{n+1}, gx_{n+2}) = \lim_{n\to\infty} d(y_n, y_{n+1}) = 0$. Following Sastry et al. [33] (see also [28])we can use induction to show that $\{y_n\}$ is a Cauchy sequence. Since $g(Y)$ is complete, there exists a point $z \in X$ such that $y_n \to z$ as $n \to \infty$. Let $t \in g^{-1}z$. Then $z = gt$ and $y_n = gx_{n+1} = fx_n \to z$. Using (10) we get

$$d(ft, fx_n) \leq \psi(d(gt, gx_n) + K[d(ft, gt) + d(fx_n, gx_n)]).$$

On letting $n \to \infty$ the above inequality reduces to

$$\begin{aligned} d(ft, z) &\leq \psi(d(gt, z) + K[d(ft, z) + d(z, z)]) \\ &\leq \psi(Kd(ft, z)) < d(ft, z), \end{aligned}$$

a contradiction. Therefore $ft = z = gt$. Now if $Y = X$ and non-trivial weak compatibility of f and g implies that $fgt = gft$. This further implies that $gft = fgt = fft = ggt$. Using (10), we obtain $d(ft, fft) \leq \psi(d(gt, gft) + K[d(ft, gt) + d(fft, gft)]) < d(ft, fft)$, a contradiction. Hence $ft = fft = gft$ and ft is a common fixed point of f and g. Uniqueness of the common fixed point follows easily. □

Some generalizations of the above proved theorems are due to Kannan [18], Geobel [10], Reich [29], Jungck [16], Górnicki [12, 13] Park and Rhoades [27], Proinov [28], Pant and Pant [23] Pant, [25], Sastry et al. [33], Singh et al. [34, 35] and many others.

Remark 2.7. *Theorem 2.1 provides a new answer to the once open question (see Rhoades [31], p.242) on the existence of contractive mappings which admit discontinuity at the common fixed point.*

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