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On Filter Convergence of Nets in Uniform Spaces

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Abstract. In this paper, we introduce $\mathcal{F}$-convergent and $\mathcal{F}_{st}$-fundamental nets in uniform spaces and study some their properties.

1. Introduction and Notations

The concept of statistical convergence was introduced by Fast [7] and Schonberg [21], and its topological properties were discussed by Fridy [8], Salat [18] and Maddox [15]. Fridy [8] also introduced the concept of statistically fundamental sequence and showed its equivalence to statistical convergence with respect to numerical sequences. This problem on the uniform space was raised in [16]. The authors [16] showed that if the sequence $\{x_n\}_{n\in\mathbb{N}}$ is statistically convergent in a uniform space, then it is statistically fundamental. Recently, Bilalov and Nazarova [3] gave the concept of $\mathcal{F}_{st}$-fundamental sequences in uniform spaces and obtain some results related with this concept.

Kostyrko et al. [12] introduced the notion of $\mathcal{I}$-convergence of sequences in a metric space and discussed some properties of such convergence. Recall that $\mathcal{I}$-convergence is a generalization of statistical convergence. Some problems about the ideals or filters can be found in [4, 5, 13, 14].

We now recall some concepts of ideal and filter [3, 12, 17].

A family of sets $\mathcal{I} \subset 2^{\mathbb{N}}$ is said to be an *ideal* if (*i*) $\emptyset \in \mathcal{I}$; (*ii*) $A, B \in \mathcal{I}$ imply $A \cup B \in \mathcal{I}$; (*iii*) $A \in \mathcal{I}$, $B \subset A$ imply $B \in \mathcal{I}$.

A family of sets $\mathcal{F} \subset 2^{\mathbb{N}}$ is said to be a *filter* if (*i*) $\emptyset \notin \mathcal{F}$; (*ii*) $A, B \in \mathcal{F}$ imply $A \cap B \in \mathcal{F}$; (*iii*) $A \in \mathcal{F}$, $A \subset B$ imply $B \in \mathcal{F}$.

If filter $\mathcal{F}$ satisfy the following axioms:

(*iv*) if $A_1 \supset A_2 \supset ...$ and $A_n \in \mathcal{F}$ for all $n \in \mathbb{N}$, then there exists $\{n_m\}_{m\in\mathbb{N}} \subset \mathbb{N}$; $n_1 < n_2 < ...$ such that $\cup_{m=1}^{\infty} \left((\alpha_m, \alpha_{m+1}] \cap A_{(m)} \right) \in \mathcal{F}$,

(*v*) $F^c\,(N \backslash F) \in \mathcal{F}$ for any finite subset $F \subset \mathbb{N}$,

then filter $\mathcal{F}$ is said to be a *monotone closed filter* and a *right filter*, respectively [2, 3].

An ideal $\mathcal{I}$ is said to be *non-trivial* if $\mathcal{I} \neq \emptyset$ and $\mathcal{I} \neq \mathbb{N}$. $\mathcal{I} \subset 2^{\mathbb{N}}$ is a non-trivial ideal if and only if $\mathcal{F} = \mathcal{F}(\mathcal{I}) = \{\mathbb{N} \backslash A : A \in \mathcal{I}\}$ is a filter. A non-trivial ideal $\mathcal{I}$ is said to be *admissible* if $\mathcal{I} \supset \{\{n\} : n \in \mathbb{N}\}$. Filter convergence was introduced in [1] and described in details in the paper [9]. Convergence with respect to

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set of filters was studied in the paper [11]. More information about filters and convergence with respect to filters can be found in [1, 12, 17, 19, 20].

Now we recall the definition of uniformity on a set X [6, 10].

$\Lambda = \{(x,x) : x \in X\}$ is said to be a diagonal or the identity relation. If $U \subset X \times X$ is a relation, then the inverse of this relation U^{-1} is defined as the set of all pairs (x,y) such that $(y,x) \in U$, that is, $U^{-1} = \{(x,y) \in X \times X : (y,x) \in U\}$. Let $U, V \subset X \times X$ be some relation. The composition $U \circ V$ of the relations U and V is defined as the set of all pairs (x,z), we get $(x,y) \in V$ and $(y,z) \in U$ for some $y \in X$, that is, $U \circ V = \{(x,z) : \exists y \in X,\ (x,y) \in V \text{ and } (y,z) \in U\}$. Let $K \subset X$ be some set and $U \subset X \times X$ be a relation. Assume $U[K] = \{y \in X : \exists x \in K \Longrightarrow (x,y) \in U\}$. For $K = \{x\}$ suppose $U[K] = U[x]$. *Uniformity* on the set X is a non-empty family $\Omega \subset 2^{X \times X}$ which satisfies the following axioms:

(a) $\Lambda \subset U, \forall U \in \Omega$;
(b) $U \in \Omega$ imply $U^{-1} \in \Omega$;
(c) $U \in \Omega$ imply $\exists V \in \Omega$ such that $V \circ V \subset U$;
(d) $U, V \in \Omega$ imply $U \cap V \in \Omega$;
(e) $U \in \Omega$ and $U \subset V \subset X \times X$ imply $V \in \Omega$.

(X, Ω) is said to be a *uniform space*. Subfamily $\Delta \subset \Omega$ of the uniformity Ω is said to be its *base* if any element of the family Ω contains an element of the family Δ.

Let (X, Ω) be a uniform space. The topology τ, associated with a uniformity Ω, is the family of all sets $K \subset X$ such that for each $x \in K$ there exists a $U \in \Omega$ such that $U[x] \subset K$.

The uniform space (X, Ω) is called Hausdorff if $\cap_{U \in \Omega} U = \Lambda$. Let (X, Ω) be a uniform space and $\{x_n\}_{n \in \mathbb{N}}$ be some sequence. $\{x_n\}_{n \in \mathbb{N}}$ is called *fundamental* if $\forall U \in \Omega$, there exists a $n_0 \in \mathbb{N}$ such that $(x_n, x_m) \in U$ for all $n, m \geq n_0$.

Throughout the paper $(D, \geq)$ will denote a directed set and $\mathcal{I}$ a non-trivial proper ideal of D. A *net* is a mapping from D to X and will be denoted by $\{s_\alpha : \alpha \in D\}$. Let $D_\alpha = \{\beta \in D : \beta \geq \alpha\}$ for $\alpha \in D$. Then the collection $\mathcal{F}_0 = \{A \subset D : A \supset D_\alpha \text{ for some } \alpha \in D\}$ forms a filter in D. Let $\mathcal{I}_0 = \{A \subset D : A^c \in \mathcal{F}_0\}$. Then $\mathcal{I}_0$ is a non-trivial ideal of D. A nontrivial ideal $\mathcal{I}$ of D will be said to be *D-admissible* if $D_\alpha \in \mathcal{F}$ for all $\alpha \in D$. A net $\{s_\alpha : \alpha \in D\}$ in a topological space (X, τ) is called *$\mathcal{F}$-convergent* to $s \in X$ if $\{\alpha \in D : s_\alpha \in U\} \in \mathcal{F}$ for any open set U containing s.

2. Main Results

In this section, we introduce $\mathcal{F}$-convergent and $\mathcal{F}_{st}$-fundamental nets in uniform spaces and study some of their properties.

Now we introduce our main definitions.

Definition 2.1. Let (X, Ω) be a uniform space and $\{s_\alpha : \alpha \in D\}$ be a net in X. The net $\{s_\alpha : \alpha \in D\}$ is said to be *$\mathcal{F}$-convergent* to s (in short, $\mathcal{F}\text{-}\lim s_\alpha = s$) if for every $U \in \Omega$, $\{\alpha \in D : (s_\alpha, s) \in U\} \in \mathcal{F}$. In other words, for $\forall U \in \Omega$, $\{\alpha \in D : s_\alpha \in U[s]\} \in \mathcal{F}$.

Definition 2.2. Let (X, Ω) be a uniform space and $\{s_\alpha : \alpha \in D\}$ be a net in X. The net $\{s_\alpha : \alpha \in D\}$ is said to be *$\mathcal{F}_{st}$-fundamental* in X if for every $U \in \Omega$, there exist a $\alpha_0 \in D$ such that $\{\alpha \in D : s_\alpha \in U[s_{\alpha_0}]\} \in \mathcal{F}$.

Lemma 2.3. *Let (X, Ω) be a Hausdorff uniform space and $\{s_\alpha : \alpha \in D\}$ be a net in X. If there exists $\mathcal{F}\text{-}\lim s_\alpha$, then it is unique.*

Proof. Let (X, Ω) be a Hausdorff uniform space. Accordingly, $\{s\} = \cap_{U \in \Omega} U[s]$. Let $\{s_\alpha : \alpha \in D\}$ be a net in X. We prove that if there exists $\mathcal{F}\text{-}\lim s_\alpha$, then it is unique. Supposed to contrary, that is, $\mathcal{F}\text{-}\lim s_\alpha$ has two values $t_1 \neq t_2$. Then it is obvious that there exists a $U_k \in \Omega$ such that $t_1 \notin U_2[t_2]$ and $t_2 \notin U_1[t_1]$. If $U = U_1 \cap U_2$, then $U \in \Omega$. Furthermore, $t_1 \notin U[t_2]$ and $t_2 \notin U[t_1]$. Since $U \in \Omega$, there exists a $V \in \Omega$ such that $V \circ V \subset U$ and $V = V^{-1}$. It is clear that $t_1 \notin V[t_2]$ and $t_2 \notin V[t_1]$. Suppose that

$$A_1 = \{\alpha \in D : s_\alpha \in V[t_1]\}$$

and

$$A_2 = \{\alpha \in D : s_\alpha \in V[t_2]\}.$$

If $A_1, A_2 \in \mathcal{F}$, then $A_1 \cap A_2 \in \mathcal{F}$. On the other hand, $A_1 \cap A_2 = \emptyset \in \mathcal{F}$. If $A_1 \cap A_2 \neq \emptyset$, then there exists a $\alpha_0 \in D$ such that $s_{\alpha_0} \in A_1 \cap A_2$. Moreover, $(s_{\alpha_0}, t_1) \in V$ and $(s_{\alpha_0}, t_2) \in V$. From the symmetry of V, we have $(t_2, s_{\alpha_0}) \in V$. Consequently, $(t_1, t_2) \in V \circ V \subset U$. This is a contradiction, that is, $\mathcal{F}$-$\lim s_\alpha$ is unique. □

Theorem 2.4. *Let (X, Ω) be a Hausdorff uniform space and $\{s_\alpha : \alpha \in D\}$ be a net in X which is $\mathcal{F}$-convergent. Then $\{s_\alpha : \alpha \in D\}$ is $\mathcal{F}_{st}$-fundamental.*

Proof. Let (X, Ω) be a uniform space, $\{s_\alpha : \alpha \in D\}$ be a net in X and $\mathcal{F}$-$\lim s_\alpha = s$. Now we prove that $\{s_\alpha : \alpha \in D\}$ is $\mathcal{F}_{st}$-fundamental. Let $U \in \Omega$. Then there exists a $V \in \Omega$ such that $V \circ V \subset U$ and $V = V^{-1}$. Take $\alpha_0 \in \{\alpha \in D : s_\alpha \in V[s]\}$. It is obvious that

$$\{\alpha \in D : s_\alpha \in V[s]\} \in \mathcal{F}.$$

If $s_\alpha \in V[s]$, then $(s_\alpha, s_{\alpha_0}) \in V \circ V \subset U$. As a result,

$$\{\alpha \in D : s_\alpha \in V[s]\} \subset \{\alpha \in D : s_\alpha \in U[s_{\alpha_0}]\}$$

and so

$$\{\alpha \in D : s_\alpha \in U[s_{\alpha_0}]\} \in \mathcal{F}.$$

Hence, the theorem is proved. □

Theorem 2.5. *Let (X, Ω) be a Hausdorff, complete uniform space with a countable base and $\{s_\alpha : \alpha \in D\}$ be a net in X. If the net $\{s_\alpha : \alpha \in D\}$ is $\mathcal{F}_{st}$-fundamental, then there exists $s \in X$ such that $\mathcal{F}$-$\lim s_\alpha = s$.*

Proof. Let (X, Ω) be a complete uniform space. We suppose that (X, Ω) has a countable base and it is Hausdorff. Then, there exists $U_\alpha \in \Omega$ such that $\cap_{\alpha \in D} U_\alpha = \Lambda$ and $U_\alpha \subset U$ for all $\alpha \in D$. Without loss of generality, we suppose that $U^{(\alpha+1)} \circ U^{(\alpha+1)} \subset U^{(\alpha)}$ and $U^{(\alpha)} = \left(U^{(\alpha)}\right)^{-1}$. Let $\{s_\alpha : \alpha \in D\}$ be $\mathcal{F}_{st}$-fundamental in X. Hence, by definition there exists $\alpha_i \in D$ such that $A_i \in \mathcal{F}$, where $A_i = \left\{\alpha \in D : s_\alpha \in U^{(i)}[s_{\alpha_i}]\right\}$ for $i = 1, 2$. It is obvious that $A_{(1)} = A_1 \cap A_2 \in \mathcal{F}$. Let $B_1 = U^{(1)}[s_{\alpha_1}] \cap U^{(2)}[s_{\alpha_2}]$. Clearly, $s_\alpha \in B_1$ for all $\alpha \in A_{(1)}$. Likewise, there exists $\alpha_3 \in D$ such that $A_3 = \left\{\alpha \in D : s_\alpha \in U^{(3)}[s_{\alpha_3}]\right\} \in \mathcal{F}$. Suppose that $A_{(2)} = A_{(1)} \cap A_3$. It is obvious that $A_{(2)} \in \mathcal{F}$. Put $B_2 = B_1 \cap U^{(3)}[s_{\alpha_3}]$. As a result, $B_2 \neq \emptyset$ and so $s_\alpha \in B_2$ for all $\alpha \in A_{(2)}$. Continuing in the same way, we get the net of open non-empty sets $\{B_\alpha\}_{\alpha \in D} \subset X$ such that

$$B_1 \supset B_2 \supset ..., \ B_n \subset U^{(\alpha+1)}[s_{k_{\alpha+1}}] \text{ for all } \alpha \in D,$$

such as $A_{(i)} \in \mathcal{F}$ such that $A_{(i)} = \{k \in D : s_k \in B_i\}$ for all $i \in D$. Take $\widetilde{s_\alpha} \in B_\alpha$ for all $\alpha \in D$. Now we prove that $\{\widetilde{s_\alpha} : \alpha \in D\}$ is a fundamental net. Let $U \in \Omega$ be an arbitrary element. Then, it is clear that there exists $\alpha_0 \in D$ such that $U^{(\alpha_0)} \subset U$ for $\alpha \geq \alpha_0$. Let $\alpha \geq \alpha_0$ be arbitrary. We obtain $\widetilde{s}_{\alpha+p} \in B_{\alpha+p} \subset B_\alpha$ for all $p \in D$. Since, we have B_α such that $B_{\alpha+p} \subset U^{(\alpha+1)}[s_{k_{\alpha+1}}]$, it is obvious that $(\widetilde{s_\alpha}, s_{k_{\alpha+1}}) \in U^{(\alpha+1)}$ and $\widetilde{s}_{\alpha+p} \in U^{(\alpha+1)}[s_{k_{\alpha+1}}]$. Moreover, $\left(\widetilde{s_\alpha}, \widetilde{s}_{\alpha+p}\right) \in U^{(\alpha+1)} \circ U^{(\alpha+1)} \subset U^{(\alpha)}$ for all $p \in D$. As a result, $\left(\widetilde{s_\alpha}, \widetilde{s}_{\alpha+p}\right) \in U$ for all $\alpha \geq \alpha_0$ and $p \in D$. Since U is arbitrary, the net $\{\widetilde{s_\alpha} : \alpha \in D\}$ is fundamental in (X, Ω) and let $\lim \widetilde{s_\alpha} = s$. Now prove that $\mathcal{F}$-$\lim s_\alpha = s$. Take $U \in \Omega$. Then, there exists a $\alpha_0 \in D$ such that $U^{(\alpha)} \subset U$ for all $\alpha \geq \alpha_0$. Since $B_\alpha \subset U^{(\alpha+1)}[s_{k_{\alpha+1}}]$, we have

$$A_{(\alpha)} \subset \left\{\alpha \in D : s_\alpha \in U^{(\alpha+1)}[s_{k_{\alpha+1}}]\right\} \in \mathcal{F}$$

for all $\alpha \in D$. Let $\alpha_1 \in D$ such that $\widetilde{s_k} \in U^{(\alpha_0+1)}[s]$ for all $k \geq \alpha_1$. Without loss of generality, we suppose that $\alpha_1 \geq \alpha_0+1$. As a result, $\widetilde{s}_{\alpha_1} \in B_{\alpha_1} \subset U^{(\alpha_1+1)}\left[s_{k_{\alpha_1+1}}\right]$. We put $\left(s_k, s_{k_{\alpha_1+1}}\right) \in U^{(\alpha_1+1)}$. Then $(s_k, \widetilde{s}_{\alpha_1}) \in U^{(\alpha_1+1)} \circ U^{(\alpha_1+1)} \subset U^{(\alpha_1)}$. Since, $\left(\widetilde{s}_{\alpha_1}, s_{k_{\alpha_1+1}}\right) \in U^{(\alpha_1+1)} \subset U^{(\alpha_1)}$, then it is obvious that

$$\left(s_k, s_{k_{\alpha_1+1}}\right) \in U^{(\alpha_1)} \circ U^{(\alpha_1)} \subset U^{(\alpha_1-1)} \in U^{(\alpha_0)} \subset U.$$

This implies that

$$\{\alpha \in D : s_\alpha \in B_{\alpha_0}\} \subset \{\alpha \in D : s_\alpha \in U[s]\}.$$

Therefore,

$$A_{(\alpha_0)} = \{\alpha \in D : s_\alpha \in B_{\alpha_0}\} \in \mathcal{F}.$$

From the previous inclusion it follows that

$$\{\alpha \in D : s_\alpha \in U[s]\} \in \mathcal{F}.$$

Since U was arbitrary, we have $\mathcal{F}\text{-}\lim s_\alpha = s$. □

Theorem 2.6. *Let (X, Ω) be a uniform space with a countable base and let $\{s_\alpha : \alpha \in D\}$ be an $\mathcal{F}_{\mathrm{st}}$-fundamental net in X. Then:*

i) if $\mathcal{F}$ is monotone closed filter and $\mathcal{F}\text{-}\lim s_\alpha = s$, then there exists $\{t_\alpha\}_{\alpha\in D} \subset X$ such that $\lim t_\alpha = s$ and $\{\alpha \in D : s_\alpha = t_\alpha\} \in \mathcal{F}$;

ii) if $\mathcal{F}$ is a right filter and $\lim t_\alpha = s$ and $\{\alpha \in D : s_\alpha = t_\alpha\} \in \mathcal{F}$, then $\mathcal{F}\text{-}\lim s_\alpha = s$.

Proof. i) Suppose that the net $\{s_\alpha : \alpha \in D\}$ is $\mathcal{F}_{st}$-fundamental, $\mathcal{F}$ is monotone closed filter and the space (X, Ω) has a countable base. Consider the net $\left\{A_{(\alpha)}\right\}_{\alpha\in D}$, constructed in the proof of Theorem 2.5. We get

$$A_{(1)} \supset A_{(2)} \supset ... \text{ and } A_{(\alpha)} \in \mathcal{F} \text{ for } \alpha \in D.$$

Then by condition (*iv*) of filter we get $\{\alpha_m : \alpha_1 < \alpha_2 < ...\}$ such that

$$\cup_{m=1}^{\infty} \left((\alpha_m, \alpha_{m+1}] \cap A_{(m)}\right) \in \mathcal{F}.$$

Suppose that

$$D_0 = \left\{k \in D : k \in (\alpha_m, \alpha_{m+1}] \cap A^c_{(m)},\ m \in D\right\} \cup [1, \alpha_1].$$

Define

$$t_k = \begin{cases} s, & k \in D_0 \\ s_k, & k \notin D_0 \end{cases},$$

where $s = \mathcal{F}\text{-}\lim s_\alpha$. Now we prove that $\lim t_k = s$. Let $U \in \Omega$ be an arbitrary element. If $k \in D_0$, then it is obvious that $t_k \in U[s]$. If $k \notin D_0$, then there exists a $m \in D$ such that $\alpha_m < k \le \alpha_{m+1}$ and $k \notin A^c_{(m)}$. Moreover, if $k \in A_{(m)}$, then $s_k \in B_m$. Let $\alpha_0 \in D$ be a number such that $U^{(\alpha_0-1)} \subset U$. Let k be sufficiently large $m \ge \alpha_0$. We get $s_k \in U^{(\alpha_0)}[s]$ and so $s_k \in U^{(\alpha_0+1)}\left[s_{k_{\alpha_0}+1}\right]$ and $s_{k_{\alpha_0}+1} \in U^{(\alpha_0+1)}[s]$. Hence, $(t_k, s) \in U^{(\alpha_0)} \subset U$, since, in this case $s_k = t_k$. Since U is arbitrary, $\lim t_k = s$. Now we prove that $\widetilde{A} = \{k \in D : s_k = t_k\} \in \mathcal{F}$. It is clear that

$$\cup_{m=1}^{\infty} \left((\alpha_m, \alpha_{m+1}] \cap A_{(m)}\right) \subset \widetilde{A}.$$

Hence, $\cup_{m=1}^{\infty} \left((\alpha_m, \alpha_{m+1}] \cap A_{(m)}\right) \in \mathcal{F}$ and we obtain $\widetilde{A} \in \mathcal{F}$ from the condition (*iii*) of filter. Therefore, if $\mathcal{F}\text{-}\lim s_\alpha = s$, then there exists an $\widetilde{A} \in \mathcal{F}$ such that $\lim t_\alpha = s$ and $s_\alpha = t_\alpha$ for all $\alpha \in \widetilde{A}$.

ii) Suppose that $\lim t_\alpha = s$, $\widetilde{A} = \{\alpha \in D : s_\alpha = t_\alpha\} \in \mathcal{F}$ and $\mathcal{F}$ is a right filter. Let $U \in \Omega$ be arbitrary. Then there exists $\alpha_0 \in D$ such that $t_\alpha \in U[s]$ for all $\alpha \ge \alpha_0$. We get

$$\left(\{\alpha \in D : \alpha \ge \alpha_0\} \cap \widetilde{A}\right) \subset \{\alpha \in D : s_\alpha \in U[s]\}.$$

It is obvious that

$$\left(\{\alpha \in D : \alpha \ge \alpha_0\} \cap \widetilde{A}\right) \in \mathcal{F}.$$

Then we have $\{\alpha \in D : s_\alpha \in U[s]\} \in \mathcal{F}$ from the condition (*iii*) of filter. □

The following results are immediate consequences of Theorems 2.5 and 2.6.

Corollary 2.7. *Let (X, Ω) be a uniform space with a countable base, $\{s_\alpha : \alpha \in D\}$ be a net in X and $\mathcal{F}$ be a monotone closed and a right filter. Then the followings are equivalent:*
i) $\mathcal{F}\text{-}\lim s_\alpha = s$,
ii) $\{s_\alpha : \alpha \in D\}$ *is* $\mathcal{F}_{st}$*-fundamental,*
iii) $\lim t_\alpha = s$ *and* $\{\alpha \in D : s_\alpha = t_\alpha\} \in \mathcal{F}$.

Corollary 2.8. *Let (X, Ω) be a uniform space with a countable base, $\{s_\alpha : \alpha \in D\}$ be an $\mathcal{F}_{\mathrm{st}}$-fundamental net in X, and $\mathcal{F}$ be a right filter. If $\mathcal{F}\text{-}\lim s_\alpha = s$, then there exists a $\{\alpha_k : \alpha_1 < \alpha_2 < \ldots\} \in \mathcal{F}$ such that $\lim s_{\alpha_k} = s$.*

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