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Some Results of the Picard-Krasnoselskii Hybrid Iterative Process

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Abstract. In this paper, we establish the strong convergence and stability results of Picard-Krasnoselskii hybrid iterative process for a general class of contractive-like operators in a hyperbolic space. Additionally, we apply this iterative process to obtain the solution of a functional equation in a Banach space.

1. Introduction and Preliminaries

Let (X, d) be a metric space and T be a self mapping on X. A point $p \in X$ is called a fixed point of T if $Tp = p$. The mapping T is said to be a contraction if $d(Tx, Ty) \leq \delta d(x, y)$ for each $x, y \in X$ and $\delta \in (0, 1)$. Osilike [15] considered the mapping T having a fixed point and satisfying the condition

$$d(Tx, Ty) \leq ad(x, y) + Ld(x, Tx) \quad \text{for each } x, y \in X, \tag{1}$$

where $L \geq 0$ and $0 \leq a < 1$. Imoru and Olatinwo [8] generalized the condition of Osilike [15] by replacing (1) with

$$d(Tx, Ty) \leq ad(x, y) + \varphi(d(x, Tx)) \quad \text{for each } x, y \in X, \tag{2}$$

where $0 \leq a < 1$ and $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ is a monotone increasing function with $\varphi(0) = 0$. Bosede and Rhoades [2] made an assumption implied by (1) and the one which renders all generalizations of the form (2) pointless. That is if $x = p$ (is a fixed point) then the inequality (1) becomes

$$d(p, Ty) \leq ad(p, y) \quad \text{for some } a \in [0, 1) \text{ and all } x, y \in X. \tag{3}$$

In 2014, Chidume and Olaleru [5] gave several examples to show that the class of mappings satisfying (3) is more general than that of (2) and (1), provided the fixed point exists. Also, they proved that every contraction mapping with a fixed point satisfies the inequality (3).

Kohlenbach [10] introduced the concept of hyperbolic space, defined below, which is more restrictive than the hyperbolic type introduced in [6] and more general than the concept of hyperbolic space in [17].

A hyperbolic space is a triple (X, d, W) where (X, d) is a metric space and $W : X \times X \times [0, 1] \to X$ is a mapping such that

(W1) $d(z, W(x, y, \eta)) \leq \eta d(z, x) + (1 - \eta)d(z, y)$,

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(W2) $d(W(x,y,\eta_1), W(x,y,\eta_2)) = \left|\eta_1 - \eta_2\right| d(x,y)$,
(W3) $W(x,y,\eta) = W(y,x,(1-\eta))$,
(W4) $d(W(x,z,\eta), W(y,w,\eta)) \leq \eta d(x,y) + (1-\eta)d(z,w)$,
for all $x,y,z,w \in X$ and $\eta, \eta_1, \eta_2 \in [0,1]$.

If a space satisfies only (W1), it coincides with the convex metric space introduced by Takahashi [18]. The class of hyperbolic spaces contains all normed linear spaces and convex subsets thereof, $\mathbb{R}$-trees, the Hilbert ball with the hyperbolic metric (see [7]), Cartesian products of Hilbert balls, Hadamard manifolds and CAT(0) spaces in the sense of Gromov (see [3]), as special cases.

A subset K of a hyperbolic space X is convex if $W(x,y,\eta) \in K$ for all $x,y \in K$ and $\eta \in [0,1]$. The following equalities hold even for the more general setting of convex metric space (see [18, Proposition 1.2]): for all $x,y \in X$ and $\eta \in [0,1]$,

$$d(y, W(x,y,\eta)) = \eta d(x,y) \quad \text{and} \quad d(x, W(x,y,\eta)) = (1-\eta)d(x,y).$$

As a consequence,

$$W(x,y,1) = x \quad \text{and} \quad W(x,y,0) = y.$$

Very recently, Okeke and Abbas [14] introduced the Picard-Krasnoselskii hybrid iterative process which is a hybrid of Picard [16] and Krasnoselskii [11] iterative processes:

$$\begin{cases} x_{n+1} = Ty_n, \\ y_n = (1-\lambda)x_n + \lambda T x_n, \quad n \in \mathbb{N} \end{cases} \tag{4}$$

where $\lambda \in (0,1)$. They showed that the iterative process (4) converges faster than all Picard [16], Mann [13], Krasnoselskii [11] and Ishikawa [9] iterative processes for contraction mappings.

Using (W3) and $W(x,y,1) = x$ in (4), we define the Picard-Krasnoselskii hybrid iterative process in a hyperbolic space:

$$\begin{cases} x_{n+1} = Ty_n, \\ y_n = W(Tx_n, x_n, \lambda), \quad n \in \mathbb{N} \end{cases} \tag{5}$$

where $\lambda \in (0,1)$.

In this paper, we prove the strong convergence and stability of the iterative process (5) for the class of mappings satisfying (3) in a hyperbolic space and also present some examples to support our results. Moreover, we show that the iterative process (4) can be used to find the solution of a functional equation in a Banach space.

2. Some Convergence and Stability Results

We begin with strong convergence theorem of the Picard-Krasnoselskii hybrid iterative process for a general class of contractive-like mappings in a hyperbolic space.

Theorem 2.1. *Let K be a nonempty closed convex subset of a hyperbolic space X, and let $T : K \to K$ be a mapping satisfying (3) with a fixed point p. Then, for $x_1 \in K$, the sequence $\{x_n\}$ defined by (5) converges strongly to p.*

Proof. Using (3), (W1) and (5), we have

$$d(x_{n+1}, p) = d(Ty_n, p) \leq a d(y_n, p) \tag{6}$$

and

$$\begin{aligned} d(y_n, p) &= d(W(Tx_n, x_n, \lambda), p) \\ &\leq \lambda d(Tx_n, p) + (1-\lambda)d(x_n, p) \\ &\leq \lambda a d(x_n, p) + (1-\lambda)d(x_n, p) \\ &= (1 - \lambda(1-a))d(x_n, p). \end{aligned} \tag{7}$$

Combining (6) and (7), we obtain

$$\begin{aligned} d(x_{n+1},p) &\leq a(1-\lambda(1-a))d(x_n,p) \\ &\vdots \\ &\leq [a(1-\lambda(1-a))]^n\, d(x_1,p). \end{aligned}$$

Since $a \in [0,1)$ and $\lambda \in (0,1)$, then we get $0 < 1-\lambda(1-a) < 1$. If $a \in (0,1)$, we have

$$\lim_{n\to\infty} d(x_{n+1},p) = 0.$$

Thus we obtain $x_n \to p$. If $a = 0$, the result is clear. This completes the proof. □

The following examples show validity of Theorem 2.1.

Example 2.2. Let $K = [0,1]$ be endowed with the usual metric. Define the mapping $T : K \to K$ by $Tx = \frac{x}{2}$ with a fixed point $p = 0$. It is clear that T satisfies the inequality (3) with $a = \frac{1}{2}$. For $\lambda = \frac{1}{2}$, the strong convergence result for the Picard-Krasnoselskii hybrid iterative process to $p = 0$ is showed in Table 1 below.

Table 1. Iterates of the Picard-Krasnoselskii hybrid iterative process

Iterate	$x_1 = 0.1$	$x_1 = 0.5$	$x_1 = 0.9$	$x_1 = 1$
x_2	0.0375000000	0.1875000000	0.3375000000	0.3750000000
x_3	0.0140625000	0.0703125000	0.1265625000	0.1406250000
x_4	0.0052734375	0.0263671875	0.0474609375	0.0527343750
x_5	0.0019775391	0.0098876953	0.0177978516	0.0197753906
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
x_{10}	0.0000146650	0.0000733249	0.0001319848	0.0001466498
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
x_{15}	0.0000001088	0.0000005438	0.0000009788	0.0000010875
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
x_{22}	0.0000000001	0.0000000006	0.0000000010	0.0000000011
x_{23}	0.0000000000	0.0000000002	0.0000000000	0.0000000004
x_{24}	0.0000000000	0.0000000001	0.0000000001	0.0000000002
x_{25}	0.0000000000	0.0000000000	0.0000000001	0.0000000001
x_{26}	0.0000000000	0.0000000000	0.0000000000	0.0000000000

Example 2.3. Let $X = l_\infty$, $K = \{x \in l_\infty : \|x\| \leq 1\}$, and let $T : K \to K$ be defined by $Tx = \frac{9}{10}(0, x_1^2, x_2^2, x_3^2, ...)$ for $x = (x_1, x_2, x_3, ...) \in K$. Clearly, the fixed point of T is $p = 0$. It is proved in [5] that T satisfies the inequality (3) with $a = \frac{9}{10}$. Set $\lambda = \frac{1}{2}$. Thus, the conditions of Theorem 2.1 are fulfilled. Therefore the result of Theorem 2.1 can be easily seen.

Now we give the following definitions which are necessary for the result related to weak w^2-stable with respect to T.

Definition 2.4. ([4]) Two sequences $\{x_n\}_{n=0}^\infty$ and $\{y_n\}_{n=0}^\infty$ are said to be equivalent sequences if $\lim_{n\to\infty} d(x_n, y_n) = 0$.

Definition 2.5. ([19]) Let (X,d) be a metric space, and let T be a self mapping on X. Let $\{x_n\}_{n=0}^\infty \subset X$ be an iterative sequence generated by the general algorithm of the form

$$\begin{cases} x_0 \in X, \\ x_{n+1} = f(T, x_n), \quad \forall n \geq 0, \end{cases} \tag{8}$$

where x_0 is an initial approximation and f is a function of T and $\{x_n\}$. Suppose that the sequence $\{x_n\}$ converges strongly to a fixed point p of T. Let $\{y_n\}_{n=0}^{\infty} \subset X$ be an equivalent sequence of $\{x_n\}_{n=0}^{\infty} \subset X$. Then, the iterative scheme (8) is said to be *weak w^2-stable with respect to T* if and only if $\lim_{n\to\infty} d\left(y_{n+1}, f(T, y_n)\right) = 0$ implies that $\lim_{n\to\infty} y_n = p$.

By using these definitions and Theorem 2.1, we now prove the stability of the Picard-Krasnoselskii hybrid iterative process (5).

Theorem 2.6. *Let X, K and T be the same as in Theorem 2.1. Then the iterative sequence $\{x_n\}$ defined by (5) is weak w^2-stable with respect to T.*

Proof. Suppose that $\{p_n\}_{n=1}^{\infty} \subset K$ is an equivalent sequence of $\{x_n\}$,

$$\varepsilon_n = d(p_{n+1}, Tq_n)$$

where $q_n = W(Tp_n, p_n, \lambda)$ and let $\lim_{n\to\infty} \varepsilon_n = 0$. Then we have

$$\begin{aligned} d(p_{n+1}, p) &\le d(p_{n+1}, x_{n+1}) + d(x_{n+1}, p) \\ &\le d(p_{n+1}, Tq_n) + d(Tq_n, Ty_n) + d(x_{n+1}, p) \\ &\le \varepsilon_n + ad(q_n, y_n) + d(x_{n+1}, p) \end{aligned} \tag{9}$$

and

$$\begin{aligned} d(q_n, y_n) &= d(W(Tp_n, p_n, \lambda), W(Tx_n, x_n, \lambda)) \\ &\le \lambda d(Tp_n, Tx_n) + (1-\lambda) d(p_n, x_n) \\ &\le \lambda a d(p_n, x_n) + (1-\lambda) d(p_n, x_n) \\ &= (1 - \lambda(1-a)) d(p_n, x_n). \end{aligned} \tag{10}$$

Using (10) in (9), we get

$$d(p_{n+1}, p) \le \varepsilon_n + a(1 - \lambda(1-a)) d(p_n, x_n) + d(x_{n+1}, p).$$

By Theorem 2.1, we have $\lim_{n\to\infty} d(x_{n+1}, p) = 0$. Since $\{x_n\}$ and $\{p_n\}$ are equivalent sequences, then we have $\lim_{n\to\infty} d(x_n, p_n) = 0$. Using these facts and the assumption $\lim_{n\to\infty} \varepsilon_n = 0$, we get

$$\lim_{n\to\infty} d(p_{n+1}, p) = 0.$$

This shows that $\{x_n\}$ is weak w^2-stable with respect to T. □

3. An Application to the Functional Equation

Let $E = \{\varphi \in C[0,1] : \varphi(0) = 0, \varphi(1) = 1\}$. Then the mapping

$$\left\|\varphi\right\| = \sup_{t \neq s} \frac{\left|\varphi(t) - \varphi(s)\right|}{|t-s|}, \quad \varphi \in E, \tag{11}$$

is a norm on E and $(E, \|.\|)$ is a Banach space (see [12]).

In this section, we consider the following functional equation:

$$\varphi(x) = x\varphi(f(x)) + (1-x)\varphi(g(x)), \quad x \in [0,1], \tag{12}$$

where $f, g : [0,1] \to [0,1]$ are contraction mappings such that $f(1) = 1$ and $g(0) = 0$.

Berinde and Khan [1] established the following result for this functional equation.

Theorem 3.1. ([1, Theorem 2.2]) *If f and g are contraction mappings on $[0,1]$ (endowed with usual norm), with the contraction coefficients α and β, respectively, satisfying $\alpha, \beta \in (0,1), \alpha \leq \beta$ and $2\alpha < 1$, then the functional equation (12) has a unique solution $\overline{\varphi}$ in E and the sequence of successive approximations $\{\varphi_n\}$ defined by*

$$\varphi_{n+1}(x) = x\varphi_n(f(x)) + (1-x)\varphi_n(g(x)), \quad x \in [0,1], n \geq 0$$

converges strongly to $\overline{\varphi}$, as $n \to \infty$, for any $\varphi_0 \in E$.

Next, we prove the following result using the Picard-Krasnoselskii hybrid iterative process (4).

Theorem 3.2. *Under the assumptions of Theorem 3.1, the functional equation (12) has a unique solution $\overline{\varphi}$ in E and the iterative sequence $\{x_n\}$ defined by (4) converges strongly to $\overline{\varphi}$.*

Proof. Let $\{x_n\}$ be an iterative sequence generated by (4) for the linear operator $T : E \to E$ defined by

$$(T\varphi)(x) = x\varphi(f(x)) + (1-x)\varphi(g(x)), \quad x \in [0,1].$$

We will show that $x_n \to \overline{\varphi}$ as $n \to \infty$. From (4) and (11), we obtain

$$\left\|x_{n+1} - \overline{\varphi}\right\| = \left\|Ty_n - T\overline{\varphi}\right\| = \left\|T\left(y_n - \overline{\varphi}\right)\right\| = \sup_{t \neq s} \frac{\left|T\left(y_n - \overline{\varphi}\right)(t) - T\left(y_n - \overline{\varphi}\right)(s)\right|}{|t-s|}.$$

We evaluate the quantity

$$\left|\triangle_{t,s}\right| = \frac{\left|T\left(y_n - \overline{\varphi}\right)(t) - T\left(y_n - \overline{\varphi}\right)(s)\right|}{|t-s|}, \quad t, s \in [0,1], t \neq s.$$

Then, we have

$$\begin{aligned}
\left|\triangle_{t,s}\right| = \frac{1}{|t-s|} &\left| \begin{array}{c} t\left(y_n - \overline{\varphi}\right)(f(t)) + (1-t)\left(y_n - \overline{\varphi}\right)(g(t)) \\ -s\left(y_n - \overline{\varphi}\right)(f(s)) - (1-s)\left(y_n - \overline{\varphi}\right)(g(s)) \end{array} \right| \\
= \quad \frac{1}{|t-s|} &\left| \begin{array}{c} t\left(y_n - \overline{\varphi}\right)(f(t)) - t\left(y_n - \overline{\varphi}\right)(f(s)) \\ +(1-t)\left(y_n - \overline{\varphi}\right)(g(t)) - (1-t)\left(y_n - \overline{\varphi}\right)(g(s)) \\ t\left(y_n - \overline{\varphi}\right)(f(s)) - s\left(y_n - \overline{\varphi}\right)(f(s)) \\ +(1-t)\left(y_n - \overline{\varphi}\right)(g(s)) - (1-s)\left(y_n - \overline{\varphi}\right)(g(s)) \end{array} \right| \\
\leq \quad & t.\frac{\left|\left(y_n - \overline{\varphi}\right)(f(t)) - \left(y_n - \overline{\varphi}\right)(f(s))\right|}{\left|f(t) - f(s)\right|}.\frac{\left|f(t) - f(s)\right|}{|t-s|} \\
& +(1-t).\frac{\left|\left(y_n - \overline{\varphi}\right)(g(t)) - \left(y_n - \overline{\varphi}\right)(g(s))\right|}{\left|g(t) - g(s)\right|}.\frac{\left|g(t) - g(s)\right|}{|t-s|} \\
& +\left|\frac{t-s}{t-s}\right|.\frac{\left|\left(y_n - \overline{\varphi}\right)(f(s)) - \left(y_n - \overline{\varphi}\right)(f(1))\right|}{\left|f(s) - f(1)\right|}.\frac{\left|f(s) - f(1)\right|}{|s-1|}.|s-1| \\
& +\left|\frac{t-s}{t-s}\right|.\frac{\left|\left(y_n - \overline{\varphi}\right)(g(s)) - \left(y_n - \overline{\varphi}\right)(g(0))\right|}{\left|g(s) - g(0)\right|}.\frac{\left|g(s) - g(0)\right|}{|s-0|}.|s-0|. \qquad (13)
\end{aligned}$$

Since f and g are contraction mappings with the contraction coefficients α and β, respectively, then we obtain

$$\left|f(t) - f(s)\right| \leq \alpha |t-s|, \quad \left|g(t) - g(s)\right| \leq \beta |t-s|$$

and

$$\left|f(s) - f(1)\right| \leq \alpha |s-1| = \alpha(1-s), \quad \left|g(s) - g(0)\right| \leq \beta |s-0| = \beta s.$$

Therefore, by (13), we get

$$\left|\triangle_{t,s}\right| \leq \alpha t \left\|y_n - \overline{\varphi}\right\| + \beta(1-t)\left\|y_n - \overline{\varphi}\right\| + \alpha(1-s)\left\|y_n - \overline{\varphi}\right\| + \beta s \left\|y_n - \overline{\varphi}\right\|, \ t \neq s.$$

and so, we have

$$\left\|x_{n+1} - \overline{\varphi}\right\| \leq \sup_{t \neq s}\left[\alpha t + \beta(1-t) + \alpha(1-s) + \beta s\right]\left\|y_n - \overline{\varphi}\right\|. \tag{14}$$

Since $\alpha, \beta \in (0,1), \alpha \leq \beta$ and $2\alpha < 1$, then we obtain

$$\begin{aligned} \alpha t + \beta(1-t) + \alpha(1-s) + \beta s &= \alpha + \beta + (\alpha - \beta)t - (\alpha - \beta)s \\ &\leq \alpha + \beta + \alpha - \beta = 2\alpha < 1. \end{aligned} \tag{15}$$

It follows from (14) and (15) that

$$\left\|x_{n+1} - \overline{\varphi}\right\| \leq 2\alpha\left\|y_n - \overline{\varphi}\right\|. \tag{16}$$

Similarly, we have

$$\begin{aligned} \left\|y_n - \overline{\varphi}\right\| &= \left\|(1-\lambda)x_n + \lambda T x_n - \overline{\varphi}\right\| \\ &\leq (1-\lambda)\left\|x_n - \overline{\varphi}\right\| + \lambda\left\|T x_n - T\overline{\varphi}\right\| \\ &\leq (1-\lambda)\left\|x_n - \overline{\varphi}\right\| + \lambda \sup_{t \neq s}\frac{\left|T\left(x_n - \overline{\varphi}\right)(t) - T\left(x_n - \overline{\varphi}\right)(s)\right|}{|t-s|} \\ &\leq (1-\lambda)\left\|x_n - \overline{\varphi}\right\| + \lambda \sup_{t \neq s}\left[\alpha t + \beta(1-t) + \alpha(1-s) + \beta s\right]\left\|x_n - \overline{\varphi}\right\| \\ &\leq (1-\lambda)\left\|x_n - \overline{\varphi}\right\| + \lambda 2\alpha\left\|x_n - \overline{\varphi}\right\| \\ &= (1-\lambda(1-2\alpha))\left\|x_n - \overline{\varphi}\right\|. \end{aligned} \tag{17}$$

Combining (16) and (17), we get

$$\begin{aligned} \left\|x_{n+1} - \overline{\varphi}\right\| &\leq 2\alpha(1-\lambda(1-2\alpha))\left\|x_n - \overline{\varphi}\right\| \\ &\ \ \vdots \\ &\leq \left[2\alpha(1-\lambda(1-2\alpha))\right]^n\left\|x_1 - \overline{\varphi}\right\|. \end{aligned}$$

Since $2\alpha(1-\lambda(1-2\alpha)) \in (0,1)$, then we obtain $\lim_{n\to\infty}\left\|x_{n+1} - \overline{\varphi}\right\| = 0$. This completes the proof. □

If we take $f(x) = (1-\alpha)x + \alpha$ and $g(x) = (1-\beta)x$ for each $x \in [0,1]$ in Theorem 3.2, we get the following corollary which is still new in the literature.

Corollary 3.3. *If f and g are given by*

$$f(x) = (1-\alpha)x + \alpha, \quad g(x) = (1-\beta)x, \quad x \in [0,1],$$

then the functional equation

$$\varphi(x) = x\varphi((1-\alpha)x + \alpha) + (1-x)\varphi((1-\beta)x), \quad x \in [0,1]$$

has a unique solution $\overline{\varphi}$ in E and the iterative sequence defined by (4) converges strongly to $\overline{\varphi}$.

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