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On the Recursive Sequence $x_{n+1} = \frac{x_{n-7}}{1+x_{n-3}}$

D. Simsek[a], P. Esengul Kyzy[b], M. Imash Kyzy[b]

[a] *Kyrgyz-Turkish Manas University, 720044 Bishkek, Kyrgyzstan, Konya Technical University, Konya, Turkey*
[b] *Kyrgyz-Turkish Manas University, 720044 Bishkek, Kyrgyzstan*

Abstract. In this paper the solutions of the following difference equation is examined:

$$x_{n+1} = \frac{x_{n-7}}{1 + x_{n-3}}, \quad n = 0, 1, 2, 3, ...$$

where the initial conditions are positive real numbers.

1. Introduction

The study of difference equations is growing continuously for the last decade. Difference equations are always attracting very much interest, because these equations appear in the mathematical models of some problems in biology, ecology and physics, and numerical solutions of differential equations [22-31]. In fact, they occupy a central position in applicable analysis and will continue undoubtedly to play an important role in mathematics as a whole. Recently, a lot of interest in studying the periodic nature of nonlinear difference equations has been revealed. We refer readers to [1, 5-19] for some recent results concerning among other problems and the periodicity of scalar nonlinear difference equations.

Cinar [2, 3, 4] has studied the following problems with positive initial values:

$$x_{n+1} = \frac{x_{n-1}}{1 + ax_n x_{n-1}},$$

$$x_{n+1} = \frac{x_{n-1}}{-1 + ax_n x_{n-1}},$$

$$x_{n+1} = \frac{ax_{n-1}}{1 + bx_n x_{n-1}},$$

for $n = 0, 1, 2, ...$, respectively.

Simsek et al. [20, 21, 22, 25] studied the following problems

$$x_{n+1} = \frac{x_{n-3}}{1 + x_{n-1}},$$

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Email addresses: dsimsek@ktun.edu.tr, dagistan.simsek@manas.edu.kg (D. Simsek), peyil.esengul@manas.edu.kg (P. Esengul Kyzy), imashkyzy@gmail.com (M. Imash Kyzy)

$$x_{n+1} = \frac{x_{n-5}}{1+x_{n-2}},$$
$$x_{n+1} = \frac{x_{n-5}}{1+x_{n-1}x_{n-3}},$$
$$x_{n+1} = \frac{x_{n-3}}{1+x_n.x_{n-1}x_{n-2}}$$

with positive initial values for $n = 0, 1, 2, ...$, respectively.

In [28], Stević solved the problem

$$x_{n+1} = \frac{x_{n-1}}{1+x_n} \quad for \;\; n = 0, 1, 2, ...,$$

where $x_{-1}, x_0 \in (0, \infty)$. This result was generalized to the equation of the following form:

$$x_{n+1} = \frac{x_{n-1}}{g(x_n)} \quad for \;\;\; n = 0, 1, 2, ...,$$

where $x_{-1}, x_0 \in (0, \infty)$.

In this paper we investigated the following nonlinear difference equation:

$$x_{n+1} = \frac{x_{n-7}}{1+x_{n-3}}, \;\; n = 0, 1, 2, 3, ... \tag{1}$$

where $x_{-7}, x_{-6}, x_{-5}, x_{-4}, x_{-3}, x_{-2}, x_{-1}, x_0 \in (0, \infty)$.

2. Main Result

Theorem 2.1. *Consider the difference equation (1). Then the following statements are true.*

a) The sequences $\{x_{8n-7}\}, \{x_{8n-6}\}, \{x_{8n-5}\}, \{x_{8n-4}\}, \{x_{8n-3}\}, \{x_{8n-2}\}, \{x_{8n-1}\}, \{x_{8n}\}$ *are decreasing and there exist* $a_1, a_2, ..., a_8 \geq 0$ *such that*

$$\lim_{n\to\infty} x_{8n-7} = a_1, \;\; \lim_{n\to\infty} x_{8n-6} = a_2, \;\; \lim_{n\to\infty} x_{8n-5} = a_3, \;\; \lim_{n\to\infty} x_{8n-4} = a_4, \;\; \lim_{n\to\infty} x_{8n-3} = a_5,$$

$$\lim_{n\to\infty} x_{8n-2} = a_6, \;\; \lim_{n\to\infty} x_{8n-1} = a_7, \;\; \lim_{n\to\infty} x_{8n} = a_8,$$

b) $\lim_{n\to\infty} x_{8n-7} \lim_{n\to\infty} x_{8n-3} = 0, \;\; \lim_{n\to\infty} x_{8n-6} \lim_{n\to\infty} x_{8n-2} = 0, \;\; \lim_{n\to\infty} x_{8n-5} \lim_{n\to\infty} x_{8n-1} = 0, \;\; \lim_{n\to\infty} x_{8n-4} \lim_{n\to\infty} x_{8n} = 0,$
or $a_1a_5 = 0, \;\; a_2a_6 = 0, \;\; a_3a_7 = 0, \;\; a_4a_8 = 0.$

c) If there exist $n_0 \in \mathbb{N} = Z_+$ *such that* $x_{n+1} \leq x_{n-3}$ *for all* $n \geq n_0$ *, then* $\lim_{n\to\infty} x_n = 0.$

d) The following formulas hold:

$$x_{8n+1} - x_{-7} = (x_1 - x_{-7}) \sum_{j=0}^{n} \prod_{i=1}^{2j} \frac{1}{1+x_{4i-3}};$$

$$x_{8n+2} - x_{-6} = (x_2 - x_{-6}) \sum_{j=0}^{n} \prod_{i=1}^{2j} \frac{1}{1+x_{4i-2}};$$

$$x_{8n+3} - x_{-5} = (x_3 - x_{-5}) \sum_{j=0}^{n} \prod_{i=1}^{2j} \frac{1}{1+x_{4i-1}};$$

$$x_{8n+4} - x_{-4} = (x_4 - x_{-4}) \sum_{j=0}^{n} \prod_{i=1}^{2j} \frac{1}{1+x_{4i}};$$

$$x_{8n+5} - x_{-3} = (x_1 - x_{-7}) \sum_{j=0}^{n} \prod_{i=1}^{2j+1} \frac{1}{1 + x_{4i-3}};$$

$$x_{8n+6} - x_{-2} = (x_2 - x_{-6}) \sum_{j=0}^{n} \prod_{i=1}^{2j+1} \frac{1}{1 + x_{4i-2}};$$

$$x_{8n+7} - x_{-1} = (x_3 - x_{-5}) \sum_{j=0}^{n} \prod_{i=1}^{2j+1} \frac{1}{1 + x_{4i-1}};$$

$$x_{8n+8} - x_{0} = (x_4 - x_{-4}) \sum_{j=0}^{n} \prod_{i=1}^{2j+1} \frac{1}{1 + x_{4i}}.$$

e) If $x_{8n+1} \to a_1 \neq 0$ then $x_{8n+5} \to 0$ as $n \to \infty$. If $x_{8n+2} \to a_2 \neq 0$ then $x_{8n+6} \to 0$ as $n \to \infty$. If $x_{8n+3} \to a_3 \neq 0$ then $x_{8n+7} \to 0$ as $n \to \infty$. If $x_{8n+4} \to a_4 \neq 0$ then $x_{8n+8} \to 0$ as $n \to \infty$.

Proof. a) Firstly, we consider the equation (1). From this equation we obtain

$$x_{n+1}(1 + x_{n-3}) = x_{n-7}.$$

If $x_{n-3} \in (0, +\infty)$, then $(1 + x_{n-3}) \in (1, +\infty)$. Since $x_{n-7} > x_{n+1},\ n \in \mathbb{N}$, we obtain that

$$\lim_{n\to\infty} x_{8n-7} = a_1,\quad \lim_{n\to\infty} x_{8n-6} = a_2,\quad \lim_{n\to\infty} x_{8n-5} = a_3,\quad \lim_{n\to\infty} x_{8n-4} = a_4,\quad \lim_{n\to\infty} x_{8n-3} = a_5,$$

$$\lim_{n\to\infty} x_{8n-2} = a_6,\quad \lim_{n\to\infty} x_{8n-1} = a_7,\quad \lim_{n\to\infty} x_{8n} = a_8.$$

b) In view of the equation (1), we obtain

$$x_{8n+1} = \frac{x_{8n-7}}{1 + x_{8n-3}}.$$

Taking limit as $n \to \infty$ on both sides of the above equality, we get

$$n = 8n \implies \lim_{n\to\infty} x_{8n-7} \lim_{n\to\infty} x_{8n-3} = 0 \ \text{ or } \ a_1 a_5 = 0.$$

Similarly,

$$n = 8n + 1 \implies \lim_{n\to\infty} x_{8n-6} \lim_{n\to\infty} x_{8n-2} = 0 \ \text{ or } \ a_2 a_6 = 0;$$

$$n = 8n + 2 \implies \lim_{n\to\infty} x_{8n-5} \lim_{n\to\infty} x_{8n-1} = 0 \ \text{ or } \ a_3 a_7 = 0;$$

$$n = 8n + 3 \implies \lim_{n\to\infty} x_{8n-4} \lim_{n\to\infty} x_{8n} = 0 \ \text{ or } \ a_4 a_8 = 0.$$

c) If there exist $n_0 \in \mathbb{N}$ such that $x_{n-3} \geq x_{n+1}$ for all $n \geq n_0$, then $a_2 \leq a_6 \leq a_2,\ a_3 \leq a_7 \leq a_3,\ a_4 \leq a_8 \leq a_4,\ a_5 \leq a_1 \leq a_5$. Using (b), we get

$$a_1 a_5 = 0,\ a_2 a_6 = 0,\ a_3 a_7 = 0,\ a_4 a_8 = 0.$$

Then, we see that

$$\lim_{n\to\infty} x_n = 0.$$

d) Subtracting x_{n-7} from the left and right-hand sides of equation (1), we obtain

$$x_{n+1} - x_{n-7} = \frac{1}{1 + x_{n-3}}(x_{n-3} - x_{n-11}). \tag{2}$$

From (2), the following formula

$$n \geq 4 \ \ for \ \begin{cases} x_{4n-15} - x_{4n-23} = (x_1 - x_{-7}) \prod_{i=1}^{n-4} \frac{1}{1+x_{4i-3}}; \\ x_{4n-14} - x_{4n-22} = (x_2 - x_{-6}) \prod_{i=1}^{n-4} \frac{1}{1+x_{4i-2}}; \\ x_{4n-13} - x_{4n-21} = (x_3 - x_{-5}) \prod_{i=1}^{n-4} \frac{1}{1+x_{4i-1}}; \\ x_{4n-12} - x_{4n-20} = (x_4 - x_{-4}) \prod_{i=1}^{n-4} \frac{1}{1+x_{4i}} \end{cases} \tag{3}$$

holds. Replacing n by $2j$ in (3) and summing from $j = 0$ to $j = n$ we obtain

$$(n = 0, 1, 2, ...,) \ \begin{cases} x_{8n+1} - x_{-7} = (x_1 - x_{-7}) \sum_{j=0}^{n} \prod_{i=1}^{2j} \frac{1}{1+x_{4i-3}}; \\ x_{8n+2} - x_{-6} = (x_2 - x_{-6}) \sum_{j=0}^{n} \prod_{i=1}^{2j} \frac{1}{1+x_{4i-2}}; \\ x_{8n+3} - x_{-5} = (x_3 - x_{-5}) \sum_{j=0}^{n} \prod_{i=1}^{2j} \frac{1}{1+x_{4i-1}}; \\ x_{8n+4} - x_{-4} = (x_4 - x_{-4}) \sum_{j=0}^{n} \prod_{i=1}^{2j} \frac{1}{1+x_{4i}}. \end{cases} \tag{4}$$

Also, replacing n by $2j + 1$ in (3) and summing up elements from $j = 0$ to $j = n$ we obtain

$$(n = 0, 1, 2, ...,) \ \begin{cases} x_{8n+5} - x_{-3} = (x_1 - x_{-7}) \sum_{j=0}^{n} \prod_{i=1}^{2j+1} \frac{1}{1+x_{4i-3}}; \\ x_{8n+6} - x_{-2} = (x_2 - x_{-6}) \sum_{j=0}^{n} \prod_{i=1}^{2j+1} \frac{1}{1+x_{4i-2}}; \\ x_{8n+7} - x_{-1} = (x_3 - x_{-5}) \sum_{j=0}^{n} \prod_{i=1}^{2j+1} \frac{1}{1+x_{4i-1}}; \\ x_{8n+8} - x_{0} = (x_4 - x_{-4}) \sum_{j=0}^{n} \prod_{i=1}^{2j+1} \frac{1}{1+x_{4i}}. \end{cases} \tag{5}$$

Now, we obtained of the above formulas,

$$\begin{cases} x_{8n+1} = x_{-7}(1 - \frac{x_{-3}}{1+x_{-3}} \sum_{j=0}^{n} \prod_{i=1}^{2j} \frac{1}{1+x_{4i-3}}); \\ x_{8n+2} = x_{-6}(1 - \frac{x_{-2}}{1+x_{-2}} \sum_{j=0}^{n} \prod_{i=1}^{2j} \frac{1}{1+x_{4i-2}}); \\ x_{8n+3} = x_{-5}(1 - \frac{x_{-1}}{1+x_{-1}} \sum_{j=0}^{n} \prod_{i=1}^{2j} \frac{1}{1+x_{4i-1}}); \\ x_{8n+4} = x_{-4}(1 - \frac{x_{0}}{1+x_{0}} \sum_{j=0}^{n} \prod_{i=1}^{2j} \frac{1}{1+x_{4i}}). \end{cases} \tag{6}$$

$$\begin{cases} x_{8n+5} = x_{-3}(1 - \frac{x_{-7}}{1+x_{-3}} \sum_{j=0}^{n} \prod_{i=1}^{2j+1} \frac{1}{1+x_{4i-3}}); \\ x_{8n+6} = x_{-2}(1 - \frac{x_{-6}}{1+x_{-2}} \sum_{j=0}^{n} \prod_{i=1}^{2j+1} \frac{1}{1+x_{4i-2}}); \\ x_{8n+7} = x_{-1}(1 - \frac{x_{-5}}{1+x_{-1}} \sum_{j=0}^{n} \prod_{i=1}^{2j+1} \frac{1}{1+x_{4i-1}}); \\ x_{8n+8} = x_{0}(1 - \frac{x_{-4}}{1+x_{0}} \sum_{j=0}^{n} \prod_{i=1}^{2j+1} \frac{1}{1+x_{4i}}). \end{cases} \tag{7}$$

e) Suppose that $a_1 = a_5 = 0$. By d) we have

$$\lim_{n\to\infty} x_{8n+1} = \lim_{n\to\infty} x_{-7}(1 - \frac{x_{-3}}{1+x_{-3}} \sum_{j=0}^{n} \prod_{i=1}^{2j} \frac{1}{1+x_{4i-3}});$$

$$a_1 = x_{-7}(1 - \frac{x_{-3}}{1+x_{-3}} \sum_{j=0}^{\infty} \prod_{i=1}^{2j} \frac{1}{1+x_{4i-3}});$$

$$a_1 = 0 \Rightarrow \frac{1+x_{-3}}{x_{-3}} = \sum_{j=0}^{\infty} \prod_{i=1}^{2j} \frac{1}{1+x_{4i-3}}. \tag{8}$$

Similarly,

$$\lim_{n\to\infty} x_{8n+5} = \lim_{n\to\infty} x_{-3}(1 - \frac{x_{-7}}{1+x_{-3}} \sum_{j=0}^{n} \prod_{i=1}^{2j+1} \frac{1}{1+x_{4i-3}});$$

$$a_5 = x_{-3}(1 - \frac{x_{-7}}{1+x_{-3}} \sum_{j=0}^{\infty} \prod_{i=1}^{2j+1} \frac{1}{1+x_{4i-3}});$$

$$a_5 = 0 \Rightarrow \frac{1+x_{-3}}{x_{-7}} = \sum_{j=0}^{\infty} \prod_{i=1}^{2j+1} \frac{1}{1+x_{4i-3}}. \tag{9}$$

From (8) and (9),

$$\frac{1+x_{-3}}{x_{-3}} = \sum_{j=0}^{\infty} \prod_{i=1}^{2j} \frac{1}{1+x_{4i-3}} > \frac{1+x_{-3}}{x_{-7}} = \sum_{j=0}^{\infty} \prod_{i=1}^{2j+1} \frac{1}{1+x_{4i-3}} \tag{10}$$

thus, $x_{-7} > x_{-3}$. If we take $x_{8n-7} > x_{8n-3}$ and are taking limit as $n \to \infty$ on both sides, we get

$$\lim_{n\to\infty} x_{8n-7} > \lim_{n\to\infty} x_{8n-3} \Rightarrow a_1 > a_5.$$

Suppose that $a_2 = a_6 = 0$. Hence similar to the previous we have

$$\frac{1+x_{-2}}{x_{-2}} = \sum_{j=0}^{\infty} \prod_{i=1}^{2j} \frac{1}{1+x_{4i-2}} > \frac{1+x_{-2}}{x_{-6}} = \sum_{j=0}^{\infty} \prod_{i=1}^{2j+1} \frac{1}{1+x_{4i-2}} \tag{11}$$

thus, $x_{-6} > x_{-2}$. If we take $x_{8n-6} > x_{8n-2}$ and are taking limit as $n \to \infty$ on both sides, we get

$$\lim_{n\to\infty} x_{8n-6} > \lim_{n\to\infty} x_{8n-2} \Rightarrow a_2 > a_6.$$

Suppose that $a_3 = a_7 = 0$. Hence similar to the previous we have

$$\frac{1+x_{-1}}{x_{-1}} = \sum_{j=0}^{\infty} \prod_{i=1}^{2j} \frac{1}{1+x_{4i-1}} > \frac{1+x_{-1}}{x_{-5}} = \sum_{j=0}^{\infty} \prod_{i=1}^{2j+1} \frac{1}{1+x_{4i-1}} \tag{12}$$

thus, $x_{-5} > x_{-1}$. If we take $x_{8n-5} > x_{8n-1}$ and are taking limit as $n \to \infty$ on both sides, we get

$$\lim_{n\to\infty} x_{8n-5} > \lim_{n\to\infty} x_{8n-1} \Rightarrow a_3 > a_7.$$

Suppose that $a_4 = a_8 = 0$. Hence similar to the previous we have

$$\frac{1+x_0}{x_0} = \sum_{j=0}^{\infty} \prod_{i=1}^{2j} \frac{1}{1+x_{4i}} > \frac{1+x_0}{x_{-4}} = \sum_{j=0}^{\infty} \prod_{i=1}^{2j+1} \frac{1}{1+x_{4i}} \tag{13}$$

thus, $x_{-4} > x_0$. If we take $x_{8n-4} > x_{8n}$ and are taking limit as $n \to \infty$ on both sides, we get

$$\lim_{n\to\infty} x_{8n-4} > \lim_{n\to\infty} x_{8n} \Rightarrow a_4 > a_8.$$

Hence we obtain $x_{-4} > x_0$, $x_{-5} > x_{-1}$, $x_{-6} > x_{-2}$, $x_{-7} > x_{-3}$. We face a contradiction which completes the proof of the theorem. □

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