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On the Existence of Solutions for Stochastic Differential Equations Driven by Fractional Brownian Motion

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Abstract. In this paper, we are concerned with a class of stochastic differential equations driven by fractional Brownian motion with Hurst parameter $1/2 < H < 1$, and a discontinuous drift. By approximation arguments and a comparison theorem, we prove the existence of solutions to this kind of equations under the linear growth condition.

1. Introduction

Consider the following stochastic differential equation:

$$X_t = X_0 + \int_0^t \sigma(s, X_s)dB_s^H + \int_0^t b(X_s)ds \quad t \in [0, T], \tag{1}$$

where $\sigma : [0, T] \times \mathbb{R} \to \mathbb{R}$ and $b : \mathbb{R} \to \mathbb{R}$ are two Borel functions, the integral $\int_0^t \sigma(s, X_s)dB_s^H$ is pathwise Riemann-Stieltjes integral, and $B^H = \{B_t^H, t \in [0, T]\}$ is a fractional Brownian motion with Hurst parameter $H \in (0, 1)$. That is, B^H is a centered Gaussian process with covariance

$$R_H(t, s) = \mathbb{E}(B_t^H B_s^H) = \frac{1}{2}(t^{2H} + s^{2H} - |t - s|^{2H}).$$

In the case $H = \frac{1}{2}$, the process B^H is the standard Brownian motion and the existence of a weak solution to (1) is well-known by the results of Zvonkin [9] and Veretennikov [8], assuming only that the coefficient $b(x)$ satisfies the following linear growth in x

$$|b(x)| \le C(1 + |x|). \tag{2}$$

In the singular case $H < \frac{1}{2}$, Nualart and Ouknine [5] established the existence of a strong solution to (1) with $\sigma \equiv 1$, also assuming only that the coefficient $b(x)$ has linear growth in x. In the regular case $H > \frac{1}{2}$, Boufoussi and Ouknine [1] proved that (1) with $\sigma \equiv 1$ has a strong solution if the coefficient b is continuous

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and has the linear growth and Li et al. [3] proved that (1) has a weak solution when the coefficient b has only the linear growth.

In [6], Nualart and Răşcanu established the existence and uniqueness of solution to (1) when b was local Lipschitz continuity and the linear growth and σ satisfied some suitable conditions. The main motivation of our work is to seek an answer to the following interesting question: when $H > \frac{1}{2}$, is there a strong solution to (1), assuming only that the coefficient $b(x)$ has linear growth in x but may be discontinuous? In this paper, by approximation arguments and a comparison theorem we will prove the existence of the strong solution to (1) when $b(x)$ has linear growth and is left-continuous and lower semi-continuous in x.

The rest of this paper is organized as follows. In Section 2, we introduce some necessary notations and preliminaries. In Section 3, we present and prove our main results.

2. Preliminaries

Let $\frac{1}{2} < H < 1, 1-H < \alpha < \frac{1}{2}$. Denote by $W_0^{\alpha,\infty}([0,T];\mathbb{R})$ the space of measurable functions $f : [0,T] \to \mathbb{R}$ such that

$$\|f\|_{\alpha,\infty} := \sup_{t\in[0,T]} \Big(|f(t)| + \int_0^t \frac{|f(t)-f(s)|}{(t-s)^{\alpha+1}} ds\Big) < \infty,$$

and for any $\lambda \geq 0$ a equivalent norm is defined by

$$\|f\|_{\alpha,\lambda} := \sup_{t\in[0,T]} e^{-\lambda t}\Big(|f(t)| + \int_0^t \frac{|f(t)-f(s)|}{(t-s)^{\alpha+1}} ds\Big) < \infty.$$

For any $0 < \alpha \leq 1$, denote by $C^\alpha([0,T];\mathbb{R})$ the space of α-Hölder continuous functions $f : [0,T] \to \mathbb{R}$, equipped with the norm

$$\|f\|_\alpha := \|f\|_\infty + \sup_{0\leq s<t\leq T} \frac{|f(t)-f(s)|}{(t-s)^\alpha} < \infty,$$

where $\|f\|_\infty := \sup_{t\in[0,T]} |f(t)|$. We have, for all $0 < \varepsilon < \alpha$

$$C^{\alpha+\varepsilon}([0,T];\mathbb{R}) \subset W_0^{\alpha,\infty}([0,T];\mathbb{R}) \subset C^{\alpha-\varepsilon}([0,T];\mathbb{R}).$$

Fix a parameter $0 < \alpha < \frac{1}{2}$. Denote by $W_T^{1-\alpha,\infty}([0,T];\mathbb{R})$ the space of measurable functions $g : [0,T] \to \mathbb{R}$ such that

$$\|g\|_{1-\alpha,\infty,T} := \sup_{0\leq s<t\leq T} \Big(\frac{|g(t)-g(s)|}{(t-s)^{1-\alpha}} + \int_s^t \frac{|g(y)-g(s)|}{(y-s)^{2-\alpha}} dy\Big) < \infty.$$

Clearly,

$$C^{1-\alpha+\varepsilon}([0,T];\mathbb{R}) \subset W_T^{1-\alpha,\infty}([0,T];\mathbb{R}) \subset C^{1-\alpha}([0,T];\mathbb{R}), \;\; \forall \varepsilon > 0.$$

Now, let us consider the following assumptions on the coefficients of (1).

(H1) $\sigma(t,x)$ is differentiable in x, and there exist some constants $M_0 > 0$, $0 < \beta, \delta \leq 1$ and for every $N \geq 0$ there exists $M_N > 0$ such that the following properties hold:

$$(H_\sigma): \begin{cases} i)\text{ Lipschitz continuity} \\ \quad |\sigma(t,x)-\sigma(t,y)| \leq M_0|x-y|, \quad \forall x,y \in \mathbb{R},\ \forall t \in [0,T], \\ ii)\text{ Local Hölder continuity} \\ \quad |\partial_x\sigma(t,x)-\partial_y\sigma(t,y)| \leq M_N|x-y|^\delta, \quad \forall |x|,|y| \leq N,\ \forall t \in [0,T], \\ iii)\text{ Hölder continuity in time} \\ \quad |\sigma(t,x)-\sigma(s,x)| + |\partial_x\sigma(t,x)-\partial_y\sigma(s,x)| \leq M_0|t-s|^\beta, \quad \forall x \in \mathbb{R},\ \forall t,s \in [0,T]. \end{cases}$$

(H2) There exists a constant $L_0 > 0$ such that the following properties hold:

$$(H_b): \begin{cases} i)\ b(\cdot)\text{ is left-continuous and lower semi-continuous, } i.e.,\text{ for each } x_0 \in \mathbb{R} \\ \quad \lim_{x\to x_0^-} b(x) = b(x_0), \quad \text{and } \liminf_{x\to x_0^+} b(x) \geq b(x_0), \\ ii)\text{ Linear growth} \\ \quad |b(x)| \leq L_0(1+|x|), \quad \forall x \in \mathbb{R}. \end{cases}$$

(H2') There exists a constant $L_0 > 0$ and for every $N \geq 0$ there exists $L_N > 0$ such that the following properties hold:

$$(H_b): \begin{cases} i) \text{ Local Lipschitz continuity} \\ \quad |b(x) - b(y)| \leq L_N|x-y|, \quad \forall |x|, |y| \leq N, \\ ii) \text{ Linear growth} \\ \quad |b(x)| \leq L_0(1+|x|), \quad \forall x \in \mathbb{R}. \end{cases}$$

Remark 2.1. *From i) and iii) of (H1), we can deduce that for all $x \in \mathbb{R}$ and $t \in [0,T]$*

$$|\sigma(t,x)| \leq |\sigma(0,0)| + M_0(|t|^\beta + |x|) \leq M_{0,T}(1+|x|),$$

where $M_{0,T} = |\sigma(0,0)| + M_0(1+T)$. Thus, the assumption (H1) implies that σ is the linear growth.

By the Theorem 2.1 of Nualart and Răşcanu [6], we can obtain the conclusion *(I)* of the following theorem. Moreover, in view of the Lemma 4.1 of Shevchenko [7], we also have the conclusion *(II)* of the following theorem.

Theorem 2.2. *Suppose that X_0 is an $\mathbb{R}$-valued random variable and $\alpha_0 = \min\{\frac{1}{2}, \beta, \frac{\delta}{1+\delta}\}$, the coefficients $\sigma(t,x)$ and $b(x)$ satisfy assumption (H1) and (H2') with $\beta > 1-H$ and $\delta > \frac{1}{H} - 1$. Then*

(I) If $\alpha \in (1-H, \alpha_0)$, then there exists a unique stochastic process $X \in L^0(\Omega, \mathcal{F}, \mathbb{P}; W_0^{\alpha,\infty}([0,T]; \mathbb{R}))$ solving the stochastic differential equation (1) and for $\mathbb{P}$-almost all $\omega \in \Omega$

$$X(\omega, \cdot) \in C^{1-\alpha}([0,T]; \mathbb{R}).$$

(II) Moreover, if $\alpha \in (1-H, \alpha_0)$, $X_0 \in L^\infty(\Omega, \mathcal{F}, \mathbb{P}; \mathbb{R})$, then the solution X satisfies

$$\mathbb{E}\|X\|_{\alpha,\infty}^p < \infty, \quad \forall p \geq 1.$$

3. Main results

To treat our main results of this section, we will use the following approximation lemma and the proof of this lemma we can refer to [2].

Lemma 3.1. *Let $b(\cdot)$ satisfies the assumption (H2). Then the sequence of functions*

$$b_n(x) = \inf_{y \in \mathbb{R}} \{b(y) + n|x-y|\}, \tag{3}$$

is well defined for $n \geq L_0$ and it satisfies

(i) convergence: if $x_n \to x^-$, then $b_n(x_n) \to b(x)$;
(ii) monotonicity in n: $\forall x \in \mathbb{R}$, $b_n(x) \leq b_{n+1}(x)$;
(iii) Lipschitz condition: $\forall x, y \in \mathbb{R}$, $|b_n(x) - b_n(y)| \leq n|x-y|$;
(iv) linear growth: $\forall x \in \mathbb{R}$, $|b_n(s)| \leq L_0(1+|x|)$.

The following comparison theorem is present in Nie and Răşcanu [4], which plays an important role in our main results.

Theorem 3.2. *Considering the two-dimensional decoupled system*

$$\begin{cases} X_t = X_0 + \int_0^t b_1(X_s)ds + \int_0^t \sigma_1(s, X_s)dB_s^H, \quad t \in [0,T], \\ Y_t = Y_0 + \int_0^t b_2(Y_s)ds + \int_0^t \sigma_2(s, Y_s)dB_s^H, \quad t \in [0,T], \end{cases}$$

where the coefficients $\sigma_i(t,x)$ and $b_i(x)$, $i = 1,2$, satisfy assumption (H1) and (H2') with $\beta > 1-H$, $\delta > \frac{1}{H} - 1$ and $\alpha \in (1-H, \alpha_0)$, then the following two assertions are equivalent:

(i) For any $t \in [0,T]$ and every $X_0 \le Y_0$: $X_t \le Y_t$;
(ii) $b_1(x) \le b_2(x)$, $\sigma_1(t,x) = \sigma_2(t,x)$, $\forall t \in [0,T]$ and $\forall x \in \mathbb{R}$.

Let $0 < \alpha < \frac{1}{2}$, $f \in W_0^{\alpha,\infty}([0,T];\mathbb{R})$ and $g \in W_T^{1-\alpha,\infty}([0,T];\mathbb{R})$. Define

$$G_t^\sigma(f) = \int_0^t \sigma(s, f(s))dg_s,$$

where σ satisfy the assumptions (H1) with constant $\beta > \alpha$. For the following estimation on $G_t^\sigma(f)$, we can refer to the Proposition 4.2 of [6].

Proposition 3.3. *If $f \in W_0^{\alpha,\infty}([0,T];\mathbb{R})$, then*

$$G^\sigma(f) \in C^{1-\alpha}([0,T];\mathbb{R}) \subset W_0^{\alpha,\infty}([0,T];\mathbb{R}).$$

Moreover, for all $\lambda \ge 1$ and $f \in W_0^{\alpha,\infty}([0,T];\mathbb{R})$:

$$(i)\ \left\|G^\sigma(f)\right\|_{1-\alpha} \le \Lambda_\alpha(g)C^{(1)}(1 + \|f\|_{\alpha,\infty}),$$

$$(ii)\ \left\|G^\sigma(f)\right\|_{\alpha,\lambda} \le \frac{\Lambda_\alpha(g)C^{(2)}}{\lambda^{1-2\alpha}}(1 + \|f\|_{\alpha,\lambda}).$$

If $f, h \in W_0^{\alpha,\infty}([0,T];\mathbb{R})$ such that $\|f\|_\infty \le N$, $\|h\|_\infty \le N$, then for all $\lambda \ge 1$,

$$\|G^\sigma(f) - G^\sigma(h)\|_{\alpha,\lambda} \le \frac{\Lambda_\alpha(g)C_N}{\lambda^{1-2\alpha}}(1 + \Delta(f) + \Delta(h))\|f - h\|_{\alpha,\lambda},$$

where

$$\Lambda_\alpha(g) := \frac{1}{\Gamma(1-\alpha)} \sup_{0\le s<t\le T} |(D_{t-}^{1-\alpha} g_{t-})(s)| \le \frac{1}{\Gamma(1-\alpha)\Gamma(\alpha)}\|g\|_{1-\alpha,\infty,T},$$

$$\Delta(f) = \sup_{r\in[0,T]} \int_0^r \frac{|f(r) - f(s)|^\delta}{(r-s)^{\alpha+1}} ds$$

and the constant $C^{(1)}$, $C^{(2)}$ and C_N is independent of λ, f, h, g and only depends on T, and constants from (H1).

Let $0 < \alpha < \frac{1}{2}$, we shall give similar estimates on the ordinary Lebesgue integrals

$$F_t^b(f) = \int_0^t b(f(s))ds,$$

where b satisfy the assumptions (H2'). For the following estimation on $F_t^b(f)$, we can refer to the Proposition 4.4 of [6].

Proposition 3.4. *If $f \in W_0^{\alpha,\infty}([0,T];\mathbb{R})$, then $F^b(f) \in C^{1-\alpha}([0,T];\mathbb{R})$ and Moreover, for all $\lambda \ge 1$ and $f \in W_0^{\alpha,\infty}([0,T];\mathbb{R})$:*

$$(j)\ \left\|F^b(f)\right\|_{1-\alpha} \le C^{(3)}(1 + \|f\|_\infty),$$

$$(jj)\ \left\|F^b(f)\right\|_{\alpha,\lambda} \le \frac{C^{(4)}}{\lambda^{1-2\alpha}}(1 + \|f\|_{\alpha,\lambda}).$$

If $f, h \in W_0^{\alpha,\infty}([0,T];\mathbb{R})$ such that $\|f\|_\infty \le N$, $\|h\|_\infty \le N$, then for all $\lambda \ge 1$,

$$\|F^b(f) - F^b(h)\|_{\alpha,\lambda} \le \frac{d_N}{\lambda^{1-2\alpha}}\|f - h\|_{\alpha,\lambda}$$

where the constant $C^{(3)}$, $C^{(4)}$ and d_N is independent of λ, f, h and only depends on T, and constants from (H2').

Now, we can state and prove our main result in this paper.

Theorem 3.5. *Suppose that* X_0 *is an* $\mathbb{R}$*-valued random variable and* $\alpha_0 = \min\{\frac{1}{2}, \beta, \frac{\delta}{1+\delta}\}$*, the coefficients* $\sigma(t,x)$ *and* $b(x)$ *satisfy assumption (H1) and (H2) with* $\beta > 1-H$ *and* $\delta > \frac{1}{H} - 1$*. Then*

(I) If $\alpha \in (1-H, \alpha_0)$*, then there exists a stochastic process* $X \in L^0(\Omega, \mathcal{F}, \mathbb{P}; W_0^{\alpha,\infty}([0,T];\mathbb{R}))$ *solving the stochastic differential equation (1) and for* $\mathbb{P}$*-almost all* $\omega \in \Omega$

$$X(\omega, \cdot) \in C^{1-\alpha}([0,T];\mathbb{R}).$$

(II) Moreover, if $\alpha \in (1-H, \alpha_0)$*,* $X_0 \in L^\infty(\Omega, \mathcal{F}, \mathbb{P}; \mathbb{R})$*, then the solution* X *satisfies*

$$\mathbb{E}\|X\|_{\alpha,\infty}^p < \infty, \quad \forall p \geq 1.$$

Proof. For any $n \geq L_0$, let b_n be as in Lemma 3.1. Consider the following stochastic differential equation

$$X_t^n = X_0 + \int_0^t \sigma(s, X_s^n) dB_s^H + \int_0^t b_n(X_s^n) ds. \tag{4}$$

Since b_n is Lipschitz and linear growth and σ satisfies the assumption (H1), by the results in the Theorem 2.1 we know

(I) If $\alpha \in (1-H, \alpha_0)$, then there exists a unique stochastic process $X^n \in L^0(\Omega, \mathcal{F}, \mathbb{P}; W_0^{\alpha,\infty}$ $([0,T];\mathbb{R}))$ solving the stochastic differential equation (4) and for $\mathbb{P}$-almost all $\omega \in \Omega$

$$X^n(\omega, \cdot) \in C^{1-\alpha}([0,T];\mathbb{R}).$$

(II) Moreover, if $\alpha \in (1-H, \alpha_0)$, $X_0 \in L^\infty(\Omega, \mathcal{F}, \mathbb{P}; \mathbb{R})$, then the solution X^n satisfies

$$\mathbb{E}\|X^n\|_{\alpha,\infty}^p < \infty, \quad \forall p \geq 1.$$

Notice that the Theorem 3.1 implies that $\{X^n\}_{n\geq 1}$ is a.s nondecreasing. Moreover, by the Lemma 4.1 of [7], we have

$$\|X^n\|_{\alpha,\infty} \leq C_1 \exp(C_2 G^{\frac{1}{1-\alpha}}) := N^*, \tag{5}$$

where C_1 and C_2 are two constants depending only T and α, and

$$G = \frac{1}{\Gamma(1-\alpha)} \sup_{0<s<t<T} |(D_{t-}^{1-\alpha} B_{t-})(s)|$$

with

$$D_{b-}^{\alpha} f(x) = \frac{(-1)^\alpha}{\Gamma(1-\alpha)} \times \Big(\frac{f(x)}{(b-x)^\alpha} + \alpha \int_x^b \frac{f(x) - f(y)}{(y - x^{\alpha+1})} dy \Big) \mathbb{I}_{(a,b)}(x).$$

Note that σ satisfies the assumption of the Proposition 3.1 and b_n satisfies the assumption of the Proposition 3.2. Then, by the Proposition 3.1 and 3.2 and (4), there exists a constant C which is only depends on $C^{(1)}$, $C^{(3)}$ and X_0 such that

$$\|X^n\|_{1-\alpha} \leq C\|X^n\|_{\alpha,\infty} \leq CN^*.$$

It means that X^n converges to X in the space $C^\beta([0,T];\mathbb{R})$ for all $\beta < 1-\alpha$. Since $\alpha < 1/2$ and for all $0 < \varepsilon < \alpha$

$$C^{\alpha+\varepsilon}([0,T];\mathbb{R}) \subset W_0^{\alpha,\infty}([0,T];\mathbb{R}) \subset C^{\alpha-\varepsilon}([0,T];\mathbb{R}),$$

we can deduce from Ascoli-Arzela theorem and the monotonicity of the sequence $\{X^n\}_{n\geq 1}$ that X^n converges uniformly to $X \in W_0^{\alpha,\infty}([0,T];\mathbb{R})$, $\|X\|_{\alpha,\infty} \leq N$ and for all $\lambda \geq 1$,

$$\|X^n - X\|_{\alpha,\lambda} \to 0, \quad \text{as } n \to \infty. \tag{6}$$

Next, we will prove that X is the solution to the equation (1). On the one hand, we have for all $\lambda \geq 1$,

$$\begin{aligned}\Big\| \int_0^{\cdot} b_n(X_s^n)ds - \int_0^{\cdot} b(X_s)ds \Big\|_{\alpha,\lambda} \leq & \Big\| \int_0^{\cdot} b_n(X_s^n)ds - \int_0^{\cdot} b_n(X_s)ds \Big\|_{\alpha,\lambda} \\ & + \Big\| \int_0^{\cdot} b_n(X_s)ds - \int_0^{\cdot} b(X_s)ds \Big\|_{\alpha,\lambda} \\ := & I_1 + I_2.\end{aligned}$$

By the Proposition 3.2 and (6), we easily know $I_1 \to 0$, as $n \to \infty$. Since $\{X^n\}_{n\geq 1}$ converges uniformly to X, we get that $I_2 \to 0$, as $n \to \infty$ by using (i) in the Lemma 3.1. Thus, we have for all $\lambda \geq 1$,

$$\lim_{n\to\infty} \Big\| \int_0^{\cdot} b_n(X_s^n)ds - \int_0^{\cdot} b(X_s)ds \Big\|_{\alpha,\lambda} = 0. \tag{7}$$

On the other hand, note that the fBm B^H admits a version whose sample paths are almost surely Hölder continuous of order strictly less than H, then using the Proposition 3.1 we have

$$\Big\| \int_0^{\cdot} \sigma(s, X_s^n)dB_s^H - \int_0^{\cdot} \sigma(s, X_s)dB_s^H \Big\|_{\alpha,\lambda} \leq \frac{\Lambda_\alpha(B^H)C_N}{\lambda^{1-2\alpha}}(1 + \Delta(X^n) + \Delta(X))\|X^n - X\|_{\alpha,\lambda}.$$

Since $\alpha_0 = \min\{\frac{1}{2}, \beta, \frac{\delta}{1+\delta}\}$ and $\alpha \in (1 - H, \alpha_0)$, we know that $\Delta(X^n)$ and $\Delta(X)$ are bounded. Then, letting $n \longrightarrow \infty$ we have

$$\lim_{n\to\infty} \Big\| \int_0^{\cdot} \sigma(s, X_s^n)dB_s^H - \int_0^{\cdot} \sigma(s, X_s)dB_s^H \Big\|_{\alpha,\lambda} = 0. \tag{8}$$

Thus, combining (7) and (8), using the equivalence of $\|\cdot\|_{\alpha,\lambda}$ and $\|\cdot\|_{\alpha,\infty}$ we get that X satisfies (1).

Moreover, by the proof of the Theorem 5.1 of [6] we know that if $X \in W_0^{\alpha,\infty}([0,T];\mathbb{R})$ is a solution of (1), then $X \in C^{1-\alpha}([0,T];\mathbb{R})$. Hence, the proof of the first assertion is complete.

Lastly, by (5), we have for all $p \geq 1$

$$\mathbb{E}\|X^n\|_{\alpha,\infty}^p \leq C_1 \mathbb{E}\exp(pC_2 G^{\frac{1}{1-\alpha}}) < \infty \tag{9}$$

provided $\frac{1}{1-\alpha} < 2$. Noticing that $0 < \alpha < 1/2$, we obtain easily that $\frac{1}{1-\alpha} < 2$. This implies that the second assertion holds. The proof is complete. □

Remark 3.6. *In the same way as in the Theorem 3.2 and the Theorem 3.3, by replacing (3) in the Lemma 3.1 with*

$$b_n(x) = sup_{y\in\mathbb{R}}\{b(y) - n|x - y|\}, \tag{10}$$

we can also prove the results of the Theorem 3.2 when b is right-continuous and upper semi-continuous, i.e., for each $x_0 \in \mathbb{R}$,

$$\lim_{x\to x_0^+} b(x) = b(x_0), \text{ and } \liminf_{x\to x_0^-} b(x) \leq b(x_0).$$

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