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Applications of (M,N)-Lucas Polynomials for Holomorphic and Bi-Univalent Functions

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Abstract. In this article, we use the (M,N)-Lucas polynomials to define a new family $H_\Sigma(\lambda;x)$ of normalized holomorphic and bi-univalent functions and to establish the bounds for $|a_2|$ and $|a_3|$, where a_2, a_3 are the initial Taylor-Maclaurin coefficients. Further we investigate Fekete-Szegö inequality for functions in the family $H_\Sigma(\lambda;x)$ which we have introduced here.

1. Introduction

Let $\mathcal{A}$ denote the family of functions which are holomorphic in the open unit disk

$$\mathbb{D} = \{z : z \in \mathbb{C} \quad \text{and} \quad |z| < 1\}$$

and have the following normalized form:

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n. \tag{1}$$

We also denote by $\mathcal{S}$ the subclass of $\mathcal{A}$ consisting of functions which are also univalent in $\mathbb{D}$. According to the Koebe-one quarter theorem [2], every function $f \in \mathcal{S}$ has an inverse f^{-1} defined by

$$f^{-1}\left(f(z)\right) = z \qquad (z \in \mathbb{D})$$

and

$$f\left(f^{-1}(w)\right) = w \qquad \left(|w| < r_0(f);\ r_0(f) \geqq \frac{1}{4}\right),$$

where

$$f^{-1}(w) = w - a_2 w^2 + \left(2a_2^2 - a_3\right) w^3 - \left(5a_2^3 - 5a_2 a_3 + a_4\right) w^4 + \cdots . \tag{2}$$

A function $f \in \mathcal{A}$ is called bi-univalent in $\mathbb{D}$ if both f and f^{-1} are univalent in $\mathbb{D}$, We indicate by Σ the class of normalized bi-univalent functions in $\mathbb{D}$ given by (1). For a brief historical account and for several interesting examples of functions in the class Σ; see the pioneering work on this subject by Srivastava *et al.* [20], which actually revived the study of bi-univalent functions in recent years. From the work of Srivastava *et al.* [20], we choose to recall the following examples of functions in the class Σ :

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$$\frac{z}{1-z}, \quad -\log(1-z) \qquad \text{and} \qquad \frac{1}{2}\log\left(\frac{1+z}{1-z}\right).$$

We notice that the class Σ is not empty. However, the Koebe function is not a member of Σ.

In a considerably large number of sequels to the aforementioned work of Srivastava *et al.* [20], several different subclasses of the bi-univalent function class Σ were introduced and studied analogously by the many authors (see, for example, [1, 5, 6, 9–16, 18, 21, 23, 24]), but only non-sharp estimates on the initial coefficients $|a_2|$ and $|a_3|$ in the Taylor Maclaurin expansion (1) were obtained in several recent papers. The problem to find the general coefficient bounds on the Taylor-Maclaurin coefficients

$$|a_n| \qquad (n \in \mathbb{N};\ n \geqq 3)$$

for functions $f \in \Sigma$ is still not completely addressed for many of the subclasses of the bi-univalent function class Σ (see, for example, [14, 21, 23]). The Fekete-Szegö functional $\left|a_3 - \delta a_2^2\right|$ for $f \in \mathcal{S}$ is well known for its rich history in the field of Geometric Function Theory. Its origin was in the disproof by Fekete and Szegö [3] of the Littlewood-Paley conjecture that the coefficients of odd univalent functions are bounded by unity. The functional has since received great attention, particularly in the study of many subclasses of the family of univalent functions. This topic has become of considerable interest among researchers in Geometric Function Theory (see, for example, [17, 19, 22]).

Let the functions f and g be analytic in $\mathbb{D}$, we say that the function f is subordinate to g, if there exists a Schwarz function ω, which is analytic in $\mathbb{D}$ with

$$\omega(0) = 0 \qquad \text{and} \qquad |\omega(z)| < 1 \quad (z \in \mathbb{D}),$$

such that

$$f(z) = g\big(\omega(z)\big).$$

This subordination is indicated by

$$f \prec g \qquad \text{or} \qquad f(z) \prec g(z) \quad (z \in \mathbb{D}).$$

The Lucas polynomials plays an important role in a variety of disciplines in the mathematical, statistical, physical and engineering sciences (see, for example [4, 8, 25]).

For the polynomials $M(x)$ and $N(x)$ with real coefficients, Lee and Aşcı [7] considered the (M,N)-Lucas polynomials $L_{M,N,k}(x)$, which are given by the following recurrence relation:

$$L_{M,N,k}(x) = M(x)L_{M,N,k-1}(x) + N(x)L_{M,N,k-2}(x) \quad (k \geqq 2),$$

with

$$L_{M,N,0}(x) = 2, \quad L_{M,N,1}(x) = M(x) \quad \textit{and} \quad L_{M,N,2}(x) = M^2(x) + 2N(x). \tag{3}$$

The generating function of the (M,N)-Lucas polynomial $L_{M,N,k}(x)$ (see [7]) is given by

$$T_{\{L_{M,N,k}(x)\}}(z) = \sum_{k=0}^{\infty} L_{M,N,k}(x)z^k = \frac{2 - M(x)z}{1 - M(x)z - N(x)z^2}.$$

2. Main Results

We begin this section by defining the new class $H_\Sigma(\lambda; x)$ as follows:

Definition 2.1. For $0 \leqq \lambda \leqq 1$, a function $f \in \Sigma$ is called in the class $H_{\Sigma}(\lambda; x)$ if it fulfills the conditions:

$$1 + \frac{zf'(z)}{f(z)} + \frac{zf''(z)}{f'(z)} - \frac{\lambda z^2 f''(z) + zf'(z)}{\lambda zf'(z) + (1-\lambda)f(z)} \prec T_{\{L_{M,N,k}(x)\}}(z) - 1$$

and

$$1 + \frac{w\left(f^{-1}(w)\right)'}{f^{-1}(w)} + \frac{w\left(f^{-1}(w)\right)''}{\left(f^{-1}(w)\right)'} - \frac{\lambda w^2\left(f^{-1}(w)\right)'' + w\left(f^{-1}(w)\right)'}{\lambda w\left(f^{-1}(w)\right)' + (1-\lambda)f^{-1}(w)} \prec T_{\{L_{M,N,k}(x)\}}(w) - 1,$$

where f^{-1} is given by (2).

Example 2.2. For $\lambda = 1$, a function $f \in \Sigma$ is called in the class $H_{\Sigma}(1; x) =: S_{\Sigma}(x)$ if it fulfills the conditions:

$$\frac{zf'(z)}{f(z)} \prec T_{\{L_{M,N,k}(x)\}}(z) - 1$$

and

$$\frac{w\left(f^{-1}(w)\right)'}{f^{-1}(w)} \prec T_{\{L_{M,N,k}(x)\}}(w) - 1,$$

where f^{-1} is given by (2).

Example 2.3. For $\lambda = 0$, a function $f \in \Sigma$ is called in the class $H_{\Sigma}(0; x) =: C_{\Sigma}(x)$ if it fulfills the conditions:

$$1 + \frac{zf''(z)}{f'(z)} \prec T_{\{L_{M,N,k}(x)\}}(z) - 1$$

and

$$1 + \frac{w\left(f^{-1}(w)\right)''}{\left(f^{-1}(w)\right)'} \prec T_{\{L_{M,N,k}(x)\}}(w) - 1,$$

where f^{-1} is given by(2).

Our first main result is asserted by Theorem 2.4 below.

Theorem 2.4. *For $0 \leqq \lambda \leqq 1$, let $f \in \mathcal{A}$ be in the class $H_{\Sigma}(\lambda; x)$. Then*

$$|a_2| \leqq \frac{|M(x)|\sqrt{|M(x)|}}{\sqrt{2\left|(\lambda-1)M^2(x) - (2-\lambda)^2 N(x)\right|}}$$

and

$$|a_3| \leqq \frac{M^2(x)}{(2-\lambda)^2} + \frac{|M(x)|}{2(3-2\lambda)}.$$

Proof. Suppose that $f \in H_{\Sigma}(\lambda; x)$. Then there are two analytic functions $\phi, \psi : \mathbb{D} \longrightarrow \mathbb{D}$ given by

$$\phi(z) = r_1 z + r_2 z^2 + r_3 z^3 + \cdots \quad (z \in \mathbb{D}) \tag{4}$$

and

$$\psi(w) = s_1 w + s_2 w^2 + s_3 w^3 + \cdots \quad (w \in \mathbb{D}), \tag{5}$$

with

$$\phi(0) = \psi(0) = 0 \qquad \text{and} \qquad \max\left\{\left|\phi(z)\right|, \left|\psi(w)\right|\right\} < 1 \quad (z, w \in \mathbb{D}),$$

such that

$$1+\frac{zf'(z)}{f(z)}+\frac{zf''(z)}{f'(z)}-\frac{\lambda z^2 f''(z)+zf'(z)}{\lambda zf'(z)+(1-\lambda)f(z)}=-1+L_{M,N,0}(x)+L_{M,N,1}(x)\phi(z)+L_{M,N,2}(x)\phi^2(z)+\cdots \tag{6}$$

and

$$\begin{aligned}&1+\frac{w\left(f^{-1}(w)\right)'}{f^{-1}(w)}+\frac{w\left(f^{-1}(w)\right)''}{(f^{-1}(w))'}-\frac{\lambda w^2\left(f^{-1}(w)\right)''+w\left(f^{-1}(w)\right)'}{\lambda w\,(f^{-1}(w))'+(1-\lambda)f^{-1}(w)}\\&=-1+L_{M,N,0}(x)+L_{M,N,1}(x)\psi(w)+L_{M,N,2}(x)\psi^2(w)+\cdots.\end{aligned} \tag{7}$$

Combining (4), (5), (6) and (7), yield

$$1+\frac{zf'(z)}{f(z)}+\frac{zf''(z)}{f'(z)}-\frac{\lambda z^2 f''(z)+zf'(z)}{\lambda zf'(z)+(1-\lambda)f(z)}=1+L_{M,N,1}(x)r_1z+\left[L_{M,N,1}(x)r_2+L_{M,N,2}(x)r_1^2\right]z^2+\cdots \tag{8}$$

and

$$\begin{aligned}&1+\frac{w\left(f^{-1}(w)\right)'}{f^{-1}(w)}+\frac{w\left(f^{-1}(w)\right)''}{(f^{-1}(w))'}-\frac{\lambda w^2\left(f^{-1}(w)\right)''+w\left(f^{-1}(w)\right)'}{\lambda w\,(f^{-1}(w))'+(1-\lambda)f^{-1}(w)}\\&=1+L_{M,N,1}(x)s_1w+\left[L_{M,N,1}(x)s_2+L_{M,N,2}(x)s_1^2\right]w^2+\cdots.\end{aligned} \tag{9}$$

It is well-known that, if

$$\max\left\{\left|\phi(z)\right|,\left|\psi(w)\right|\right\}<1 \qquad (z,w\in\mathbb{D}),$$

then

$$\left|r_j\right|\leqq 1 \qquad \text{and} \qquad \left|s_j\right|\leqq 1 \qquad (\forall\ j\in\mathbb{N}). \tag{10}$$

Now, by comparing the corresponding coefficients in (8) and (9), and after simplifying, we find that

$$(2-\lambda)a_2=L_{M,N,1}(x)r_1, \tag{11}$$

$$2(3-2\lambda)a_3-\left(5-(\lambda+1)^2\right)a_2^2=L_{M,N,1}(x)r_2+L_{M,N,2}(x)r_1^2, \tag{12}$$

$$(\lambda-2)a_2=L_{M,N,1}(x)s_1 \tag{13}$$

and

$$\left(7-8\lambda+(\lambda+1)^2\right)a_2^2-2(3-2\lambda)a_3=L_{M,N,1}(x)s_2+L_{M,N,2}(x)s_1^2. \tag{14}$$

It follows from (11) and (13) that

$$r_1=-s_1 \tag{15}$$

and

$$2\,(2-\lambda)^2\,a_2^2=L^2_{M,N,1}(x)(r_1^2+s_1^2). \tag{16}$$

If we add (12) to (14), we obtain

$$2\left(1+(\lambda-1)^2\right)a_2^2=L_{M,N,1}(x)(r_2+s_2)+L_{M,N,2}(x)(r_1^2+s_1^2). \tag{17}$$

Substituting the value of $r_1^2+s_1^2$ from (16) in the right hand side of (17), we deduce that

$$2\left[1+(\lambda-1)^2-\frac{L_{M,N,2}(x)}{L^2_{M,N,1}(x)}\,(2-\lambda)^2\right]a_2^2=L_{M,N,1}(x)(r_2+s_2). \tag{18}$$

Moreover computations using (3), (10) and (18), we find that

$$|a_2| \leqq \frac{|M(x)|\sqrt{|M(x)|}}{\sqrt{2\left|(\lambda-1)\,M^2(x)-(2-\lambda)^2\,N(x)\right|}}.$$

Next, if we subtract (14) from (12), we can easily see that

$$4(3-2\lambda)(a_3-a_2^2)=L_{M,N,1}(x)(r_2-s_2)+L_{M,N,2}(x)(r_1^2-s_1^2). \tag{19}$$

In view of (15) and (16), we get from (19)

$$a_3=\frac{L^2_{M,N,1}(x)}{2\,(2-\lambda)^2}(r_1^2+s_1^2)+\frac{L_{M,N,1}(x)}{4(3-2\lambda)}(r_2-s_2).$$

Thus applying (3), we conclude that

$$|a_3| \leqq \frac{M^2(x)}{(2-\lambda)^2}+\frac{|M(x)|}{2(3-2\lambda)}.$$

□

Putting $\lambda = 1$ in Theorem 2.4, we obtain the following result:

Corollary 2.5. *If $f \in \mathcal{A}$ be in the class $S_\Sigma(x)$, then*

$$|a_2| \leqq |M(x)|\sqrt{\left|\frac{M(x)}{2N(x)}\right|}$$

and

$$|a_3| \leqq M^2(x)+\frac{|M(x)|}{2}.$$

Putting $\lambda = 0$ in Theorem 2.4, we obtain the following result:

Corollary 2.6. *If $f \in \mathcal{A}$ be in the class $C_\Sigma(x)$, then*

$$|a_2| \leqq \frac{|M(x)|\sqrt{|M(x)|}}{\sqrt{2\left|M^2(x)+4N(x)\right|}}$$

and

$$|a_3| \leqq \frac{M^2(x)}{4}+\frac{|M(x)|}{6}.$$

In the next theorem, we present the "Fekete-Szegö inequality" for $f \in H_\Sigma(\lambda; x)$.

Theorem 2.7. *For $0 \leqq \lambda \leqq 1$ and $\delta \in \mathbb{R}$, let $f \in \mathcal{A}$ be in the class $H_\Sigma(\lambda; x)$. Then*

$$\left|a_3-\delta a_2^2\right| \leqq \begin{cases} \frac{|M(x)|}{2(3-2\lambda)} \\ \left(|\delta-1| \leqq \frac{1}{3-2\lambda}\left|\lambda-1-\frac{(2-\lambda)^2N(x)}{M^2(x)}\right|\right) \\ \frac{|M(x)|^3|\delta-1|}{2\left|(\lambda-1)M^2(x)-(2-\lambda)^2N(x)\right|} \\ \left(|\delta-1| \geqq \frac{1}{3-2\lambda}\left|\lambda-1-\frac{(2-\lambda)^2N(x)}{M^2(x)}\right|\right). \end{cases}$$

Proof. By making use of (18) and (19), we conclude that

$$a_3 - \delta a_2^2 = (1-\delta)\frac{L_{M,N,1}^3(x)(r_2+s_2)}{2\left[\left((\lambda-1)^2+1\right)L_{M,N,1}^2(x)-(2-\lambda)^2 L_{M,N,2}(x)\right]} + \frac{L_{M,N,1}(x)(r_2-s_2)}{4(3-2\lambda)}$$
$$= L_{M,N,1}(x)\left[\left(\varphi(\delta;x)+\frac{1}{4(3-2\lambda)}\right)r_2 + \left(\varphi(\delta;x)-\frac{1}{4(3-2\lambda)}\right)s_2\right],$$

where

$$\varphi(\delta;x) = \frac{L_{M,N,1}^2(x)(1-\delta)}{2\left[\left((\lambda-1)^2+1\right)L_{M,N,1}^2(x)-(2-\lambda)^2 L_{M,N,2}(x)\right]}.$$

Thus, according to (3), we find that

$$\left|a_3-\delta a_2^2\right| \leqq \begin{cases} \frac{|M(x)|}{2(3-2\lambda)} \\ \left(0 \leqq \left|\varphi(\delta;x)\right| \leqq \frac{1}{4(3-2\lambda)}\right) \\ 2\,|M(x)|\,.\,\left|\varphi(\delta;x)\right| \\ \left(\left|\varphi(\delta;x)\right| \geqq \frac{1}{4(3-2\lambda)}\right), \end{cases}$$

which, after some computations, yields

$$\left|a_3-\delta a_2^2\right| \leqq \begin{cases} \frac{|M(x)|}{2(3-2\lambda)} \\ \left(|\delta-1| \leqq \frac{1}{3-2\lambda}\left|\lambda-1-\frac{(2-\lambda)^2 N(x)}{M^2(x)}\right|\right) \\ \frac{|M(x)|^3|\delta-1|}{2\left|(\lambda-1)M^2(x)-(2-\lambda)^2N(x)\right|} \\ \left(|\delta-1| \geqq \frac{1}{3-2\lambda}\left|\lambda-1-\frac{(2-\lambda)^2 N(x)}{M^2(x)}\right|\right). \end{cases}$$

□

Putting $\lambda = 1$ in Theorem 2.7, we obtain the following result:

Corollary 2.8. *If $f \in \mathcal{A}$ be in the class $S_\Sigma(x)$, then*

$$\left|a_3-\delta a_2^2\right| \leqq \begin{cases} \frac{|M(x)|}{2} \\ \left(|\delta-1| \leqq \frac{|N(x)|}{M^2(x)}\right) \\ \frac{|M(x)|^3|\delta-1|}{2|N(x)|} \\ \left(|\delta-1| \geqq \frac{|N(x)|}{M^2(x)}\right). \end{cases}$$

Putting $\lambda = 0$ in Theorem 2.7, we obtain the following result:

Corollary 2.9. *If $f \in \mathcal{A}$ be in the class $C_\Sigma(x)$, then*

$$\left|a_3 - \delta a_2^2\right| \leqq \begin{cases} \frac{|M(x)|}{6} \\ \left(|\delta - 1| \leqq \frac{1}{3}\left|1 + \frac{4N(x)}{M^2(x)}\right|\right) \\ \frac{|M(x)|^3|\delta-1|}{2\left|M^2(x)+4N(x)\right|} \\ \left(|\delta - 1| \geqq \frac{1}{3}\left|1 + \frac{4N(x)}{M^2(x)}\right|\right). \end{cases}$$

Putting $\delta = 1$ in Theorem 2.7, we obtain the following result:

Corollary 2.10. *If $f \in \mathcal{A}$ be in the class $H_\Sigma(\lambda; x)$, then*

$$\left|a_3 - a_2^2\right| \leqq \frac{|M(x)|}{2(3-2\lambda)}.$$

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