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Distances from B^{α} Functions to $F(p, q, s)$ Space

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Abstract. In this paper, we consider several equivalent formulas for the distances from B^{α} functions to $F(p, q, s)$ space.

1. The first section

Let $\Delta = \{z \in \mathbb{C} : |z| < 1\}$ be the unit disk and S^1 be its boundary. Denote by $H(\Delta)$ the space of all analytic functions on Δ.

For $0 < p < \infty$, $-2 < q < \infty$, $0 < s < \infty$, $F(p, q, s)$ is the space of all functions $f \in H(\Delta)$ satisfying

$$\|f\|_{F_{p,q,s}} = \Big(\sup_{a\in\Delta} \iint_{\Delta} |f'(z)|^p (1-|z|^2)^q (1-|\sigma_a(z)|^2)^s dxdy\Big)^{1/p} < \infty,$$

where $\sigma_a(z) = \frac{a-z}{1-\bar{a}z}$ is a Möbius transformation of Δ mapping a to 0. $F_0(p, q, s)$ is the subspace of $F(p, q, s)$ such that

$$\lim_{|a|\to 1} \iint_{\Delta} |f'(z)|^p (1-|z|^2)^q (1-|\sigma_a(z)|^2)^s dxdy = 0.$$

$F(p, q, s)$ was introduced by Zhao[11] and it is trivial if $q + s \le -1$. As we known, $F(p, q, s)$ is a Banach space under the following norm

$$\|f\|^*_{F_{p,q,s}} = |f(0)| + \|f\|_{F_{p,q,s}}.$$

It is proved that $F(p, q, s)$ and $F_0(p, q, s)$ are respectively contained in the Bloch-type space $B^{\frac{q+2}{p}}$ and $B_0^{\frac{q+2}{p}}$. Here B^{α} $(\alpha > 0)$ is the space of all functions $f \in H(\Delta)$ with

$$\|f\|_{B^{\alpha}} = \sup_{z\in\Delta} (1-|z|^2)^{\alpha} |f'(z)| < \infty.$$

And B_0^{α} consists of all functions $f \in B^{\alpha}$ such that

$$\lim_{|z|\to 1} (1-|z|^2)^{\alpha} |f'(z)| = 0.$$

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It is well known that B^1 is classical Bloch space $\mathcal{B}$ and B^α is also a Banach space if it is equipped with the following norm

$$\|f\|^*_{B^\alpha} = |f(0)| + \|f\|_{B^\alpha}.$$

Moreover B^α_0 is the closure of polynomials in B^α.

It is clear that $F(p, q, s)$ contains lots of the special function spaces. For example, $F(2, 0, 1) = BMOA$, $F(2, 0, s) = Q_s$ ($s > 1$) and $F(p, p-2, s) = \mathcal{B}$ if $0 < p < \infty$, $s > 1$ (in detail see [1–3, 11]).

Suppose $X \subset \mathcal{B}$ is an analytic function space. The distance from a Bloch function f to X is defined as follows:

$$dist_{\mathcal{B}}(f, X) = \inf_{g\in X} \|f - g\|^*_{\mathcal{B}}.$$

In Ghatage-Zheng[6], Xu[9] and Zhao[10], they studied the distances from $BMOA$ and $F(p, p-2, s)$ to the Bloch space, separately in the following theorems.

For $f \in \mathcal{B}$ and $\epsilon > 0$, set

$$\Omega_\epsilon(f) = \{z \in \Delta : |f'(z)|(1 - |z|^2) \geq \epsilon\}.$$

Theorem A. [6] *Suppose $f \in \mathcal{B}$. The following quantities are equivalent:*

1. $dist_{\mathcal{B}}(f, BMOA)$;
2. $inf\{\epsilon : \frac{\chi_{\Omega_\epsilon(f)}(z)}{1-|z|^2} dxdy$ *is a Carleson measure*$\}$, *where χ is the characteristic function.*

Theorem B. [10] *Suppose $2 \leq p < \infty$, $0 \leq q < \infty$, $0 \leq s \leq 1$, and $f \in \mathcal{B}$. The following quantities are equivalent:*

1. $dist_{\mathcal{B}}(f, F(p, p-2, s))$;
2. $inf\{\epsilon : \frac{\chi_{\Omega_\epsilon(f)}(z)}{(1-|z|^2)^{2-s}} dxdy$ *is an s-Carleson measure*$\}$;
3. $inf\{\epsilon : \sup_{a\in\Delta} \iint_{\Omega_\epsilon(f)} |f'(z)|^q (1 - |z|^2)^{q-2} (1 - |\sigma_a(z)|^2)^s dxdy < \infty\}$;
4. $inf\{\epsilon : \sup_{a\in\Delta} \iint_{\Omega_\epsilon(f)} |f'(z)|^q (1 - |z|^2)^{q-2} g^s(z, a) dxdy < \infty\}$, *where $g(z, a) = log \frac{1}{|\sigma_a(z)|}$ is the Green function of Δ with the pole at a.*

In this paper, we aim to extend the results about the distance from $F(p, q, s)$ to B^α in Section 3.

This paper is organized as follows. Some relevant notations and important results are given in section 2. In section 3, we prove some equivalent quantities of the distance from $F(p, q, s)$ to B^α.

Through this paper $f \approx g$ always means $f \lesssim g \lesssim f$, where $f \lesssim g$ means that there is a constant $C > 0$, independent of functions f and g, such that $f \leq Cg$.

2. Preliminaries

Given an arc I on S^1, the Carleson box $S(I)$ on Δ is defined as

$$S(I) = \{z \in \Delta : 1 - |I| \leq |z| \leq 1, z/|z| \in I\},$$

where $|I|$ is the Lebesgue measure of I. A positive measure λ on Δ is called an s-Carleson measure ($s > 0$) if

$$\|\lambda\|_{C,s} = \sup_{I\subset S^1} \frac{\lambda(S(I))}{|I|^s} < \infty,$$

and a compact s-Carleson measure in addition if

$$\lim_{|I|\to 0} \frac{\lambda(S(I))}{|I|^s} = 0.$$

Obviously, 1-Carleson measure is the classical Carleson measure (see [4]). Denoted by $CM_s(\Delta)$ (or $CM_{s,0}(\Delta)$) the set of all (compact) s-Carleson measures on Δ.

Lemma A. [5] *Let $\alpha > 0, \beta > 0$ and $s < 1 + \frac{\alpha}{2}$. For a positive measure λ on Δ, set*

$$\widetilde{\lambda}(z) = \iint_{\Delta} \frac{(1-|z|^2)^{\alpha}(1-|w|^2)^{\beta}}{|1-\bar{z}w|^{\alpha+\beta+2}} \lambda(w)dudv.$$

If $\lambda \in CM_s(\Delta)$, then $\widetilde{\lambda} \in CM_s(\Delta)$ and there exists a constant $C > 0$ such that $\|\widetilde{\lambda}\|_{C,s} \leq C\|\lambda\|_{C,s}$, while $\widetilde{\lambda} \in CM_{s,0}(\Delta)$ if $\lambda \in CM_{s,0}(\Delta)$.

Lemma B. [8] *Let $s > 0$, a positive measure λ on Δ is an s-Carleson measure if and only if*

$$\sup_{a\in\Delta} \iint_{\Delta} \Big(\frac{1-|a|^2}{|1-\bar{a}z|^2}\Big)^s \lambda(z)dxdy < \infty, \tag{1}$$

and is a compact s-Carleson measure if and only if

$$\lim_{|a|\to 1} \iint_{\Delta} \Big(\frac{1-|a|^2}{|1-\bar{a}z|^2}\Big)^s \lambda(z)dxdy = 0. \tag{2}$$

Lemma C. [10] *Suppose that $k > -1$, $r, t > 0$, and $r + t - k > 2$. If $t < k + 2 < r$, then there exists a universal constant $C > 0$, such that for all $z, \zeta \in \Delta$,*

$$\iint_{\Delta} \frac{(1-|w|^2)^k}{|1-\overline{w}z|^r|1-\overline{w}\zeta|^t} dudv \leq C\frac{(1-|z|^2)^{2+k-r}}{|1-\overline{\zeta}z|^t}.$$

3. Main Results

Suppose $X \subset B^{\alpha}$ is an analytic function space. The distance from a function $f \in B^{\alpha}$ to X is defined as follows:

$$dist_{B^{\alpha}}(f, X) = \inf_{g\in X} \|f-g\|^*_{B^{\alpha}}.$$

For $f \in B^{\alpha}$ and $\epsilon > 0$, set

$$\Omega_{\epsilon}(f) = \{z \in \Delta : |f'(z)|(1-|z|^2)^{\alpha} \geq \epsilon\}.$$

The idea of establishing equivalent forms is from Lou-Chen[7] and we state it in detail as follows. The distance formula between a function $f \in B^{\alpha}$ and the subspace $X \in B^{\alpha}$ is to decompose f properly into two parts so that one part is in the space X and the α-Bloch norm of the other part is equivalent to the distance $dist_{B^{\alpha}}(f, X)$. It is expected that such a decomposition is nonlinear and is not unique.

For $f \in B^{\alpha}$, it is easy to get the following formula (see Lemma 4.2.8 in [12])

$$f(z) = f(0) + \iint_{\Delta} \frac{f'(w)(1-|w|^2)}{(1-z\overline{w})^2\overline{w}} dudv \quad z \in \Delta.$$

Let

$$E_{\epsilon}(f)(z) = \iint_{\Delta\setminus\Omega_{\epsilon}(f)} \frac{f'(w)(1-|w|^2)}{(1-z\overline{w})^2\overline{w}} dudv + C_{\epsilon,f}$$

and

$$P_{\epsilon}(f)(z) = f(z) - E_{\epsilon}(f)(z),$$

where $C_{\epsilon,f}$ is a constant such that $E_{\epsilon}(f)(0) = 0$. It is clear that

$$P_{\epsilon}(f)(z) = f(0) - C_{\epsilon,f} + \iint_{\Omega_{\epsilon}(f)} \frac{f'(w)(1-|w|^2)}{(1-z\overline{w})^2\overline{w}} dudv, \quad z \in \Delta. \tag{3}$$

Lemma 1. *If* $f \in B^\alpha$ *and* $0 < \alpha < 2$. *Then* $P_\epsilon(f) \in B^\alpha$.

Proof. From (3) and $f \in B^\alpha$, we have

$$|P'_\epsilon(f)| = |\iint_{\Omega_\epsilon(f)} \frac{2f'(w)(1-|w|^2)}{(1-z\overline{w})^3} dudv| \le 2\|f\|_{B^\alpha} \iint_\Delta \frac{(1-|w|^2)^{1-\alpha}}{|1-z\overline{w}|^3} dudv. \tag{4}$$

By Lemma 4.2.2 in [12], we get

$$\iint_\Delta \frac{(1-|w|^2)^{1-\alpha}}{|1-z\overline{w}|^3} dudv \approx \frac{1}{(1-|z|^2)^\alpha}.$$

Hence $|P'_\epsilon(f)|(1-|z|^2)^\alpha \le C\|f\|_{B^\alpha}$, which implies $P_\epsilon(f) \in B^\alpha$. □

Theorem 1. *Suppose* $0 < p < \infty$, $-2 < q < \infty$, $0 < s < \infty$ *and* $0 < s < \alpha = \frac{q+2}{p} < 2$. *If* $f \in B^\alpha$ *and* $1 < \alpha + s < 3$. *Then the following quantities are equivalent:*

1. $dist_{B^\alpha}(f, F(p,q,s))$;
2. $inf\{\epsilon : \frac{\chi_{\Omega_\epsilon(f)}(z)}{(1-|z|^2)^{2-s}} dxdy$ *is an s-Carleson measure*$\}$;
3. $inf\{\epsilon : \sup_{a\in\Delta} \iint_{\Omega_\epsilon(f)} |f'(z)|^p(1-|z|^2)^q g^s(z,a)dxdy < \infty\}$;
4. $inf\{\epsilon : \sup_{a\in\Delta} \iint_{\Omega_\epsilon(f)} |f'(z)|^p(1-|z|^2)^q(1-|\sigma_a(z)|^2)^s dxdy < \infty\}$.

Proof. Let d_1, d_2, d_3 and d_4 be the quantities of (1), (2), (3) and (4) in Theorem 1, respectively. We would show that $d_1 \approx d_2$, $d_2 = d_4$ and $d_3 \approx d_4$ by three parts.

Part 1: (1) To prove $d_1 \le Cd_2$.

We firstly prove that $P_\epsilon(f) \in F(p,q,s)$ if $\epsilon > d_2$.

Note that

$$1 - |\sigma_a(z)|^2 = \frac{(1-|a|^2)(1-|z|^2)}{|1-\overline{a}z|^2}. \tag{5}$$

By Lemma 1, (4) and (5), we have

$$\begin{aligned} L &:= \sup_{a\in\Delta} \iint_\Delta |P'_\epsilon(f)|^p(1-|z|^2)^q(1-|\sigma_a(z)|^2)^s dxdy \\ &\le \|P_\epsilon(f)\|_{B^\alpha}^{p-1} \sup_{a\in\Delta} \iint_\Delta \Big(\iint_{\Omega_\epsilon(f)} \tfrac{2|f'(w)|(1-|w|^2)}{|1-z\overline{w}|^3} dudv\Big) \times (1-|z|^2)^{\alpha-2}(1-|\sigma_a(z)|^2)^s dxdy \\ &\le C\|f\|_{B^\alpha}^p \sup_{a\in\Delta} \iint_{\Omega_\epsilon(f)} (1-|w|^2)^{1-\alpha}(1-|a|^2)^s \times (\iint_\Delta \tfrac{(1-|z|^2)^{\alpha-2+s}}{|1-z\overline{w}|^3|1-z\overline{a}|^{2s}} dxdy) dudv \end{aligned}$$

From Lemma C, we get

$$\iint_\Delta \frac{(1-|z|^2)^{\alpha-2+s}}{|1-z\overline{w}|^3|1-z\overline{a}|^{2s}} dxdy \le C\frac{(1-|w|^2)^{\alpha+s-3}}{|1-w\overline{a}|^{2s}} \tag{6}$$

From (6), we know

$$L \le C\|f\|_{B^\alpha}^p \sup_{a\in\Delta} \iint_{\Omega_\epsilon(f)} \frac{(1-|w|^2)^{s-2}(1-|a|^2)^s}{|1-w\overline{a}|^{2s}} dudv$$

Since $\frac{\chi_{\Omega_\epsilon(f)}(w)}{(1-|w|^2)^{2-s}} dxdy$ is an s-Carleson measure, by Lemma B, we know L is finite. So $P_\epsilon(f) \in F(p,q,s)$ if $\epsilon > d_2$.

Since $E_\epsilon(f)(0) = 0$, we have

$$d_1 = dist_{B^\alpha}(f, F(p,q,s))) \le \|f - P_\epsilon(f)\|_{B^\alpha}^* = \|E_\epsilon(f)\|_{B^\alpha}^* = \|E_\epsilon(f)\|_{B^\alpha}$$

To conclude $d_1 \le Cd_2$, we left to show that $\|E_\epsilon(f)\|_{B^\alpha} \le C\epsilon$ if $\epsilon > d_2$.

Since $|f'(z)|(1-|z|^2)^\alpha \geq \epsilon$, by Lemma 4.2.2 in [12], we have

$$|E_\epsilon(f)'(z)| = |\iint_{\Delta\setminus\Omega_\epsilon(f)} \frac{2f'(w)(1-|w|^2)}{(1-z\overline{w})^3}dudv| \leq 2\epsilon \iint_{\Delta} \frac{(1-|w|^2)^{1-\alpha}}{|1-z\overline{w}|^3}dudv \approx \frac{2\epsilon}{(1-|z|^2)^\alpha}$$

By $\|E_\epsilon(f)\|_{B^\alpha} \leq C\epsilon$ and the definition of d_2, we have $d_1 \lesssim d_2$.

(2) We now prove $d_2 \leq d_1$ by contradiction.

Indeed, suppose that $d_1 < d_2$, then there exists $0 < \epsilon_1 < \epsilon$ and $f_{\epsilon_1} \in F(p,q,s)$ such that $\frac{X_{\Omega_\epsilon(f)}(z)}{(1-|z|^2)^{2-s}}dxdy$ is not an s-Carleson measure and $\|f - f_{\epsilon_1}\|^*_{B^\alpha} \leq \epsilon_1$.

For $z \in \Delta$, we have

$$|f'(z)|(1-|z|^2)^\alpha \leq |f'_{\epsilon_1}(z)|(1-|z|^2)^\alpha + \|f - f_{\epsilon_1}\|_{B^\alpha} \leq |f'_{\epsilon_1}(z)|(1-|z|^2)^\alpha + \epsilon_1$$

This means $\Omega_\epsilon(f) \subset \Omega_{\epsilon-\epsilon_1}(f_{\epsilon_1})$ and

$$X_{\Omega_\epsilon(f)}(z) \leq \frac{|f'_{\epsilon_1}(z)|(1-|z|^2)^\alpha}{\epsilon - \epsilon_1},$$

which implies

$$\frac{X_{\Omega_\epsilon(f)}(z)}{(1-|z|^2)^{2-s}} \leq \frac{|f'_{\epsilon_1}(z)|^p(1-|z|^2)^{q+s}}{(\epsilon-\epsilon_1)^p}.$$

Note that $|f'_{\epsilon_1}(z)|^p(1-|z|^2)^{q+s}dxdy$ is an s-Carleson measure, by Lemma B and (5), which implies that $\frac{X_{\Omega_\epsilon(f)}(z)}{(1-|z|^2)^{2-s}}dxdy$ is also an s-Carleson measure. This is a contradiction. Hence $d_2 \leq d_1$. Moreover $d_1 \approx d_2$.

Part 2: (1) To prove $d_4 \leq d_2$, we start with the assumption that $\frac{X_{\Omega_\epsilon(f)}(z)}{(1-|z|^2)^{2-s}}dxdy$ is an s-Carleson measure. By Lemma B and 5, we know that

$$\sup_{a\in\Delta} \iint_{\Omega_\epsilon(f)} \frac{(1-|\sigma_a(z)|^2)^s}{(1-|z|^2)^2} < \infty.$$

Hence we have

$$\sup_{a\in\Delta} \iint_{\Omega_\epsilon(f)} |f'(z)|^p(1-|z|^2)^q(1-|\sigma_a(z)|^2)^s dxdy \leq \|f\|^p_{B^\alpha} \sup_{a\in\Delta} \iint_{\Omega_\epsilon(f)} \frac{(1-|\sigma_a(z)|^2)^s}{(1-|z|^2)^2} < \infty.$$

(2) $d_2 \leq d_4$ is obvious from the following estimate and Lemma B. Because $|f'(z)|(1-|z|^2)^\alpha \geq \epsilon$, we have

$$\sup_{a\in\Delta} \iint_{\Omega_\epsilon(f)} \frac{(1-|\sigma_a(z)|^2)^s}{(1-|z|^2)^2}dxdy \leq (\frac{1}{\epsilon})^p \sup_{a\in\Delta} \iint_{\Omega_\epsilon(f)} |f'(z)|^p(1-|z|^2)^q(1-|\sigma_a(z)|^2)^s dxdy.$$

Part 3: Since $1-|z|^2 \leq 2log\frac{1}{|z|}$ for $z \in \Delta$, we have $d_4 \leq Cd_3$. We only need to prove $d_3 \leq Cd_4$. Set

$$I = \iint_{\Omega_\epsilon(f)} |f'(z)|^p(1-|z|^2)^q g^s(z,a)dxdy := I_1 + I_2,$$

where $\Delta(a,r) = \{z \in \Delta : |z-a| < r\}$ and

$$I_1 = \iint_{\Omega_\epsilon(f)\cap\Delta(0,\frac{1}{4})} |f'(z)|^p(1-|z|^2)^q g^s(z,a)dxdy,$$

$$I_2 = \iint_{\Omega_\epsilon(f)\setminus\Delta(0,\frac{1}{4})} |f'(z)|^p(1-|z|^2)^q g^s(z,a)dxdy.$$

By the following inequality

$$g(z,a) = log\frac{1}{|\sigma_a(z)|}\begin{cases} \geq log4 \geq 1, & |\sigma_a(z)| \leq \frac{1}{4}; \\ \leq 4(1-|\sigma_a(z)|^2), & |\sigma_a(z)| \geq \frac{1}{4}. \end{cases}$$

Hence

$$I_2 \le 4 \iint_{\Omega_\epsilon(f)} |f'(z)|^p(1-|z|^2)^q g^s(z,a)dxdy$$

and

$$I_1 \le \iint_{\Omega_\epsilon(f)} |f'(z)|^p(1-|z|^2)^q g^s(z,a)dxdy \le \|f\|^p_{B^\alpha} \iint_{\Omega_\epsilon(f)} (1-|z|^2)^{-2} g^2(z,a)dxdy \le C,$$

where C is a constant number independent of a. Therefore $d_3 \le Cd_4$. □

Corollary 1. *Suppose* $0 < p < \infty$, $-2 < q < \infty$, $0 < s < \infty$ *and* $0 < s < \alpha = \frac{q+2}{p} < 2$. *If* $f \in H(\Delta)$ *and* $1 < \alpha + s < 3$. *Then the following conditions are equivalent:*

1. f *is in the closure of* $F(p,q,s)$ *in* B^α*;*
2. $\frac{\chi_{\Omega_\epsilon(f)}(z)}{(1-|z|^2)^{2-s}}dxdy$ *is an s-Carleson measure for all* $\epsilon > 0$*;*
3. $\sup_{a\in\Delta} \iint_{\Omega_\epsilon(f)} |f'(z)|^p(1-|z|^2)^q g^s(z,a)dxdy < \infty\}$ *for all* $\epsilon > 0$*;*
4. $\sup_{a\in\Delta} \iint_{\Omega_\epsilon(f)} |f'(z)|^p(1-|z|^2)^q(1-|\sigma_a(z)|^2)^s dxdy < \infty$ *for all* $\epsilon > 0$.

Corollary 2. *Suppose* $0 < p < \infty$, $-2 < q < \infty$, $0 < s < \infty$ *and* $0 < s < \alpha = \frac{q+2}{p} < 2$. *If* $f \in H(\Delta)$, $1 < \alpha + s < 3$ *and* $s_1 < s_2$. *Then*

$$dist_{B^\alpha}(f, F(p,q,s_1)) = dist_{B^\alpha}(f, F(p,q,s_2)).$$

Remark:

1. If $s > 1$, $F(p,q,s) = B^\alpha$.
2. If $q + s \le -1$, $F(p,q,s)$ is trivial.

Similarly, we have the following result.

Theorem 2. *Suppose* $0 < p < \infty$, $-2 < q < \infty$, $0 < s < \infty$ *and* $0 < s < \alpha = \frac{q+2}{p} < 2$. *If* $f \in B^\alpha$ *and* $1 < \alpha + s < 3$. *Then the following quantities are equivalent:*

1. $dist_{B^\alpha}(f, F_0(p,q,s))$*;*
2. $dist_{B^\alpha}(f, B^\alpha_0)$*;*
3. $inf\{\epsilon : \frac{\chi_{\Omega_\epsilon(f)}(z)}{(1-|z|^2)^{2-s}}dxdy$ *is a compact s-Carleson measure*$\}$*;*
4. $inf\{\epsilon : \lim_{|a|\to 1} \iint_{\Omega_\epsilon(f)} |f'(z)|^p(1-|z|^2)^q g^s(z,a)dxdy = 0\}$*;*
5. $inf\{\epsilon : \lim_{|a|\to 1} \iint_{\Omega_\epsilon(f)} |f'(z)|^p(1-|z|^2)^q(1-|\sigma_a(z)|^2)^s dxdy = 0\}$.

Corollary 3. *Suppose* $0 < p < \infty$, $-2 < q < \infty$, $0 < s < \infty$ *and* $0 < s < \alpha = \frac{q+2}{p} < 2$. *If* $f \in B^\alpha_0$ *and* $1 < \alpha + s < 3$. *Then* $f \in F_0(p,q,s)$ *if and only if* $\frac{\chi_{\Omega_\epsilon(f)}(z)}{(1-|z|^2)^{2-s}}dxdy$ *is a compact s-Carleson measure for all* $\epsilon > 0$.

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