Filomat 35:12 (2021), 4095–4103
https://doi.org/10.2298/FIL2112095K

Published by Faculty of Sciences and Mathematics,
University of Niš, Serbia
Available at: http://www.pmf.ni.ac.rs/filomat

Some Results on Soft Equicontinuity and Soft Uniform Equicontinuity

Sandeep Kaur[a], **Alkan Özkan**[b]

[a]*Department of Mathematics and Statistics, Central University of Punjab, Bathinda, India*
[b]*Department of Mathematics, Faculty of Arts and Sciences, Iğdır University, Iğdır, Turkey*

Abstract. Equicontinuity plays a vital role in general metric spaces and multiple studies have tried to characterize equicontinuity since its origin. Advent of soft set theory leads us to study these mappings in terms of soft sets, as well. In this paper, first we introduce the concept of soft equicontinuity, soft pointwise equicontinuity and soft uniform equicontinuity with examples. Moreover, we explore soft continuity of soft pointwise limit of a sequence of soft mappings when the family of the given soft mapping is a soft equicontinuous mapping and then discuss soft pointwise convergence of sequence of soft mappings which are a soft pointwise equicontinuous mappings when co-domain is soft complete space. We also give characterizations of soft pointwise equicontinuity and soft uniform equicontinuity in terms of convergent sequence of soft points in soft dense subset.

1. Introduction

Molodtsov [18] introduced the concept of soft set theory for dealing with uncertainities which is the generalization of set theory and fuzzy set theory. Many applications have found in various directions showing the rich potential of the concept of soft sets. Maji et al. [16] have given a detailed theoretical study on soft set operators by means of examples. Shabir and Naz [26] introduced the study of soft topological space as a parameterized form of topological structures using soft set theory. Consequently, they defined some basic notions of soft topological space such as soft open and closed sets, soft closure, soft separation axioms and established their elementary properties. Kharal and Ahmad [15], defined the notion of a soft mapping on soft classes and studied some properties of images and preimages of soft sets under soft mapping. Aygünoğlu and Aygün [10], introduced soft continuity of soft mappings and soft product topology. Since its advent a lot of work has been done in this field by various authors in [2–4, 6–9, 21–24, 27]. In these studies, the concept of soft points were studied in various forms. The concept of soft points that we use in this paper was defined in [20]. Das and Samanta introduced the notion of soft real sets, soft real numbers in [12] and detailed concept of soft metric spaces by using the notion of soft points with properties in [13]. Al-shami [1] introduced the notion of soft somewhere dense sets as a new class of generalized soft open sets and studied its properties in detail. In [5], Al-shami et al. introduced some new types of soft maps and studied its characterizations.

2020 *Mathematics Subject Classification*. Primary 54E35; Secondary 54C50, 54E40, 54A05
Keywords. Soft metric spaces, soft dense sets, soft continuous mapping, soft equicontinuous mapping, soft uniform equicontinuity, soft pointwise equicontinuity
Received: 19 September 2020; Revised: 18 March 2021; Accepted: 22 March 2021
Communicated by Ljubiša D.R. Kočinac
Email addresses: sandybrar86@gmail.com (Sandeep Kaur), alkan.ozkan@igdir.edu.tr (Alkan Özkan)

It is well known that detailed concept of equicontinuity of mappings and its generalizations have been studied in literature in various forms. Many necessary and sufficient conditions for continuous mappings to be equicontinuous were also investigated. Rapid growth in soft set theory leads us to study these mappings in terms of soft sets, as well. Characterizations of soft continuity in terms of soft dense subsets and in terms of convergent sequences of soft mappings taken from soft dense subsets were obtained by Kaur et al. [14] in 2020.

In this paper, we first introduce soft equicontinuity at a soft point and give its characterizations in terms of soft dense subsets. We also introduce the definition of soft pointwise equicontinuity on a soft subset of an absolute soft set $\widetilde{X}$ and discuss soft pointwise convergence of sequence of soft mappings which are soft pointwise equicontinuous when co-domain is soft complete space. Later, we introduce the definitions of soft uniformly equicontinuous family of soft mappings and soft uniform convergence of soft mappings. We prove that limit of soft uniformly convergent sequence of soft continuous mappings is a soft continuous map.

2. Preliminaries

In this section we will give some basic definitions and results that we need in our further sections.

Let X be an initial universe set, $P(X)$ the power set of X and A a set of parameters. A pair F_A is called a *soft set* [18] over X, where F is a mapping given by $F : A \to P(X)$. In other words, a soft set over X is a parameterized family of subsets of the universe X. The family of all soft sets F_A over X is denoted by $S(X, A)$. The complement of a soft set F_A [25] is denoted by F^c_A and is defined by $(F_A)^c = (F^c, A)$ where $F^c : A \to P(X)$ is a mapping given by $F^c(a) = X - F(a)$, for all $a \in A$. F^c is said to be the *soft complement* function [16] of F. $(F^c)^c$ is the same as F and $((F^c_A))^c = F_A$. A soft set F_A over X is said to be a *null soft set* [16], denoted by Φ_A , if for all $a \in A$, $F(a) = \phi$. A soft set F_A over X is said to be an *absolute soft set* [16], denoted by $\widetilde{X}$, if for all $a \in A$, $F(a) = X$. For two soft sets F_A and G_A over a common universe X, F_A is a *soft subset* [25] of G_A if $\forall a \in A$, $F(a) \subseteq G(a)$ for all $a \in A$ and is denoted by $F_A \widetilde{\subseteq} G_A$. F_A is said that to be a *soft superset* [25] of G_A, if G_A is a soft subset of F_A and is denoted by $F_A \widetilde{\supseteq} G_A$. For two soft sets F_A and G_A over a common universe X, *union* [16] of two soft sets of F_A and G_A is the soft set H_A, where $H(a) = F(a) \cup G(a)$ for all $a \in A$ and is denoted by $F_A \widetilde{\cup} G_A = H_A$. For two soft sets F_A and G_A over a common universe X, *intersection* [25] of two soft sets of F_A and G_A is the soft set H_A, where $H(a) = F(a) \cap G(a)$ for all $a \in A$ and is denoted by $F_A \widetilde{\cap} G_A = H_A$.

We shall make use of the fundamental notion of soft points introduced in [20].

Definition 2.1. A soft set P_A over X is said to be a soft point, denoted by P^x_a, if for the element $a \in A$, $P(a) = \{x\}$ and $P(a') = \phi$, for all $a' \neq a$.

The soft point P^x_a is said to be in the soft set F_A, denoted by $P^x_a \widetilde{\in} F_A$, if $x \in F(a)$. Two soft points $P^{x_1}_{a_1}, P^{x_2}_{a_2}$ are said to be equal if $a_1 = a_2$ and $x_1 = x_2$. Thus, $P^{x_1}_{a_1} \neq P^{x_2}_{a_2} \Leftrightarrow a_1 \neq a_2$ or $x_1 \neq x_2$.

The family of all soft points over X will be denoted by SP(X,A). The union of any collection of soft points can be considered as a soft set and every soft set can be expressed as union of all soft points belonging to it; $F_A = \bigcup_{P^x_a \widetilde{\in} F_A} P^x_a$.

Now, we introduce the definition of soft metric space defined in [13] which is based on soft point of soft set and the basic definitions and theorems defined on this space which is taken from [11, 14, 17, 19] to use in the sequel. Let $\mathbf{R}$ be the set of real numbers and $\mathfrak{B}(\mathbf{R})$ be the collection of all non-empty bounded subsets of $\mathbf{R}$ and A be the set of parameters. Then a mapping $F : A \to \mathfrak{B}(\mathbf{R})$ is a special case of soft set called a soft real set. It is denoted by (F,A). If specifically (F,A) is a singleton soft set, i.e. there exists $x \in \mathbf{R}$ such that $F(a) = \{x\}$ for all a in A, then after identifying (F,A) with the corresponding soft element, it will be called a soft real number. We use notations $\widetilde{r}, \widetilde{s}, \widetilde{t}$ to denote soft real numbers whereas $\bar{r}, \bar{s}, \bar{t}$ will denote a particular type of soft real numbers such that $\bar{r}(\lambda) = \{r\}$, for all $\lambda \in A$. In such a case we also use $\bar{r}(\lambda) = r$ instead of $\bar{r}(\lambda) = \{r\}$. For example, $\bar{0}$ is the soft real number where $\bar{0}(\lambda) = 0$, for all $\lambda \in A$.

Definition 2.2. ([13]) For two soft real numbers $\bar{r}, \bar{s}$ we define:

1. $\widetilde{r} \widetilde{\leq} \widetilde{s}$ if $\widetilde{r}(\lambda) \leq \widetilde{s}(\lambda)$, for all $\lambda \in A$;
2. $\widetilde{r} \widetilde{\geq} \widetilde{s}$ if $\widetilde{r}(\lambda) \geq \widetilde{s}(\lambda)$, for all $\lambda \in A$;
3. $\widetilde{r} \widetilde{<} \widetilde{s}$ if $\widetilde{r}(\lambda) < \widetilde{s}(\lambda)$, for all $\lambda \in A$;
4. $\widetilde{r} \widetilde{>} \widetilde{s}$ if $\widetilde{r}(\lambda) > \widetilde{s}(\lambda)$, for all $\lambda \in A$.

Let $\widetilde{X}$ be the absolute soft set. Let $SP(\widetilde{X})$ be the collection of all soft points of $\widetilde{X}$ and $\mathbf{R}(A)^*$ denote the set of all non-negative soft real numbers.

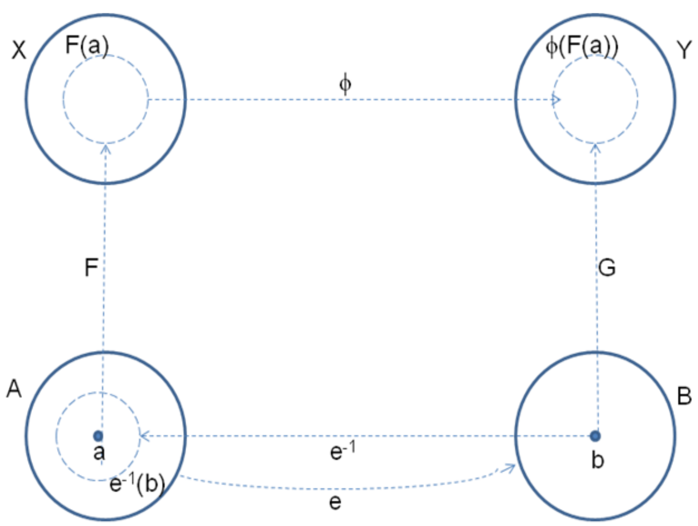
Definition 2.3. ([13])

1. A mapping $\widetilde{d} : SP(\widetilde{X}) \times SP(\widetilde{X}) \to \mathbf{R}(A)^*$, is said to be a *soft metric* on the soft set $\widetilde{X}$ if $\widetilde{d}$ satisfies the following conditions:
   (a) $\widetilde{d}(P^y_b, P^x_a) \widetilde{\geq} \bar{0}$ for all $P^y_b, P^x_a \widetilde{\in} \widetilde{X}$.
   (b) $\widetilde{d}(P^y_b, P^x_a) = \bar{0}$ if and only if $P^y_b = P^x_a$.
   (c) $\widetilde{d}(P^y_b, P^x_a) = \widetilde{d}(P^x_a, P^y_b)$ for all $P^y_b, P^x_a \widetilde{\in} \widetilde{X}$.
   (d) For all $P^x_a, P^y_b, P^z_c \widetilde{\in} \widetilde{X}$, $\widetilde{d}(P^x_a, P^z_c) \widetilde{\leq} \widetilde{d}(P^x_a, P^y_b) + \widetilde{d}(P^y_b, P^z_c)$.

   The soft set $\widetilde{X}$ with a soft metric $\widetilde{d}$ on $\widetilde{X}$ is called a *soft metric space* and denoted by $(\widetilde{X}, \widetilde{d}, A)$.
2. Let $(\widetilde{X}, \widetilde{d}, A)$ be a soft metric space and (Y, A) be a non-null soft subset of $\widetilde{X}$. Then the mapping $\widetilde{d}_Y : SP(Y, A) \times SP(Y, A) \to \mathbf{R}(A)^*$ given by $\widetilde{d}_Y(P^y_b, P^x_a) = \widetilde{d}(P^y_b, P^x_a)$ for all $P^y_b, P^x_a \widetilde{\in} (Y, A)$ is a soft metric on (Y,A). This metric $\widetilde{d}_Y$ is known as the relative metric induced on (Y, A) by $\widetilde{d}$. The soft metric $(\widetilde{Y}, \widetilde{d}_Y, A)$ is called a metric subspace or simply a subspace of the soft metric space $(\widetilde{X}, \widetilde{d}, A)$.
3. Let $\{P^{x_n}_{a_n}\}_n$ be a sequence of soft points in a soft metric space $(\widetilde{X}, \widetilde{d}, A)$. The soft sequence $\{P^{x_n}_{a_n}\}_n$ is called as convergent soft sequence in $(\widetilde{X}, \widetilde{d}, A)$ if there is a soft point $P^x_a \widetilde{\in} \widetilde{X}$ such that $\widetilde{d}(P^{x_n}_{a_n}, P^x_a) \longrightarrow \bar{0}$ as $n \longrightarrow \infty$, that is for given $\widetilde{\epsilon} \widetilde{>} \bar{0}$, there is a natural number N such that $\bar{0} \widetilde{\leq} \widetilde{d}(P^{x_n}_{a_n}, P^x_a) \widetilde{<} \widetilde{\epsilon}$, whenever $n \geq N$.
   We denote this by $P^{x_n}_{a_n} \longrightarrow P^x_a$ as $n \longrightarrow \infty$.

Let $(\widetilde{X}, \widetilde{d}, A)$ and $(\widetilde{Y}, \widetilde{\rho}, B)$ be two soft metric spaces. The mapping $(\varphi, e) : (\widetilde{X}, \widetilde{d}, A) \to (\widetilde{Y}, \widetilde{\rho}, B)$ is a soft mapping defined in [17], where $\varphi : X \to Y$ and $e : A \to B$ are two mappings.

Concept of soft mapping defined in [17] is illustrated by the following diagram.



Definition 2.4. ([19]) A soft mapping (φ, e) is a soft continuous mapping at the soft point $P_a^x \in \widetilde{X}$ if for every $\widetilde{\epsilon} \widetilde{>} \bar{0}$ there exists $\widetilde{\delta} \widetilde{>} \bar{0}$ such that $\widetilde{d}(P_b^y, P_a^x) \widetilde{<} \widetilde{\delta}$ implies that $\widetilde{\rho}((\varphi, e)(P_b^y), (\varphi, e)(P_a^x)) \widetilde{<} \widetilde{\epsilon}$ for every $P_b^y \in \widetilde{X}$. If (φ, e) is a soft continuous mapping at every soft point P_a^x of $(\widetilde{X}, \widetilde{d}, A)$, then it is said to be a soft continuous mapping on $(\widetilde{X}, \widetilde{d}, A)$.

Definition 2.5. ([19]) Let $(\widetilde{X}, \widetilde{d}, A)$ be a soft metric space. A mapping $(\varphi, e) : (\widetilde{X}, \widetilde{d}, A) \to (\widetilde{Y}, \widetilde{\rho}, B)$ is called a soft contraction mapping if there exists a soft real number $\alpha \in \mathbf{R}$, $\bar{0} \leq \alpha \leq \bar{1}$ such that $\widetilde{\rho}((\varphi, e)(P_b^y), (\varphi, e)(P_a^x))$ $\widetilde{\leq} \alpha\widetilde{d}(P_b^y, P_a^x)$.

Definition 2.6. ([11]) Let $(\widetilde{X}, \widetilde{d}, A)$ be a soft metric space. $(\widetilde{X}, \widetilde{d}, A)$ is called soft sequential compact metric space if every soft sequence has a soft subsequence that converges in $\widetilde{X}$.

Definition 2.7. ([11]) Let $(\widetilde{X}, \widetilde{d}, A)$ and $(\widetilde{Y}, \widetilde{\rho}, B)$ be two soft metric spaces. A soft mapping $(\varphi, e) : (\widetilde{X}, \widetilde{d}, A) \to (\widetilde{Y}, \widetilde{\rho}, B)$ is a *soft uniformly continuous mapping* if for given any $\widetilde{\epsilon} \widetilde{>} \bar{0}$, there exists $\widetilde{\delta} \widetilde{>} \bar{0}$ (depending only on $\widetilde{\epsilon}$) such that for any soft points P_a^x, $P_b^y \widetilde{\in} \widetilde{X}$, $\widetilde{d}(P_a^x, P_b^y) \widetilde{<} \widetilde{\delta}$ implies $\widetilde{\rho}((\varphi, e)(P_a^x), (\varphi, e)(P_b^y)) \widetilde{<} \widetilde{\epsilon}$.

Definition 2.8. ([1, 14]) A soft set (Z, A) of $\widetilde{X}$ is called soft dense (Soft somewhere dense sets in sense of [1]) in $\widetilde{X}$ if $\overline{(Z, A)} = \widetilde{X}$. In other words, for every $P_a^x \widetilde{\in} \widetilde{X}$ and $\widetilde{\epsilon} \widetilde{>} \bar{0}$ there exists soft point $P_b^y \widetilde{\in} (Z, A)$ such that $\widetilde{d}(P_b^y, P_a^x) \widetilde{<} \widetilde{\epsilon}$.

Throughout this paper, (Z, A) will denote an arbitrarily fixed soft dense subset of $\widetilde{X}$.
The following results will be utilized in this paper.

Theorem 2.9. ([19]) *A soft mapping (φ, e) is a soft continuous mapping at the soft point $P_a^x \in \widetilde{X}$ if and only if for every sequence of soft points $\{P_{a_n}^{x_n}\}_n$ converging to the soft point P_a^x in the soft metric space $(\widetilde{X}, \widetilde{d}, A)$, the sequence $\{(\varphi, e)(P_{a_n}^{x_n})\}_n$ in $(\widetilde{Y}, \widetilde{\rho}, B)$ converges to a soft point $(\varphi, e)(P_a^x)$ in $\widetilde{Y}$."*

Theorem 2.10. ([19]) *Every soft contraction mapping is a soft continuous mapping.*

Theorem 2.11. ([14]) *A soft set (Z, A) of $\widetilde{X}$ is said to be a soft dense set in $\widetilde{X}$ if and only if for every $P_a^x \widetilde{\in} \widetilde{X}$ there exists a sequence $\{P_{a_n}^{d_n}\}_n$ of soft points in (Z, A) converging to P_a^x.*

Theorem 2.12. ([14]) *Let $(\widetilde{X}, \widetilde{d}, A)$ and $(\widetilde{Y}, \widetilde{\rho}, B)$ be two soft metric spaces. A soft mapping $(\varphi, e) : (\widetilde{X}, \widetilde{d}, A) \to (\widetilde{Y}, \widetilde{\rho}, B)$ is a soft continuous mapping if and only if $(\varphi, e)|_{(Z,A)}$ is a soft continuous mapping and (φ, e) is a soft continuous mapping at each point of $(Z, A)^c$ where (Z, A) is soft dense in $\widetilde{X}$.*

3. Soft Equicontinuity

In this section, we introduce soft equicontinuous mapping, soft pointwise equicontinuous mapping, soft pointwise convergence and their basic properties.

Definition 3.1. Let $\mathcal{A}$ be a family of soft mappings from $\widetilde{X}$ to $\widetilde{Y}$. Then the family $\mathcal{A}$ will said to be

1. *soft equicontinuous mapping* at a soft point $P_a^x \widetilde{\in} \widetilde{X}$ if for $\widetilde{\epsilon} \widetilde{>} \bar{0}$ there exist $\widetilde{\delta}$ (depending on $\widetilde{\epsilon}$ and P_a^x) $\widetilde{>} \bar{0}$ such that $\widetilde{d}(P_b^y, P_a^x) \widetilde{<} \widetilde{\delta}$ then $\widetilde{\rho}((\varphi, e)(P_b^y), (\varphi, e)(P_a^x)) \widetilde{<} \widetilde{\epsilon}$ for every $(\varphi, e) \widetilde{\in} \mathcal{A}$ and $P_b^y \widetilde{\in} \widetilde{X}$.
2. *soft pointwise equicontinuous mapping* on a soft subset (F, A) of $\widetilde{X}$ if it is a soft equicontinuous mapping at each soft point of (F, A).

We illustrate this with the following example.

Example 3.2. Let $\mathcal{F}$ be a set of all soft contraction mappings $(\varphi, e) : (\widetilde{X}, \widetilde{d}, A) \to (\widetilde{Y}, \widetilde{\rho}, B)$. Then $\mathcal{F}$ is a soft equicontinuous mapping, since we can choose $\widetilde{\delta} = \widetilde{\epsilon}$. To see this, we note that $\widetilde{d}(P_b^y, P_a^x) \widetilde{<} \widetilde{\delta} = \widetilde{\epsilon}$ then $\widetilde{\rho}((\varphi, e)(P_b^y), (\varphi, e)(P_a^x)) \widetilde{\leq} \widetilde{d}(P_b^y, P_a^x) \widetilde{<} \widetilde{\epsilon}$ for all $P_b^y, P_a^x \widetilde{\in} \widetilde{X}$ and $(\varphi, e) \in \mathcal{F}$.

Next we introduce soft pointwise convergence of sequence of soft mappings:

Definition 3.3. A sequence $\{(\varphi, e)_n\}_n$ of soft mappings from $\widetilde{X}$ to $\widetilde{Y}$ is a soft pointwise converges to a soft mapping $(\varphi, e) : (\widetilde{X}, \widetilde{d}, A) \to (\widetilde{Y}, \widetilde{\rho}, B)$ if for all $P_a^x \widetilde{\in} \widetilde{X}$ and $\widetilde{\epsilon} \widetilde{>} \bar{0}$ there exist $N = N(\widetilde{\epsilon})$ such that $\widetilde{\rho}((\varphi, e)_n(P_a^x), (\varphi, e)(P_a^x)) \widetilde{<} \widetilde{\epsilon}$.

We illustrate this with the following example.

Example 3.4. Let $A = \mathfrak{R}$ be parameter set and $X = \mathfrak{R}$. Consider usual metric on these sets and define soft metric on $\widetilde{X}$ by $\widetilde{d}(P_b^y, P_a^x) = |\bar{x} - \bar{y}| + |\bar{a} - \bar{b}|$. Define $(\varphi, e)_n : (\widetilde{X}, \widetilde{d}, A) \to (\widetilde{X}, \widetilde{d}, A)$ by $(\varphi, e)_n(P_a^x) = (P_a^{\frac{x}{n}})$ and $(\varphi, e) : (\widetilde{X}, \widetilde{d}, A) \to (\widetilde{X}, \widetilde{d}, A)$ by $(\varphi, e)(P_a^x) = P_a^0$. Therefore, $(\varphi, e)_n \longrightarrow (\varphi, e)$.

The following theorem shows the soft continuity of soft pointwise limit of a sequence of soft mappings when the family of that soft mappings is a soft equicontinuous mapping.

Theorem 3.5. *Let $\{(\varphi, e)_n\}_n$ be a sequence of soft mappings from $\widetilde{X}$ to $\widetilde{Y}$ with the property that $(\varphi, e)_n(P_a^x)$ converges to $(\varphi, e)(P_a^x)$ for $P_a^x \widetilde{\in} \widetilde{X}$. Suppose further the family $\{(\varphi, e)_n\}_{n=1}^{\infty}$ is a soft equicontinuous mapping. Then (φ, e) is a soft continuous mapping and the family $\{(\varphi, e), (\varphi, e)_1, (\varphi, e)_2, ...\}$ is also a soft equicontinuous mapping.*

Proof. Fix any $\widetilde{\epsilon} \widetilde{>} \bar{0}$ and $P_a^x \widetilde{\in} \widetilde{X}$, there exist $\widetilde{\delta}$ (depending *on $\widetilde{\epsilon}$ and* P_a^x) such that whenever $\widetilde{d}(P_b^y, P_a^x) \widetilde{<} \widetilde{\delta}$ then $\widetilde{\rho}((\varphi, e)_n(P_b^y), (\varphi, e)_n(P_a^x)) \widetilde{<} \frac{\widetilde{\epsilon}}{3}$ for every $P_b^y \widetilde{\in} \widetilde{X}$ and $n \in \mathbf{N}$. Further as $(\varphi, e)_n(P_a^x) \longrightarrow (\varphi, e)(P_a^x)$ for every $P_a^x \widetilde{\in} \widetilde{X}$, there exist N such that $\widetilde{\rho}((\varphi, e)_n(P_a^x), (\varphi, e)(P_a^x)) \widetilde{<} \frac{\widetilde{\epsilon}}{3}$ and $\widetilde{\rho}((\varphi, e)_n(P_b^y), (\varphi, e)(P_b^y)) \widetilde{<} \frac{\widetilde{\epsilon}}{3}$ for all $n \geq \mathbf{N}$. Fix a value n, then for all $P_b^y \widetilde{\in} \widetilde{X}$ with $\widetilde{d}(P_b^y, P_a^x) \widetilde{<} \widetilde{\delta}$ we have
$\widetilde{\rho}((\varphi, e)(P_b^y), (\varphi, e)(P_a^x)) \widetilde{\leq} \widetilde{\rho}((\varphi, e)_n(P_a^x), (\varphi, e)(P_a^x)) + \widetilde{\rho}((\varphi, e)_n(P_b^y), (\varphi, e)_n(P_a^x)) + \widetilde{\rho}((\varphi, e)_n(P_b^y), (\varphi, e)(P_b^y)) \widetilde{<} \frac{\widetilde{\epsilon}}{3} + \frac{\widetilde{\epsilon}}{3} + \frac{\widetilde{\epsilon}}{3} = \widetilde{\epsilon}$. □

The following example shows that converse of above theorem need not be true.

Example 3.6. Consider soft metric defined in Example 3.4. Define $(f, e)_n : (\widetilde{X}, \widetilde{d}, A) \to (\widetilde{X}, \widetilde{d}, A)$ by $(f, e)_n(P_a^x) = P_a^{nx}$ and $(f, e) : (\widetilde{X}, \widetilde{d}, A) \to (\widetilde{X}, \widetilde{d}, A)$ by $(f, e)(P_a^x) = P_a^0$. Here, (f, e) is a soft continuous mapping but $(f, e)_n(P_a^x)$ does not converge to $(f, e)(P_a^x)$.

For our next results, we need the following definitions.

Definition 3.7. ([13]) A sequence $\{(\varphi, e)_n\}_n$ of soft mappings from $\widetilde{X}$ to $\widetilde{Y}$ is said to be a Cauchy sequence if for every $P_a^x \widetilde{\in} \widetilde{X}$ and $\widetilde{\epsilon} \widetilde{>} \bar{0}$ there exist N such that for all $n, m \geq N$, $\widetilde{\rho}((\varphi, e)_n(P_a^x), (\varphi, e)_m(P_a^x)) \widetilde{<} \widetilde{\epsilon}$.

Definition 3.8. ([13]) A soft metric space $(\widetilde{X}, \widetilde{d}, A)$ is a complete if every Cauchy sequence in $\widetilde{X}$ converges to a soft point of $\widetilde{X}$.

In the following theorem, soft pointwise convergence of a sequence of soft mappings which are soft pointwise equicontinuous mapping are discussed, when co-domain space is a soft complete metric space.

Theorem 3.9. *Let $\{(\varphi, e)_n\}_{n=1}^{\infty}$ be a sequence of soft mappings from $\widetilde{X}$ to $\widetilde{Y}$ such that family $\{(\varphi, e)_n(P_a^x)\}_n$ is a soft pointwise equicontinuous mapping on $(Z, A)^c$ and $(\widetilde{Y}, \widetilde{\rho}, B)$ is a soft complete metric space. If $\{(\varphi, e)_n(P_a^x)\}_n$ converges for all $P_a^x \widetilde{\in} (Z, A)$ then $\{(\varphi, e)_n(P_a^x)\}_n$ converges for all $P_a^x \widetilde{\in} \widetilde{X}$.*
In particular, this theorem holds if $\{(\varphi, e)_n\}_{n=1}^{\infty}$ is a soft equicontinuous mapping family of soft mappings.

Proof. Let $P_a^x \,\widetilde{\in}\, (Z,A)^c$ and $\widetilde{\epsilon} \,\widetilde{>}\, \bar{0}$. Since $\{(\varphi,e)_n\}_n$ is a soft pointwise equicontinuous mapping at P_a^x then there exist $\widetilde{\delta} \,\widetilde{>}\, \bar{0}$ such that for every n, $\widetilde{\rho}((\varphi,e)_n(P_a^x),(\varphi,e)_n(P_b^d)) \,\widetilde{<}\, \frac{\widetilde{\epsilon}}{3}$ whenever $\widetilde{d}(P_a^x,P_b^d) \,\widetilde{<}\, \widetilde{\delta}$ for all $P_b^d \,\widetilde{\in}\, (Z,A)$. Fix one such $P_{b_0}^{d_0}$ in (Z,A). Then since $\{(\varphi,e)_n(P_{b_0}^{d_0})\}_n$ is convergent in $\widetilde{Y}$, there exist a positive integer $N_0 = N_0(\widetilde{\epsilon})$ such that for all $m,n \geq N_0$, $\widetilde{\rho}((\varphi,e)_n(P_{b_0}^{d_0}),(\varphi,e)_m(P_{b_0}^{d_0})) \,\widetilde{<}\, \frac{\widetilde{\epsilon}}{3}$. It follows that $\widetilde{\rho}((\varphi,e)_n(P_a^x),(\varphi,e)_m(P_a^x)) \,\widetilde{\leq}\, \widetilde{\rho}((\varphi,e)_n(P_a^x),(\varphi,e)_n(P_{b_0}^{d_0})) + \widetilde{\rho}((\varphi,e)_n(P_{b_0}^{d_0}),(\varphi,e)_m(P_{b_0}^{d_0})) + \widetilde{\rho}((\varphi,e)_m(P_{b_0}^{d_0}),(\varphi,e)_m(P_a^x)) \,\widetilde{<}\, \frac{\widetilde{\epsilon}}{3} + \frac{\widetilde{\epsilon}}{3} + \frac{\widetilde{\epsilon}}{3} = \widetilde{\epsilon}$ for all $m,n \geq N$. Thus, $\{(\varphi,e)_n(P_a^x)\}_n$ is a cauchy sequence in $\widetilde{X}$. Hence it converges, since $(\widetilde{Y},\widetilde{\rho},B)$ is a soft complete metric space, $\square$

The following example shows that assumption of soft completeness cannot be dropped from the above theorem.

Example 3.10. Let $X = \mathfrak{R}$ and $Y = \mathfrak{R} - \{0\}$ and $A = \mathfrak{R}$ be parameter set. Define $(\varphi,e)_n : (\widetilde{X},\widetilde{d},A) \to (\widetilde{Y},\widetilde{d},A)$ by $(\varphi,e)_n(P_a^x) = P_{a,n}^y$ where $P_{a,n}^y(a) = x + \frac{1}{n}$ and $P_{a,n}^y(a') = \phi$ *for all* $a' \in A - \{a\}$. Now $(Z,A) = \mathfrak{R} - \{0\}$ is soft dense in $\widetilde{X}$. For $P_a^x \,\widetilde{\in}\, (Z,A)$ then sequence $\{(\varphi,e)_n(P_a^x)\}_n$ converges. But for $P_a^0 \,\widetilde{\in}\, \widetilde{X}$ then limit of $\{(\varphi,e)_n(P_a^0)\}_n$ does not exist in $\widetilde{Y}$.

Finally, we give characterization of soft pointwise equicontinuity of soft mapping in terms of convergence sequence of soft points in soft dense subset:

Theorem 3.11. *For a family $\mathcal{A}$ of soft mappings from $\widetilde{X}$ to $\widetilde{Y}$, the following conditions are equivalent:*

1. *$\mathcal{A}$ is a soft pointwise equicontinuous mapping on $\widetilde{X}$.*
2. *the family $\mathcal{A}|_{(Z,A)} = \{(\varphi,e)|_{(Z,A)} | (\varphi,e) \in \mathcal{A}\}$ is a soft pointwise equicontinuous mapping on $(Z,A)^c$.*
3. *for any sequence of soft points $\{P_{a_n}^{d_n}\}_n$ in (Z,A) converging to a soft point $P_a^x \,\widetilde{\in}\, \widetilde{X}$ implies that for every $\widetilde{\epsilon} \,\widetilde{>}\, \bar{0}$, there exists a positive integer $n_0 = n_0$ (depending on $\widetilde{\epsilon}$ and P_a^x) such that for every (φ,e) in $\mathcal{A}$ and for all $n \geq n_0$, $\widetilde{\rho}((\varphi,e)(P_{a_n}^{d_n}),(\varphi,e)(P_a^x)) \,\widetilde{<}\, \widetilde{\epsilon}$.*
4. *for any sequence of soft points $\{P_{a_n}^{x_n}\}_n$ in $\widetilde{X}$ converging to a soft point $P_a^x \,\widetilde{\in}\, \widetilde{X}$ implies that for every $\widetilde{\epsilon} \,\widetilde{>}\, \bar{0}$, there exists a positive integer $n_0 = n_0$ (depending on $\widetilde{\epsilon}$ and P_a^x) such that for every (φ,e) in $\mathcal{A}$ and for all $n \geq n_0$, $\widetilde{\rho}((\varphi,e)(P_{a_n}^{x_n}),(\varphi,e)(P_a^x)) \,\widetilde{<}\, \widetilde{\epsilon}$.*

Proof. (1) $\Rightarrow$ (4): Let $\widetilde{\epsilon} \,\widetilde{>}\, \bar{0}$ and $P_a^x \,\widetilde{\in}\, \widetilde{X}$, then since $\mathcal{A}$ is a soft pointwise equicontinuous mapping there exist $\widetilde{\delta} = \widetilde{\delta}(\widetilde{\epsilon},P_a^x) \,\widetilde{>}\, \bar{0}$ such that for every $P_b^y \,\widetilde{\in}\, \widetilde{X}$, $\widetilde{\rho}((\varphi,e)(P_b^y),(\varphi,e)(P_a^x)) \,\widetilde{<}\, \widetilde{\epsilon}$ whenever $\widetilde{d}(P_b^y,P_a^x) \,\widetilde{<}\, \widetilde{\delta}$ for every (φ,e) in $\mathcal{A}$. Now, since $P_{a_n}^{x_n} \longrightarrow P_a^x$, there exist positive integer $N_0 = N_0(\widetilde{\delta},P_a^x) = N_0(\widetilde{\epsilon},P_a^x)$ such that for every $n \geq N_0$, $\widetilde{d}(P_{a_n}^{x_n},P_a^x) \,\widetilde{<}\, \widetilde{\delta}$. It implies that $\widetilde{\rho}((\varphi,e)(P_{a_n}^{x_n}),(\varphi,e)(P_a^x)) \,\widetilde{<}\, \widetilde{\epsilon}$ for every (φ,e) in $\mathcal{A}$.

(4) $\Rightarrow$ (1): Assume that (1) does not hold. Then there exist $P_a^x \,\widetilde{\in}\, \widetilde{X}$ and $\widetilde{\epsilon} \,\widetilde{>}\, \bar{0}$ such that for every $\widetilde{\delta} \,\widetilde{>}\, \bar{0}$, there exist (f,e) in $\mathcal{A}$ and $P_b^y \,\widetilde{\in}\, \widetilde{X}$ depending on $\widetilde{\delta}$ such that $\widetilde{d}(P_b^y,P_a^x) \,\widetilde{<}\, \widetilde{\delta}$ but $\widetilde{\rho}((f,e)(P_b^y),(f,e)(P_a^x)) \,\widetilde{\geq}\, \widetilde{\epsilon}$. In particular, by taking $\widetilde{\delta} = \frac{\bar{1}}{n}$, then there exists $(f,e)_n$ in $\mathcal{A}$ and $P_{a_n}^{x_n}$ in $\widetilde{X}$ for every n satisfying $\widetilde{d}(P_{a_n}^{x_n},P_a^x) \,\widetilde{<}\, \frac{\bar{1}}{n} \longrightarrow \bar{0}$ as $n \longrightarrow \infty$. So $P_{a_n}^{x_n} \longrightarrow P_a^x$ but $\widetilde{\rho}((f,e)_n(P_{a_n}^{x_n}),(f,e)_n(P_a^x)) \,\widetilde{\geq}\, \widetilde{\epsilon}$ which contradicts (4).

(1) $\Rightarrow$ (2) is obvious. (2) $\Rightarrow$ (3) follows from (1) $\Rightarrow$ (4).

(3) $\Rightarrow$ (4): Let $P_{a_n}^{x_n} \longrightarrow P_a^x$ and $\widetilde{\epsilon} \,\widetilde{>}\, \bar{0}$. Since (Z,A) is soft dense in $\widetilde{X}$, for every n, there exists a sequence of soft points $\{P_{a_{n_k}}^{d_{n_k}}\}$ in (Z,A) such that $P_{a_{n_k}}^{d_{n_k}} \longrightarrow P_{a_n}^{x_n}$. Therefore, there exist $k_1(n)$ such that $\widetilde{d}(P_{a_{n_k}}^{d_{n_k}},P_{a_n}^{x_n}) \,\widetilde{<}\, \frac{\bar{1}}{n}$ for all $k \geq k_1(n)$. Then by (3), it follows that for every n, there exist $k_2(n)$ such that for every (φ,e) in $\mathcal{A}$, $\widetilde{\rho}((\varphi,e)(P_{a_{n_k}}^{d_{n_k}}),(\varphi,e)(P_{a_n}^{x_n})) \,\widetilde{<}\, \widetilde{\epsilon}$. In particular for $k' = max\{k_1(n),k_2(n)\}$ and for every (φ,e) in $\mathcal{A}$, $\widetilde{\rho}((\varphi,e)(P_{a_{n_{k'}}}^{d_{n_{k'}}}),(\varphi,e)(P_{a_n}^{x_n})) \,\widetilde{<}\, \frac{\widetilde{\epsilon}}{2}$ and $\widetilde{d}(P_{a_{n_{k'}}}^{d_{n_{k'}}},P_{a_n}^{x_n}) \,\widetilde{<}\, \frac{\bar{1}}{n} \longrightarrow \bar{0}$ as $n \longrightarrow \infty$. Since $P_{a_n}^{x_n} \longrightarrow P_a^x$ and $P_{a_{n_{k'}}}^{d_{n_{k'}}} \longrightarrow P_{a_n}^{x_n}$ implies $P_{a_{n_{k'}}}^{d_{n_{k'}}} \longrightarrow P_a^x$. Therefore by (3) again, there exist $n_0(\widetilde{\epsilon},P_a^x)$ such that for every (φ,e) in $\mathcal{A}$ and for all

$n \geq n_0$ $\widetilde{\rho}((\varphi,e)(P^{d_{n_{k'}}}_{a_{n_{k'}}}),(\varphi,e)(P^x_a)) < \frac{\widetilde{\epsilon}}{2}$. Hence for all $n \geq n_0$ and for all (φ,e) in $\mathcal{A}$, $\widetilde{\rho}((\varphi,e)(P^{x_n}_{a_n}),(\varphi,e)(P^x_a)) \widetilde{\leq}$ $\widetilde{\rho}((\varphi,e)(P^{x_n}_{a_n}),(\varphi,e)(P^{d_{n_{k'}}}_{a_{n_{k'}}})) + \widetilde{\rho}((\varphi,e)(P^{d_{n_{k'}}}_{a_{n_{k'}}}),(\varphi,e)(P^x_a)) \widetilde{<} \frac{\widetilde{\epsilon}}{2} + \frac{\widetilde{\epsilon}}{2} = \widetilde{\epsilon}$. □

From Theorems 2.12 and 3.11 we have the following:

Theorem 3.12. *Let Let $\{(\varphi,e)_n\}_n$ be a sequence of soft mappings from $\widetilde{X}$ to $\widetilde{Y}$ with the property that $(\varphi,e)_n(P^x_a)$ converges to $(\varphi,e)(P^x_a)$ for $P^x_a \widetilde{\in} \widetilde{X}$. If the family $\{(\varphi,e)_n|_{(Z,A)}\}_n$ is a soft pointwise equicontinuous mapping on (Z,A) then (φ,e) is a soft continuous mapping if and only if (φ,e) is a soft continuous mapping at each soft point in $(Z,A)^c$.*

4. Soft Uniform Equicontinuity

In this section, we introduce soft uniformly equicontinuous, soft uniformly converges and their basic properties.

Definition 4.1. A family $\mathcal{A}$ be a family of soft mappings from $\widetilde{X}$ to $\widetilde{Y}$ is said to be *soft uniformly equicontinuous mapping* on a soft subset (F,A) of $\widetilde{X}$ if for all $\widetilde{\epsilon} \widetilde{>} \bar{0}$ there exist $\widetilde{\delta}$ (depending on $\widetilde{\epsilon}$) such that for every (φ,e) in $\mathcal{A}$ and P^x_a, $P^y_b \widetilde{\in} (F,A)$, $\widetilde{\rho}((\varphi,e)(P^y_b),(\varphi,e)(P^x_a)) \widetilde{<} \widetilde{\epsilon}$ whenever $\widetilde{d}(P^y_b,P^x_a) \widetilde{<} \widetilde{\delta}$.

Next, we introduce soft uniform convergence of sequence of soft mappings.

Definition 4.2. A sequence $\{(\varphi,e)_n\}_n$ of soft mappings from $\widetilde{X}$ to $\widetilde{Y}$ is a soft uniformly converges to a soft mapping $(\varphi,e) : (\widetilde{X},\widetilde{d},A) \to (\widetilde{Y},\widetilde{\rho},B)$ if for all $\widetilde{\epsilon} \widetilde{>} \bar{0}$ there exists $N = N$ (depending only on $\widetilde{\epsilon}$) such that $\widetilde{\rho}((\varphi,e)_n(P^x_a),(\varphi,e)(P^x_a)) \widetilde{<} \widetilde{\epsilon}$ for every $n \geq N$ and for every $P^x_a \widetilde{\in} \widetilde{X}$.

The following theorem shows that limit of soft uniformly convergent sequence of soft continuous mappings is a soft continuous mapping.

Theorem 4.3. *Let $(\widetilde{X},\widetilde{d},A)$ and $(\widetilde{Y},\widetilde{\rho},B)$ be two soft metric spaces and assume that $\{(\varphi,e)_n\}_n$ is a sequence of soft continuous mappings, where $(\varphi,e)_n : (\widetilde{X},\widetilde{d},A) \to (\widetilde{Y},\widetilde{\rho},B)$ is a soft uniformly converges to a soft mappings (φ,e). Then (φ,e) is a soft continuous mapping.*

Proof. Let $P^x_a \widetilde{\in} \widetilde{X}$. Given an $\widetilde{\epsilon} \widetilde{>} \bar{0}$, we find $\widetilde{\delta} \widetilde{>} \bar{0}$ such that $\widetilde{\rho}((\varphi,e)(P^y_b),(\varphi,e)(P^x_a)) \widetilde{<} \widetilde{\epsilon}$ whenever $\widetilde{d}(P^y_b,P^x_a) \widetilde{<} \widetilde{\delta}$ for every $P^y_b \widetilde{\in} \widetilde{X}$. Since $(\varphi,e)_n$ is a soft uniformly converges to (φ,e), there is natural number N such that when n $\geq$ N, $\widetilde{\rho}((\varphi,e)_n(P^y_b),(\varphi,e)(P^y_b)) \widetilde{<} \frac{\widetilde{\epsilon}}{3}$ for all $P^y_b \widetilde{\in} \widetilde{X}$. Also as $(\varphi,e)_n$ is a soft continuous mapping at P^x_a, there is a $\widetilde{\delta} \widetilde{>} \bar{0}$ such that $\widetilde{\rho}((\varphi,e)_n(P^y_b),(\varphi,e)_n(P^x_a)) \widetilde{<} \frac{\widetilde{\epsilon}}{3}$ whenever $\widetilde{d}(P^y_b,P^x_a) \widetilde{<} \widetilde{\delta}$. It follows that $\widetilde{\rho}((\varphi,e)(P^y_b),(\varphi,e)(P^x_a)) \widetilde{\leq}$ $\widetilde{\rho}((\varphi,e)(P^y_b),(\varphi,e)_n(P^y_b)) + \widetilde{\rho}((\varphi,e)_n(P^y_b),(\varphi,e)_n(P^x_a)) + \widetilde{\rho}((\varphi,e)_n(P^x_a),(\varphi,e)(P^x_a)) \widetilde{<} \frac{\widetilde{\epsilon}}{3} + \frac{\widetilde{\epsilon}}{3} + \frac{\widetilde{\epsilon}}{3} = \widetilde{\epsilon}$. Hence, (φ,e) is a soft continuous mapping at P^x_a. □

In the following theorem, we show that a soft equicontinuous family of soft mappings with soft sequentially compact domain space is a soft uniformly equicontinuous mapping.

Theorem 4.4. *A soft equicontinuous family of soft mappings from soft sequentially compact metric space to any soft metric space is a soft uniformly equicontinuous mapping.*

Proof. Suppose $(\widetilde{X},\widetilde{d},A)$ be a soft sequential compact metric space and $\mathcal{F}$ is a family of soft mappings $(\varphi,e) : (\widetilde{X},\widetilde{d},A) \to (\widetilde{Y},\widetilde{\rho},B)$ that is not a soft uniformly equicontinuous mapping then there is $\widetilde{\epsilon} \widetilde{>} \bar{0}$, such that for every n $\in$ N, there are soft points $P^{x_n}_{a_n}$, $P^{y_n}_{b_n} \widetilde{\in} \widetilde{X}$ and a soft mapping $(\varphi,e)_n \widetilde{\in} \mathcal{F}$ with $\widetilde{d}(P^{x_n}_{a_n},P^{y_n}_{b_n})$ $\widetilde{<} \widetilde{\frac{1}{n}}$ and $\widetilde{\rho}((\varphi,e)_n(P^{x_n}_{a_n}),(\varphi,e)_n(P^{y_n}_{b_n})) \widetilde{\geq} \widetilde{\epsilon}$. Since $(\widetilde{X},\widetilde{d},A)$ is a soft sequential compact, the sequence $\{P^{x_n}_{a_n}\}_n$ of soft points has a subsequence $\{P^{x_{n_k}}_{a_{n_k}}\}$ converging to a soft point $P^x_a \widetilde{\in} \widetilde{X}$. Also by $\widetilde{d}(P^{x_n}_{a_n}, P^{y_n}_{b_n}) \widetilde{<} \widetilde{\frac{1}{n}}$, we get corresponding subsequence $\{P^{y_{n_k}}_{b_{n_k}}\}$ of $\{P^{y_n}_{b_n}\}_n$ also converges to P^x_a. Hence for all $\widetilde{\epsilon} \widetilde{>} \bar{0}$, there are soft points $P^{x_{n_k}}_{a_{n_k}}, P^{y_{n_k}}_{b_{n_k}}$ such that $\widetilde{d}(P^{x_{n_k}}_{a_{n_k}}, P^x_a) \widetilde{<} \widetilde{\delta}$ and $\widetilde{d}(P^{y_{n_k}}_{b_{n_k}}, P^x_a) \widetilde{<} \widetilde{\delta}$. Then we have either $\widetilde{\rho}((\varphi,e)_n(P^{x_{n_k}}_{a_{n_k}}), (\varphi,e)_n(P^x_a)) \widetilde{\geq} \widetilde{\epsilon}$ or $\widetilde{\rho}((\varphi,e)_n(P^{y_{n_k}}_{b_{n_k}}), (\varphi,e)_n(P^x_a)) \widetilde{\geq} \widetilde{\epsilon}$ which implies $\mathcal{F}$ is not soft equicontinuous at P^x_a, which is a contradiction. Hence, $\mathcal{F}$ is a soft uniformly equicontinuous mapping. □

Finally, we get analogous result of Theorem 3.11 for soft uniform equicontinuity.

Theorem 4.5. *For a family $\mathcal{A}$ of soft mappings from $\tilde{X}$ to $\tilde{Y}$, the following conditions are equivalent:*

1. *$\mathcal{A}$ is a soft uniformly equicontinuous mapping on $\widetilde{X}$.*
2. *for any sequence of soft points $\{P^{x_n}_{a_n}\}_n$, $\{P^{y_n}_{b_n}\}_n$ of soft points in $\widetilde{X}$, $\widetilde{d}(P^{x_n}_{a_n}, P^{y_n}_{b_n}) \longrightarrow \bar{0}$ implies that for every $\widetilde{\epsilon} \widetilde{>} \bar{0}$, there exists a positive integer N (depending on $\widetilde{\epsilon}$) such that for all $n \geq N$ and for every (φ,e) in $\mathcal{A}$, $\widetilde{\rho}((\varphi,e)(P^{x_n}_{a_n}), (\varphi,e)(P^{y_n}_{b_n})) \widetilde{<} \widetilde{\epsilon}$.*
3. *for any sequence of soft points $\{P^{d_n}_{e_n}\}_n$, $\{P^{y_n}_{b_n}\}_n$ in (Z,A) and $\widetilde{X}$ respectively, $\widetilde{d}(P^{d_n}_{e_n}, P^{y_n}_{b_n}) \longrightarrow \bar{0}$ implies that for every $\widetilde{\epsilon} \widetilde{>} \bar{0}$, there exists a positive integer N (depending on $\widetilde{\epsilon}$) such that for all $n \geq N$ and for every (φ,e) in $\mathcal{A}$, $\widetilde{\rho}((\varphi,e)(P^{d_n}_{e_n}), (\varphi,e)(P^{y_n}_{b_n})) \longrightarrow \bar{0}$.*
4. *the family $\mathcal{A}|_{(Z,A)} = \{(\varphi,e)|_{(Z,A)} | (\varphi,e) \in \mathcal{A}\}$ is a soft uniformly equicontinuous mapping on (Z,A) and every (φ,e) in $\mathcal{A}$ is a soft continuous mapping at each soft point of $(Z,A)^c$.*

Proof. (1) ⇒ (2): If (1) holds then for given $\widetilde{\epsilon} \widetilde{>} \bar{0}$, there exists $\widetilde{\delta} = \widetilde{\delta}(\widetilde{\epsilon}) \widetilde{>} \bar{0}$ such that for any (φ,e) in $\mathcal{A}$, $\widetilde{\rho}((\varphi,e)(P^x_a), (\varphi,e)(P^y_b)) \widetilde{<} \widetilde{\epsilon}$ whenever $\widetilde{d}(P^x_a, P^y_b) \widetilde{<} \widetilde{\delta}$. Then, if $\widetilde{d}(P^{x_n}_{a_n}, P^{y_n}_{b_n}) \longrightarrow \bar{0}$ there exists a positive integer N=N($\widetilde{\delta}$) =N($\widetilde{\epsilon}$) such that for every n ≥ N, $\widetilde{d}(P^{x_n}_{a_n}, P^{y_n}_{b_n}) \widetilde{<} \widetilde{\delta}$. Therefore, for every (φ,e) in $\mathcal{A}$ and n ≥ N, $\widetilde{\rho}((\varphi,e)(P^{x_n}_{a_n}), (\varphi,e)(P^{y_n}_{b_n})) \widetilde{<} \widetilde{\epsilon}$ and hence (2) holds.

(2) ⇒ (1): If $\mathcal{A}$ is not a soft uniformly equicontinuous mapping on $\widetilde{X}$, then there exist $\widetilde{\epsilon} \widetilde{>} \bar{0}$ such that for every positive integer n, there exists $(\varphi,e)_n$ in $\mathcal{A}$ and soft points $P^{x_n}_{a_n}$ and $P^{y_n}_{b_n}$ in $\widetilde{X}$ such that $\widetilde{d}(P^{x_n}_{a_n}, P^{y_n}_{b_n}) \widetilde{<} \widetilde{\frac{1}{n}}$ and so, $\widetilde{d}(P^{x_n}_{a_n}, P^{y_n}_{b_n}) \longrightarrow \bar{0}$ but $\widetilde{\rho}((\varphi,e)_n(P^{x_n}_{a_n}), (\varphi,e)_n(P^{y_n}_{b_n})) \widetilde{\geq} \tilde{\epsilon}$, which is a contradiction.

(2) ⇒ (3) is obvious.

(3) ⇒ (4): by proof of (2) ⇒ (1), we see that if (3) holds then $\mathcal{A}|_{(Z,A)}$ is a soft uniformly equicontinuous mapping on (Z,A). Now let $P^x_a \in (Z,A)^c$ and (φ,e) in $\mathcal{A}$. Take $P^{y_n}_{b_n} = P^x_a$ for every n in (3), then we get a sequence $\{P^{d_n}_{e_n}\}_n$ of soft points in (Z,A) converging to P^x_a which implies $(\varphi,e)(P^{d_n}_{e_n})) \longrightarrow (\varphi,e)(P^x_a)$. Hence (φ,e) is a soft continuous mapping by Theorem 2.11.

(4) ⇒ (1): Let $\widetilde{\epsilon} \widetilde{>} \bar{0}$. By soft uniform equicontinuity of $\mathcal{A}|_{(Z,A)}$, there exists $\widetilde{\delta} = \widetilde{\delta}(\widetilde{\epsilon}) \widetilde{>} \bar{0}$ such that for every (φ,e) in $\mathcal{A}$, $\widetilde{\rho}((\varphi,e)(P^d_e), (\varphi,e)(P^t_f)) \widetilde{<} \frac{\tilde{\epsilon}}{3}$, for any pair of soft points P^d_e and P^t_f in (Z,A) satisfying $\widetilde{d}(P^d_e, P^t_f) \widetilde{<} \widetilde{\delta}$. Let (φ,e) in $\mathcal{A}$ and P^x_a, $P^y_b \in \widetilde{X}$ with $\widetilde{d}(P^x_a, P^y_b) \widetilde{<} \frac{\tilde{\delta}}{3}$. Now as (Z,A) is soft dense in $\widetilde{X}$, there exist P^d_e and P^t_f in (Z,A) such that $\widetilde{d}(P^x_a, P^d_e) \widetilde{<} \frac{\tilde{\delta}}{3}$ and $\widetilde{d}(P^y_b, P^t_f) \widetilde{<} \frac{\tilde{\delta}}{3}$. Since (φ,e) is a soft continuous mapping at each point of $(Z,A)^c$ implies that $\widetilde{\rho}((\varphi,e)_n(P^x_a), (\varphi,e)_n(P^d_e)) \widetilde{<} \widetilde{\frac{\epsilon}{3}}$ and $\widetilde{\rho}((\varphi,e)(P^y_b), (\varphi,e)(P^t_f)) \widetilde{<} \widetilde{\frac{\epsilon}{3}}$. Therefore, we get $\widetilde{d}(P^d_e, P^t_f) \widetilde{<} \widetilde{d}(P^x_a, P^d_e) + \widetilde{d}(P^x_a, P^y_b) + \widetilde{d}(P^y_b, P^t_f) \widetilde{<} \widetilde{\frac{\delta}{3}} + \frac{\tilde{\delta}}{3} + \widetilde{\frac{\delta}{3}} = \widetilde{\delta}$ and hence, $\widetilde{\rho}((\varphi,e)(P^d_e), (\varphi,e)(P^t_f)) \widetilde{<} \widetilde{\rho}((\varphi,e)(P^x_a), (\varphi,e)(P^d_e)) + \widetilde{\rho}((\varphi,e)(P^d_e), (\varphi,e)(P^t_f)) + \widetilde{\rho}((\varphi,e)(P^t_f), (\varphi,e)(P^y_b)) \widetilde{<} \widetilde{\frac{\epsilon}{3}} + \widetilde{\frac{\epsilon}{3}} + \widetilde{\frac{\epsilon}{3}} = \widetilde{\epsilon}$. □

5. Conclusion

In this paper, we introduce the definitions of soft equicontinuity at a soft point, soft pointwise equicontinuity on a soft subset of an absolute soft set $\widetilde{X}$ and soft uniformly equicontinuous family of soft mappings.

These definitions help in finding some new conditions of equivalence and in describing the structure of soft equicontinuous mapping and soft uniformly equicontinuous mappings. Providing a theoretical approach for further studies of soft equicontinuity, these can also provide a powerful tool to study the optimization and approximation theory, information system and various fields of engineering.

Acknowledgement

The authors are very grateful to the reviewer's valuable inputs which were of great help to improve the quality of the paper.

References

[1] T.M. Al-shami, Soft somewhere dense sets on soft topological spaces, Communications of the Korean Mathematical Society 33 (2018) 1341–1356.
[2] T.M. Al-shami, Comments on some results related to soft separation axioms, Afrika Matematika 31 (2020) 1105–1119.
[3] T.M. Al-shami, Compactness on soft topological ordered spaces and its application on the information system, Journal of Mathematics 2021 (2021), Article ID 6699092, 12 pages.
[4] T.M. Al-shami, On soft separation axioms and their applications on decisionmaking problem, Mathematical Problems in Engineering 2021 (2021), Article ID 8876978.
[5] T.M. Al-shami, I. Alshammari, B.A. Asaad, Soft maps via soft somewhere dense sets, Filomat 34 (2020) 3429–3440.
[6] T.M. Al-shami, M.E. El-Shafei, T-soft equality relation, Turkish Journal of Mathematics 44 (2020) 1427-1441.
[7] T.M. Al-shami, M.E. El-Shafei, Partial belong relation on soft separation axioms and decision-making problem, two birds with one stone, Soft Computing 24 (2020) 5377–5387.
[8] T.M. Al-shami, Lj.D.R. Kočinac, The equivalence between the enriched and extended soft topologies, Applied and Computational Mathematics 18 (2019) 149–162.
[9] I. Altintas, K. Taskopru, Compactness of soft cone metric space and fixed point theorems related to diametrically contractive mapping, Turkish Journal of Mathematics 44 (2020) 2199–2216.
[10] A. Aygünoğlu, H. Aygün, Some notes on soft topological spaces, Neural Computing and Applications 21 (2012) 113–119.
[11] S. Bayramov, C. Gunduz, M. Yazar, Some notes on Compact sets in Soft Metric spaces, arXiv preprint arXiv:1308.3390: 2013.
[12] S. Das, S.K. Samanta, Soft real sets, soft real numbers and their properties, The Journal of Fuzzy Mathematics 20 (2012) 551–576.
[13] S. Das, S.K. Samanta, Soft metric, Annals of Fuzzy Mathematics and informatics 6 (2013) 77–94.
[14] S. Kaur, A. Singh, N.S. Noorie, Apllications of soft dense sets to soft continuity, Advances in Mathematics: Scientific Journal 9 (2020) 3623-=3630.
[15] A. Kharal, B. Ahmad, Mappings on soft classes, New Mathematics and Natural Computation 7 (2011) 471–481.
[16] P.K. Maji, R. Biswas, A.R. Roy, Soft set theory, Computers and Mathematics with Applications 45 (2003) 555–562.
[17] P. Majumdar, S.K. Samanta, On soft mappings, Computers and Mathematics with Applications 60 (2010) 2666–2672.
[18] D. Molodtsov, Soft set theory-First results, Computers and Mathematics with Applications 37(4-5) (1999) 19–31.
[19] I. Murat, G. Cigdem, S. Bayramov, Fixed point theorems of soft contractive mappings, Filomat 30 (2016) 269–279.
[20] S. Nazmul, S. Samanta, Neighbourhood properties of soft topological spaces, Annals of Fuzzy Mathematics and Informatics 6 (2013) 1–15.
[21] A. Özkan, On near soft sets, Turkish Journal of Mathematics 43 (2019) 1005–1017.
[22] A. Özkan, M. Akdag, F. Erol, Soft b-compact spaces, New Trends in Mathematical Sciences 4 (2016) 211–219.
[23] T.Y. Özturk, S. Bayramov, Soft mappings space, The Scientific World Journal (2014) (2014), Article ID 307292, 8 pages.
[24] T.Y. Özturk, A. Benek, A. Özkan, Neutrosophic soft compact spaces, Afrika Matematika 32 (2021) 301–316.
[25] D. Pei, D. Miao, From soft sets to information system, In: Proceedings of the IEEE International Conference on Granular Computing 2 (2005) 617–621.
[26] M. Shabir, M. Naz, On soft topological spaces, Computers and Mathematics with Applications 61(7) (2011) 1786–1799.
[27] I. Zorlutuna, M. Akdag, W.K. Min, S. Atmaca, Remarks on soft topological spaces, Annals of Fuzzy Mathematics and Informatics 3 (2021) 171–185.