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# Erratum: A Companion of Ostrowski Type Integral Inequality Using a 5-Step Kernel with Some Applications

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**Abstract.** The aim of this paper is to correct results from the published paper: A companion of Ostrowski Type Integral Inequality Using a 5-Step Kernel with Some Applications, Filomat 30:13 (2016), 3601-3614.

## 1. Introduction

In the recent paper [4] authors obtained the following identity

$$\frac{1}{b-a}\int_a^b P(x,t)\, f'(t)\, dt \tag{1}$$
$$= \frac{1}{4}\left[f(x) + f(a+b-x) + f\left(\frac{a+x}{2}\right) + f\left(\frac{a+2b-x}{2}\right)\right] - \frac{1}{b-a}\int_a^b f(t)\, dt$$

where $x \in \left[a, \frac{a+b}{2}\right]$ and $P(x,t)$ is the kernel, defined by

$$P(x,t) = \begin{cases} t-a, & a \le t \le \frac{a+x}{2}, \\ t - \frac{3a+b}{4} & \frac{a+x}{2} < t \le x, \\ t - \frac{a+b}{2}, & x < t \le a+b-x, \\ t - \frac{a+3b}{4} & a+b-x < t \le \frac{a+2b-x}{2}, \\ t-b & \frac{a+2b-x}{2} < t \le b. \end{cases}$$

Starting from this identity involving 5-step kernel, authors have obtained five new 4-point Ostrowski type inequalities for a differentiable function $f$. Three theorems hold for $f$ such that $f' \in L^1[a,b]$, one theorem holds for $f' \in L^2[a,b]$ and one for $f'' \in L^2[a,b]$. Special cases are considered to obtain midpoint,

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trapezoidal and higher order quadrature rules. Finally, results are applied to obtain composite quadrature rules and to probability cumulative distribution function.

Unfortunately, in the proof of the Theorem 2.2 Grüss inequality has been applied for function with upper and lower bounding function instead of upper and lower bounding constant. In the proof of the Theorem 2.8, Theorem 2.14 and Theorem 2.19 there are two calculation mistakes in (16) and (35). Thus the results of four theorems are not correct, consequently also of all corollaries and applications. Mistake is obvious in corollaries in which negative upper bound is obtained for the absolute value of the term on the left-hand side.

In present paper we give corrections of all results in four theorems and an improvement of the fifth theorem.

## 2. Main results

Here the symbol $L^p_{[a,b]}$ $(p \geq 1)$ denotes the space of $p$-power integrable functions on the interval $[a, b]$ equipped with the norm

$$\left\|f\right\|_p = \left(\int_a^b \left|f(t)\right|^p dt\right)^{\frac{1}{p}}$$

and $L^\infty_{[a,b]}$denotes the space of essentially bounded functions on $[a, b]$ with the norm

$$\left\|f\right\|_\infty = ess \sup_{t\in[a,b]} \left|f(t)\right|.$$

The following Ostrowski inequality is well known, proved in 1938 in [3]:

$$\left|f(x) - \frac{1}{b-a}\int_a^b f(t)dt\right| \leq \left[\frac{1}{4} + \frac{\left(x - \frac{a+b}{2}\right)^2}{(b-a)^2}\right](b-a)\left\|f'\right\|_\infty .$$

It holds for every $x \in [a, b]$ whenever $f : [a, b] \to \mathbb{R}$ is continuous on $[a, b]$ and differentiable on $(a, b)$ with derivative $f' : (a, b) \to \mathbb{R}$ bounded on $(a, b)$.

For two Lebesgue integrable functions $f, g : [a, b] \to \mathbb{R}$, Čebyšev functional is given by

$$T(f,g) = \frac{1}{b-a}\int_a^b f(x)g(x)dx - \frac{1}{(b-a)^2}\left(\int_a^b f(x)dx\right)\left(\int_a^b g(x)dx\right).$$

In 1934, Grüss in his paper [1] proved that

$$\left|T(f,g)\right| \leq \frac{1}{4}(M-m)(N-n), \tag{2}$$

provided that there exists the real numbers $m, M, n, N$ such that

$$m \leq f(t) \leq M, \quad n \leq g(t) \leq N$$

for a.e. $t \in [a, b]$.

*2.1. Case $f' \in L^1_{[a,b]}$*

**Theorem 2.1.** *Let $I \subset \mathbb{R}$ be an open interval, $a, b \in I$, $a < b$ and let $f : I \to \mathbb{R}$ be differentiable on $(a, b)$. If $f' \in L^1_{[a,b]}$ and $\gamma \leq f'(t) \leq \Gamma$ for all $t \in [a, b]$, then the inequality*

$$\left|\frac{1}{4}\left[f(x) + f(a+b-x) + f\left(\frac{a+x}{2}\right) + f\left(\frac{a+2b-x}{2}\right)\right] - \frac{1}{b-a}\int_a^b f(t)\,dt\right| \leq \frac{1}{4}(b-a)(\Gamma - \gamma) \tag{3}$$

*holds for all $x \in \left[a, \frac{a+b}{2}\right]$.*

*Proof.* For $x \in \left[a, \frac{a+b}{2}\right]$ and $t \in [a, b]$ we have

$$-\frac{b-a}{2} \leq P(x,t) \leq \frac{b-a}{2}.$$

By applying Grüss inequality to the mappings $P(x, \cdot)$ and $f'$ we obtain

$$\left|\frac{1}{b-a}\int_a^b P(x,t) f'(t)\,dt - \frac{1}{(b-a)^2}\left(\int_a^b P(x,t)\,dt\right)\left(\int_a^b f'(t)\,dt\right)\right| \leq \frac{1}{4}(b-a)(\Gamma-\gamma).$$

Using the fact that $\int_a^b P(x,t)\,dt = 0$ and (1), we get required result. □

**Remark 2.2.** *The previous theorem is the correct version of Theorem 2.2. from [4]. In Theorem 2.2. Grüss inequality was improperly applied with bounds for $P(x, t)$ which are not constants*

$$x - \frac{3a+b}{2} \leq P(x,t) \leq x - a.$$

*In such way constant $\frac{1}{16}$ instead of $\frac{1}{4}$ has been obtained. Consequently, all obtained inequalities in corollaries are also not correct.*

**Corollary 2.3.** *Suppose that all the assumptions of Theorem 2.1 hold. Then the following inequalities hold*

$$\left|\frac{1}{2}\left[f(a) + f(b)\right] - \frac{1}{b-a}\int_a^b f(t)\,dt\right| \leq \frac{1}{4}(b-a)(\Gamma-\gamma),$$

$$\left|\frac{1}{4}\left[2f\left(\frac{a+b}{2}\right) + f\left(\frac{3a+b}{4}\right) + f\left(\frac{a+3b}{4}\right)\right] - \frac{1}{b-a}\int_a^b f(t)\,dt\right| \leq \frac{1}{4}(b-a)(\Gamma-\gamma),$$

$$\left|\frac{1}{4}\left[f\left(\frac{3a+b}{4}\right) + f\left(\frac{a+3b}{4}\right) + f\left(\frac{7a+b}{8}\right) + f\left(\frac{a+7b}{8}\right)\right] - \frac{1}{b-a}\int_a^b f(t)\,dt\right| \leq \frac{1}{4}(b-a)(\Gamma-\gamma).$$

*Proof.* By applying (3) with $x = a$, $x = \frac{a+b}{2}$ and $x = \frac{3a+b}{4}$. □

**Theorem 2.4.** *Let $I \subset \mathbb{R}$ be an open interval, $a, b \in I$, $a < b$ and let $f : I \to \mathbb{R}$ be differentiable on $(a, b)$. If $f' \in L^1_{[a,b]}$ and $\gamma \leq f'(t) \leq \Gamma$ for all $t \in [a, b]$, then the inequality holds for all $x \in \left[a, \frac{a+b}{2}\right]$*

$$\left|\frac{1}{4}\left[f(x) + f(a+b-x) + f\left(\frac{a+x}{2}\right) + f\left(\frac{a+2b-x}{2}\right)\right] - \frac{1}{b-a}\int_a^b f(t)\,dt\right| \leq \frac{\Gamma-\gamma}{2(b-a)}\alpha(x) \tag{4}$$

*where*

$$\alpha(x) = \begin{cases} \left(\frac{a+b-2x}{2}\right)\left(\frac{b-x}{2}\right), & x \in \left[a, \frac{3a+b}{4}\right], \\ \left(\frac{x-a}{2}\right)^2 + \left(x - \frac{3a+b}{4}\right)^2 + \frac{5}{4}\left(\frac{a+b}{2} - x\right)^2, & x \in \left[\frac{3a+b}{4}, \frac{a+b}{2}\right]. \end{cases}$$

*Proof.* In [4] following is obtained for arbitrary constant $C \in \mathbb{R}$

$$\frac{1}{b-a}\int_a^b P(x,t) f'(t)\,dt = \frac{1}{b-a}\int_a^b P(x,t)(f'(t) - C)\,dt =$$
$$= \frac{1}{4}\left[f(x) + f(a+b-x) + f\left(\frac{a+x}{2}\right) + f\left(\frac{a+2b-x}{2}\right)\right] - \frac{1}{b-a}\int_a^b f(t)\,dt$$

and

$$\left|\frac{1}{b-a}\int_a^b P(x,t)(f'(t) - C)\,dt\right| \leq \frac{1}{(b-a)}\max_{t\in[a,b]}\left|f'(t) - C\right|\left(\int_a^b |P(x,t)|\,dt\right). \tag{5}$$

Since $\gamma \leq f'(t) \leq \Gamma$ for all $t \in [a, b]$, we have

$$\max_{t\in[a,b]}\left|f'(t) - \frac{\gamma+\Gamma}{2}\right| \leq \frac{\Gamma-\gamma}{2}.$$

In case $x \in \left[a, \frac{3a+b}{4}\right]$ we have

$$\begin{aligned}\int_a^b |P(x,t)|\,dt &= \left(\frac{x-a}{2}\right)^2 + \left(\frac{3a+b}{4} - \frac{a+x}{2}\right)^2 - \left(\frac{3a+b}{4} - x\right)^2 + \left(\frac{a+b}{2} - x\right)^2 \\ &= \left(\frac{x-a}{2}\right)^2 + \left(\frac{2a+b-3x}{2}\right)\left(\frac{x-a}{2}\right) + \left(\frac{a+b}{2} - x\right)^2 \\ &= \left(\frac{x-a}{2}\right)\left(\frac{a+b-2x}{2}\right) + \left(\frac{a+b}{2} - x\right)^2 \\ &= \left(\frac{a+b-2x}{2}\right)\left(\frac{b-x}{2}\right).\end{aligned}$$

In case $x \in \left[\frac{3a+b}{4}, \frac{a+b}{2}\right]$ we have

$$\begin{aligned}\int_a^b |P(x,t)|\,dt &= \left(\frac{x-a}{2}\right)^2 + \left(\frac{3a+b}{4} - \frac{a+x}{2}\right)^2 + \left(x - \frac{3a+b}{4}\right)^2 + \left(\frac{a+b}{2} - x\right)^2 \\ &= \left(\frac{x-a}{2}\right)^2 + \left(x - \frac{3a+b}{4}\right)^2 + \frac{5}{4}\left(\frac{a+b}{2} - x\right)^2.\end{aligned}$$

Applying (5) with $C = \frac{\gamma+\Gamma}{2}$ we obtain (4). □

**Remark 2.5.** *The previous theorem is the correct version of Theorem 2.8. from [4]. In Theorem 2.8. incorrect inequality*

$$\max_{t\in[a,b]}\left|f'(t)-\frac{\gamma+\Gamma}{2}\right|\leq\frac{\gamma+\Gamma}{2}$$

*was applied. Also, calculation of* $\int_a^b |P(x,t)|\,dt$ *was not correct. Consequently, all obtained inequalities in corollaries are also not correct.*

**Corollary 2.6.** *Suppose that all the assumptions of Theorem 2.4 hold. Then the following inequalities hold*

$$\left|\frac{1}{2}[f(a)+f(b)]-\frac{1}{b-a}\int_a^b f(t)\,dt\right|\leq\frac{1}{8}(b-a)(\Gamma-\gamma),$$

$$\left|\frac{1}{4}\left[2f\left(\frac{a+b}{2}\right)+f\left(\frac{3a+b}{4}\right)+f\left(\frac{a+3b}{4}\right)\right]-\frac{1}{b-a}\int_a^b f(t)\,dt\right|\leq\frac{1}{16}(b-a)(\Gamma-\gamma),$$

$$\left|\frac{1}{4}\left[f\left(\frac{3a+b}{4}\right)+f\left(\frac{a+3b}{4}\right)+f\left(\frac{7a+b}{8}\right)+f\left(\frac{a+7b}{8}\right)\right]-\frac{1}{b-a}\int_a^b f(t)\,dt\right|$$
$$\leq\frac{3}{64}(b-a)(\Gamma-\gamma).$$

*Proof.* By applying (3) with $x=a$, $x=\frac{a+b}{2}$ and $x=\frac{3a+b}{4}$. □

The next theorem is the improvement of Theorem 2.13. from [4].

**Theorem 2.7.** *Let $I\subset\mathbb{R}$ be an open interval, $a,b\in I$, $a<b$ and let $f:I\to\mathbb{R}$ be differentiable on $(a,b)$. If $f'\in L^1_{[a,b]}$ and $\gamma\leq f'(t)\leq\Gamma$ for all $t\in[a,b]$, then the inequality holds for all $x\in\left[a,\frac{a+b}{2}\right]$*

$$\left|\frac{1}{4}\left[f(x)+f(a+b-x)+f\left(\frac{a+x}{2}\right)+f\left(\frac{a+2b-x}{2}\right)\right]-\frac{1}{b-a}\int_a^b f(t)\,dt\right|$$
$$\leq\Omega(x)\cdot\min\{S-\gamma,\Gamma-S\} \tag{6}$$

*where $S=\frac{f(b)-f(a)}{b-a}$ and*

$$\Omega(x)=\begin{cases}\frac{a+b}{2}-x, & x\in\left[a,\frac{3a+b}{4}\right],\\ \max\left\{\frac{x-a}{2},\frac{a+b}{2}-x,x-\frac{3a+b}{4}\right\}, & x\in\left[\frac{3a+b}{4},\frac{a+b}{2}\right].\end{cases}$$

*Proof.* Let's denote

$$R(x)=\frac{1}{b-a}\int_a^b P(x,t)\,f'(t)\,dt-\frac{1}{(b-a)^2}\left(\int_a^b P(x,t)\,dt\right)\left(\int_a^b f'(t)\,dt\right). \tag{7}$$

In [4] following is obtained for arbitrary constant $C\in\mathbb{R}$

$$R(x)=\frac{1}{4}\left[f(x)+f(a+b-x)+f\left(\frac{a+x}{2}\right)+f\left(\frac{a+2b-x}{2}\right)\right]-\frac{1}{b-a}\int_a^b f(t)\,dt, \tag{8}$$

$$|R(x)|\leq\frac{1}{b-a}\max_{t\in[a,b]}|P(x,t)-0|\left(\int_a^b\left|f'(t)-C\right|dt\right)$$

and in [5]

$$\int_a^b\left|f'(t)-\gamma\right|dt=(S-\gamma)(b-a),$$

$$\int_a^b\left|f'(t)-\Gamma\right|dt=(\Gamma-S)(b-a).$$

Further, for $x\in\left[a,\frac{3a+b}{4}\right]$ we have

$$\Omega(x)=\max_{t\in[a,b]}|P(x,t)|=\frac{a+b}{2}-x$$

and for $x\in\left[\frac{3a+b}{4},\frac{a+b}{2}\right]$

$$\begin{aligned}\Omega(x)=\max_{t\in[a,b]}|P(x,t)|&=\max\left\{\frac{x-a}{2},\frac{a+b}{2}-x,x-\frac{3a+b}{4},\frac{3a+b}{4}-\frac{a+x}{2}\right\}\\&=\max\left\{\frac{x-a}{2},\frac{a+b}{2}-x,x-\frac{3a+b}{4}\right\}.\end{aligned}$$

Thus (6) is obtained. □

**Corollary 2.8.** *Suppose that all the assumptions of Theorem 2.7 hold. Then the following inequalities hold*

$$\left|\frac{1}{2}[f(a)+f(b)]-\frac{1}{b-a}\int_a^b f(t)\,dt\right|\leq\frac{b-a}{2}\min\{S-\gamma,\Gamma-S\},$$

$$\left|\frac{1}{4}\left[2f\left(\frac{a+b}{2}\right)+f\left(\frac{3a+b}{4}\right)+f\left(\frac{a+3b}{4}\right)\right]-\frac{1}{b-a}\int_a^b f(t)\,dt\right|$$
$$\leq\frac{b-a}{4}\min\{S-\gamma,\Gamma-S\},$$

$$\left|\frac{1}{4}\left[f\left(\frac{3a+b}{4}\right)+f\left(\frac{a+3b}{4}\right)+f\left(\frac{7a+b}{8}\right)+f\left(\frac{a+7b}{8}\right)\right]-\frac{1}{b-a}\int_a^b f(t)\,dt\right|$$
$$\leq\frac{b-a}{4}\min\{S-\gamma,\Gamma-S\}.$$

*Proof.* By applying (3) with $x=a$, $x=\frac{a+b}{2}$ and $x=\frac{3a+b}{4}$. □

*2.2. Case $f' \in L^2_{[a,b]}$*

**Theorem 2.9.** *Let $f : [a,b] \to \mathbb{R}$ be an absolutely continuous mapping in $(a,b)$. If $f' \in L^2_{[a,b]}$, then for all $x \in \left[a, \frac{a+b}{2}\right]$ the following inequality holds*

$$\left|\frac{1}{4}\left[f(x) + f(a+b-x) + f\left(\frac{a+x}{2}\right) + f\left(\frac{a+2b-x}{2}\right)\right] - \frac{1}{b-a}\int_a^b f(t)\,dt\right| \leq \frac{1}{b-a}\sqrt{\sigma(f')\,\beta(x)} \tag{9}$$

*where*

$$\sigma(f') = \left\|f'\right\|_2^2 - \frac{(f(b)-f(a))^2}{b-a}$$

*and*

$$\beta(x) = \frac{2}{3}\left[\left(\frac{x-a}{2}\right)^3 + \left(x - \frac{3a+b}{4}\right)^3 + \frac{9}{8}\left(\frac{a+b}{2} - x\right)^3\right]. \tag{10}$$

*Proof.* Let $R(x)$ be as in (7). In [4] following inequality is obtained using the Cauchy inequality

$$\begin{aligned} |R(x)| &\leq \frac{1}{b-a}\left[\int_a^b \left(f'(t) - \frac{1}{b-a}\int_a^b f'(s)\,ds\right)^2 dt\right]^{\frac{1}{2}} \\ &\quad \cdot \left[\int_a^b \left(P(x,t) - \frac{1}{b-a}\int_a^b P(x,s)\,ds\right)^2 dt\right]^{\frac{1}{2}} \\ &= \frac{1}{b-a}\sqrt{\sigma(f')}\left[\int_a^b \left(P(x,t) - \frac{1}{b-a}\int_a^b P(x,s)\,ds\right)^2 dt\right]^{\frac{1}{2}}. \end{aligned}$$

Further, for $x \in \left[a, \frac{a+b}{2}\right]$ we have

$$\begin{aligned} \beta(x) &= \int_a^b \left(P(x,t) - \frac{1}{b-a}\int_a^b P(x,s)\,ds\right)^2 dt = \int_a^b (P(x,t))^2\,dt \\ &= 2\left[\int_a^{\frac{a+x}{2}} (t-a)^2\,dt + \int_{\frac{a+x}{2}}^x \left(t - \frac{3a+b}{4}\right)^2 dt + \int_x^{\frac{a+b}{2}} \left(t - \frac{a+b}{2}\right)^2 dt\right] \\ &= \frac{2}{3}\left[\left(\frac{x-a}{2}\right)^3 + \left(x - \frac{3a+b}{4}\right)^3 + \frac{9}{8}\left(\frac{a+b}{2} - x\right)^3\right]. \end{aligned}$$

Then from (8) we obtain (9). □

**Remark 2.10.** *The previous theorem is the correct version of Theorem 2.14. from [4]. In Theorem 2.14. calculation of $\int_a^b (P(x,t))^2\,dt$ was not correct. Consequently, all obtained inequalities in corollaries are not correct.*

**Corollary 2.11.** *Suppose that all the assumptions of Theorem 2.9 hold. Then the following inequalities hold*

$$\left|\frac{1}{2}[f(a) + f(b)] - \frac{1}{b-a}\int_a^b f(t)\,dt\right| \leq \sqrt{\sigma(f')\,\frac{b-a}{12}},$$

$$\left|\frac{1}{4}\left[2f\left(\frac{a+b}{2}\right)+f\left(\frac{3a+b}{4}\right)+f\left(\frac{a+3b}{4}\right)\right]-\frac{1}{b-a}\int_a^b f(t)\,dt\right|\leq\sqrt{\sigma(f')\frac{b-a}{48}},$$

$$\left|\frac{1}{4}\left[f\left(\frac{3a+b}{4}\right)+f\left(\frac{a+3b}{4}\right)+f\left(\frac{7a+b}{8}\right)+f\left(\frac{a+7b}{8}\right)\right]-\frac{1}{b-a}\int_a^b f(t)\,dt\right|$$
$$\leq\sqrt{\sigma(f')\frac{10(b-a)}{768}}.$$

*Proof.* By applying (3) with $x=a,\ x=\frac{a+b}{2}$ and $x=\frac{3a+b}{4}$. □

*2.3. Case $f''\in L^2_{[a,b]}$*

**Theorem 2.12.** *Let $f:[a,b]\to\mathbb{R}$ be a twice continuously differentiable mapping in $(a,b)$ with $f''\in L^2_{[a,b]}$, then or all $x\in\left[a,\frac{a+b}{2}\right]$ the following inequality holds*

$$\left|\frac{1}{4}\left[f(x)+f(a+b-x)+f\left(\frac{a+x}{2}\right)+f\left(\frac{a+2b-x}{2}\right)\right]-\frac{1}{b-a}\int_a^b f(t)\,dt\right|$$
$$\leq\frac{1}{\pi}\left\|f'\right\|_2\sqrt{\beta(x)} \tag{11}$$

*where $\beta(x)$ is defined with (10).*

*Proof.* Let $R(x)$ be as in (7). In [4] following inequality is obtained using the Cauchy and Diaz-Metkalf inequality (see [2])

$$|R(x)|\leq\frac{1}{b-a}\left[\int_a^b\left(f'(t)-f'\left(\frac{a+b}{2}\right)\right)^2dt\right]^{\frac{1}{2}}\left[\int_a^b\left(P(x,t)-\frac{1}{b-a}\int_a^b P(x,s)\,ds\right)^2dt\right]^{\frac{1}{2}}$$
$$=\frac{1}{\pi}\left\|f'\right\|_2\left[\int_a^b(P(x,t))^2\,dt\right]^{\frac{1}{2}}=\frac{1}{\pi}\left\|f'\right\|_2\sqrt{\beta(x)}.$$

□

**Remark 2.13.** *The previous theorem is the correct version of the Theorem 2.19. from [4]. In the Theorem 2.19. the same mistake as in the Theorem 2.14 was made, that is the incorrect calculation of $\int_a^b(P(x,t))^2\,dt$. Consequently, all obtained inequalities in corollaries are not correct.*

**Corollary 2.14.** *Suppose that all the assumptions of Theorem 2.9 hold. Then the following inequalities hold*

$$\left|\frac{1}{2}[f(a)+f(b)]-\frac{1}{b-a}\int_a^b f(t)\,dt\right|\leq\frac{b-a}{\pi}\left\|f'\right\|_2\sqrt{\frac{b-a}{12}},$$

$$\left|\frac{1}{4}\left[2f\left(\frac{a+b}{2}\right)+f\left(\frac{3a+b}{4}\right)+f\left(\frac{a+3b}{4}\right)\right]-\frac{1}{b-a}\int_a^b f(t)\,dt\right|$$
$$\leq\frac{b-a}{\pi}\left\|f'\right\|_2\sqrt{\frac{b-a}{48}},$$

$$\left|\frac{1}{4}\left[f\left(\frac{3a+b}{4}\right)+f\left(\frac{a+3b}{4}\right)+f\left(\frac{7a+b}{8}\right)+f\left(\frac{a+7b}{8}\right)\right]-\frac{1}{b-a}\int_a^b f(t)\,dt\right|$$
$$\leq \frac{b-a}{\pi}\left\|f'\right\|_2\sqrt{\frac{10(b-a)}{192}}.$$

*Proof.* By applying (3) with $x = a$, $x = \frac{a+b}{2}$ and $x = \frac{3a+b}{4}$. □

Results of the last five theorems were applied in [4] to obtain composite quadrature rules and for results involving probability cumulative distribution function. Corresponding corrected results are omitted here.

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