Filomat 35:5 (2021), 1687–1691
https://doi.org/10.2298/FIL2105687Z

Published by Faculty of Sciences and Mathematics,
University of Niš, Serbia
Available at: http://www.pmf.ni.ac.rs/filomat

Corrigendum to "On (L, M)-Fuzzy Convex Structures"

Hu Zhao[a], Qiao-Ling Song[a], O.R. Sayed[b], E. El-Sanousy[c], Y.H.Ragheb Sayed[c], Gui-Xiu Chen[d]

[a] *School of Science, Xi'an Polytechnic University, 710048 Xi'an, P.R. China*
[b] *Department of Mathematics, Faculty of Science Assiut University, Assiut 71516, Egypt*
[c] *Department of Mathematics, Faculty of Science, Sohag University, Sohag 82524, Egypt*
[d] *School of Mathematics and Statistics, Qinghai Normal University, 810008 Xining, P. R. China*

Abstract. In this paper, we point out that the proof of Theorem 2.4(5), Proposition 2.6(1) and Proposition 2.8(1) in the paper titled "On (L, M)-fuzzy convex structures" (Filomat 33(13): 4151-4163, 2019) are not true in general. Then, we give three correct proofs of these results.

1. Introduction

Sayed et al.[4] defined a new class of L-fuzzy sets called r-L-fuzzy biconvex sets in (L, M)-fuzzy convex structures. The transformation method between L-fuzzy hull operators and (L, M)-fuzzy convex structures were introduced, and a characterization of the product of the L-fuzzy hull operator was obtained. The aim of this article is to correct some errors in the proof of Theorem 2.4(5),Proposition 2.6(1) and Proposition 2.8(1) proposed by Sayed et al. ([4]).

2. Preliminaries

Throughout this paper, let X be a non-empty set, both L and M be two completely distributive lattices with order reversing involution $'$ where $\perp_M$ $(\perp_L)$ and $\top_M(\top_L)$ denote the least and the greatest elements in $M(L)$ respectively, and $M_{\perp_M} = M - \{\perp_M\}(L_{\perp_L} = L - \{\perp_L\})$. Recall that an order-reversing involution $'$ on L is a map $(-)' : L \longrightarrow L$ such that for any $a, b \in L$, the following conditions hold: (1) $a \leq b$ implies $b' \leq a'$. (2) $a'' = a$. The following properties hold for any subset $\{b_i : i \in I\} \in L$: (1) $(\bigvee_{i\in I} b_i)' = \bigwedge_{i\in I} b_i'$; (2) $(\bigwedge_{i\in I} b_i)' = \bigvee_{i\in I} b_i'$. An L-fuzzy subset of X is a mapping $\mu : X \longrightarrow L$ and the family L^X denoted the set of all fuzzy subsets of a given X ([1]). The least and the greatest elements in L^X are denoted by $\chi_\emptyset$ and χ_X, respectively. For each $\alpha \in L$, let $\underline{\alpha}$ denote the constant L-fuzzy subset of X with the value α. The complementation of a fuzzy subset are defined as $\mu'(x) = (\mu(x))'$ for all $x \in X$, (e.g. $\mu'(x) = 1 - \mu(x)$ in the case of $L = [0, 1]$). We say $\{\mu_i : i \in \Gamma\}$

2010 *Mathematics Subject Classification*. Primary 03E72; Secondary 52A01, 54A40
Keywords. (L, M)-fuzzy convex structure, L-fuzzy hull operator, (L, M)-fuzzy convexity preserving function
Received: 29 April 2020; Revised: 24 July 2020; Accepted 25 July 2020
Communicated by Ljubiša D.R. Kočinac
Research supported by the National Natural Science Foundation of China (Grant No. 12171386), the Applied Basic Research Program Funded by Qinghai Province (Program No.2019-ZJ-7078), the Scientific Research Program Funded by Shaanxi Provincial Education Department (Program No.18JK0360), and the Doctoral Scientific Research Foundation of Xi'an Polytechnic University (Grant No. BS1426).
Email addresses: zhaohu@xpu.edu.cn (Hu Zhao), songqlaa@139.com (Qiao-Ling Song), *o_r_sayed*@yahoo.com (O.R. Sayed), elsanowsy@yahoo.com (E. El-Sanousy), yh_raghp2011@yahoo.com (Y.H.Ragheb Sayed), cgx0510@yahoo.com.cn (Gui-Xiu Chen)

is a directed (resp. co-directed) subset of L^X, in symbols $\{\mu_i : i \in \Gamma\} \overset{dir}{\subseteq} L^X$ (resp. $\{\mu_i : i \in \Gamma\} \overset{cdir}{\subseteq} L^X$) if for each $\mu_1, \mu_2 \in \{\mu_i : i \in \Gamma\}$, there exists $\mu_3 \in \{\mu_i : i \in \Gamma\}$ such that $\mu_1, \mu_2 \leq \mu_3$ (resp. $\mu_1, \mu_2 \geq \mu_3$). An element $a \neq \perp_M$ in a lattice is called coprime if $a \leq b \vee c$ implies $a \leq b$ or $a \leq c$ for all $b, c \in M$. Further, a is said to be join-irreducible if $a = b \vee c$ implies $a = b$ or $a = c$ for all $b, c \in M$. The set of all non-zero coprime elements (resp. join-irreducible elements) of M is denoted $\mathrm{Copr}(M)$ (resp. $J(M)$). It can be verified that if M is distributive, then $a \in M$ is coprime iff it is join-irreducible, which means $\mathrm{Copr}(M) = J(M)$. So, for convenience, we usually use $J(M)$ to stand for the set of all coprime elements of M if M is distributive. If M is a completely distributive lattice and $x \triangleleft \bigvee_{t \in T} y_t$, then there must be $t^\star \in T$ such that $x \triangleleft y_{t^\star}$ (here $x \triangleleft a$ means: $K \subset M, a \leq \bigvee K \Rightarrow \exists y \in K$ such that $x \leq y$), and for each $b \in M$, $b = \bigvee\{a \in M : a \triangleleft b\} = \bigvee\{a \in J(M) : a \triangleleft b\}$. Some more properties of $\triangleleft$ can be found in [2] and [6].

First, we recall two definitions which will be used in this paper.

Definition 2.1. ([5]) The pair $(X, \mathcal{C})$ is called an (L, M)-fuzzy convex structure ((L, M)-fcs, for short), where $\mathcal{C} : L^X \longrightarrow M$ satisfying the following axioms:

(LMC1) $\mathcal{C}(\underline{0}) = \mathcal{C}(\underline{1}) = \top_M$.

(LMC2) If $\{\mu_i : i \in \Gamma\} \subseteq L^X$ is nonempty, then

$$\mathcal{C}(\bigwedge_{i\in\Gamma} \mu_i) \geq \bigwedge_{i\in\Gamma} \mathcal{C}(\mu_i).$$

(LMC3) If $\{\mu_i : i \in \Gamma\} \subseteq L^X$ is nonempty and totally ordered by inclusion, then

$$\mathcal{C}(\bigvee_{i\in\Gamma} \mu_i) \geq \bigwedge_{i\in\Gamma} \mathcal{C}(\mu_i).$$

The mapping $\mathcal{C}$ is called an (L, M)-fuzzy convexity on X and $\mathcal{C}(\mu)$ can be regarded as the degree to which μ is an L-convex fuzzy set.

Definition 2.2. ([3]) Let $f : X \longrightarrow Y$. Then the image $f^{\rightarrow}(\mu)$ of $\mu \in L^X$ and the preimage $f^{\leftarrow}(\nu)$ of $\nu \in L^Y$ are defined by:

$$f^{\rightarrow}(\mu)(y) = \bigvee\{\mu(x) : x \in X, f(x) = y\}$$

and $f^{\leftarrow}(\nu) = \nu \circ f$, respectively. It can be verified that the pair $(f^{\rightarrow}, f^{\leftarrow})$ is a Galois connection on $(L^X, \leq)$ and $(L^Y, \leq)$.

Next, we recall Theorem 2.4, Proposition 2.6 and Proposition 2.8 of [4] as follows.

Theorem 2.3. ([4, Theorem 2.4]) *Let $(X, \mathcal{C})$ be an (L, M)-fuzzy convex structure. For each $\mu \in L^X$ and $r \in M_{\perp_M}$, we define a mapping $CO_{\mathcal{C}} : L^X \times M_{\perp_M} \longrightarrow L^X$ as follows:*

$$CO_{\mathcal{C}}(\mu, r) = \bigwedge\{\nu \in L^X : \mu \leq \nu,\ \mathcal{C}(\nu) \geq r\}.$$

For $\mu, \nu \in L^X$ and $r, s \in M_{\perp_M}$ the operator $CO_{\mathcal{C}}$ satisfies the following conditions:

(1) $CO_{\mathcal{C}}(\underline{0}, r) = \underline{0}$.

(2) $\mu \leq CO_{\mathcal{C}}(\mu, r)$.

(3) If $\mu \leq \nu$, then $CO_{\mathcal{C}}(\mu, r) \leq CO_{\mathcal{C}}(\nu, r)$.

(4) If $r \leq s$, then $CO_{\mathcal{C}}(\mu, r) \leq CO_{\mathcal{C}}(\mu, s)$.

(5) $CO_{\mathcal{C}}(CO_{\mathcal{C}}(\mu, r), r) = CO_{\mathcal{C}}(\mu, r)$.

(6) For $\{\mu_i : i \in \Gamma\} \subseteq L^X$ is nonempty and totally ordered by inclusion,

$$CO_{\mathcal{C}}(\bigvee_{i\in\Gamma} \mu_i, r) = \bigvee_{i\in\Gamma} CO_{\mathcal{C}}(\mu_i, r).$$

A mapping $CO_{\mathcal{C}}$ is called L-fuzzy hull operator generated by an (L, M)-fuzzy convex structure.

Proposition 2.4. ([4, Proposition 2.6(1)]) *Let $(X, \mathcal{C}_1, \mathcal{C}_2)$ be an (L, M)-fbcs. For each $r \in M_{\perp_M}$ and $\mu \in L^X$, a mapping $\mathcal{C}_{CO_{12}} : L^X \longrightarrow M$ is defined as follows*

$$\mathcal{C}_{CO_{12}}(\mu) = \bigvee\{r \in M_{\perp_M} : \mu = CO_{12}(\mu, r)\},$$

where $CO_{12}(\mu, r) = CO_{\mathcal{C}1}(\mu, r) \wedge CO_{\mathcal{C}2}(\mu, r)$ satisfies the conditions (1)-(6) of Theorem 2.3 (see [4]). Then $\mathcal{C}_{CO_{12}}$ is an (L, M)-fuzzy convexity on X.

Proposition 2.5. ([4, Proposition 2.8]) *Let $(X, \mathcal{C})$ and $(Y, \mathcal{D})$ be (L, M)-fuzzy convex structures. Then $f : X \longrightarrow Y$ is*

(1) An (L, M)-fuzzy convexity preserving function if and only if $f^{\rightarrow}(CO_{\mathcal{C}}(\mu, r)) \leq CO_{\mathcal{D}}(f^{\rightarrow}(\mu), r)$ for all $\mu \in L^X$ and $r \in M_{\perp_M}$.

(2) An (L, M)-fuzzy convex-to-convex function if and only if $CO_{\mathcal{D}}(f^{\rightarrow}(\mu), r) \leq f^{\rightarrow}(CO_{\mathcal{C}}(\mu, r))$ for all $\mu \in L^X$ and $r \in M_{\perp_M}$.

3. Main Results

First, we point out that the proof of Theorem 2.4(5), Proposition 2.6(1) and Proposition 2.8(1) are not true in general (see [4]). Here is why:

Notice that L (M) is a completely distributive lattice, not a unit interval [0,1]. So, if $a \nleq b$, it doesn't imply $a > b$. Because there exists another case that a and b may are not compparable, i.e., $a \parallel b$.

Now, we provide three correct proofs of these results as follows.

Proposition 3.1. ([4, Theorem 2.4(5)]) *Let $(X, \mathcal{C})$ be an (L, M)-fuzzy convex structure. For each $\mu \in L^X$ and $r \in M_{\perp_M}$, we define a mapping $CO_{\mathcal{C}} : L^X \times M_{\perp_M} \longrightarrow L^X$ as follows:*

$$CO_{\mathcal{C}}(\mu, r) = \bigwedge\{\nu \in L^X : \mu \leq \nu,\ \mathcal{C}(\nu) \geq r\}.$$

Then

$$CO_{\mathcal{C}}(CO_{\mathcal{C}}(\mu, r), r) = CO_{\mathcal{C}}(\mu, r).$$

Proof. For all $\mu \in L^X$ and $r \in M_{\perp_M}$. By the definition of $CO_{\mathcal{C}}(\mu, r)$, we have $\mu \leq CO_{\mathcal{C}}(\mu, r)$. Hence, $CO_{\mathcal{C}}(CO_{\mathcal{C}}(\mu, r), r) \geq CO_{\mathcal{C}}(\mu, r)$.

On the other hand,

$$\begin{aligned} CO_{\mathcal{C}}(CO_{\mathcal{C}}(\mu, r), r) &= CO_{\mathcal{C}}\left(\wedge\left\{\nu \in L^X : \mu \leq \nu,\ \mathcal{C}(\nu) \geq r\right\}, r\right) \\ &\leq \bigwedge_{\mu \leq \nu,\ \mathcal{C}(\nu) \geq r} CO_{\mathcal{C}}(\nu, r) \\ &= \bigwedge_{\mu \leq \nu,\ \mathcal{C}(\nu) \geq r}\ \bigwedge_{\nu \leq \varpi,\ \mathcal{C}(\varpi) \geq r} \varpi \\ &= \bigwedge_{\mu \leq \varpi,\ \mathcal{C}(\varpi) \geq r} \varpi \\ &= CO_{\mathcal{C}}(\mu, r). \end{aligned}$$

Hence $CO_{\mathcal{C}}(CO_{\mathcal{C}}(\mu, r), r) = CO_{\mathcal{C}}(\mu, r)$. □

Proposition 3.2. ([4, Proposition 2.6(1)]) *Let $(X, \mathcal{C}_1, \mathcal{C}_2)$ be an (L, M)-fbcs. For each $r \in M_{\perp_M}$ and $\mu \in L^X$, a mapping $\mathcal{C}_{CO_{12}} : L^X \longrightarrow M$ is defined as follows*

$$\mathcal{C}_{CO_{12}}(\mu) = \bigvee\{r \in M_{\perp_M} : \mu = CO_{12}(\mu, r)\}.$$

Then $\mathcal{C}_{CO_{12}}$ is an (L, M)-fuzzy convexity on X.

Proof. (LMC1) Since for all $r \in M_{\perp_M}$, $CO_{12}(\underline{1}, r) \geq \underline{1}$ and $CO_{12}(\underline{0}, r) = \underline{0}$, we have

$$\mathcal{C}_{CO_{12}}(\underline{0}) = \mathcal{C}_{CO_{12}}(\underline{1}) = \top_M.$$

(LMC2) Suppose that $b \in J(M)$ and $b \triangleleft \bigwedge_{i\in\Gamma} \mathcal{C}_{CO_{12}}(\mu_i)$. Then $b \triangleleft \mathcal{C}_{CO_{12}}(\mu_i)$ for all $i \in \Gamma$. There exists $r_0^i \in M_{\perp_M}$ such that $\mu_i = CO_{12}(\mu_i, r_0^i)$ and $b \triangleleft r_0^i$ (thus $b \leq r_0^i$). Put $r_0 = \bigwedge_{i\in\Gamma} r_0^i$, then $b \leq r_0$. Since CO_{12} satisfies the conditions (1)-(6) of Theorem 2.3, we have $CO_{12}(\bigwedge_{i\in\Gamma} \mu_i, r_0^i) \leq CO_{12}(\mu_i, r_0^i)$ for all $i \in \Gamma$. Then it follows that

$$CO_{12}(\bigwedge_{i\in\Gamma} \mu_i, r_0) \leq \bigwedge_{i\in\Gamma} CO_{12}(\bigwedge_{i\in\Gamma} \mu_i, r_0^i) \leq \bigwedge_{i\in\Gamma} CO_{12}(\mu_i, r_0^i) = \bigwedge_{i\in\Gamma} \mu_i.$$

On the other hand, by Theorem 2.3 (2), we have

$$CO_{12}(\bigwedge_{i\in\Gamma} \mu_i, r_0) \geq \bigwedge_{i\in\Gamma} \mu_i.$$

So, we obtain

$$CO_{12}(\bigwedge_{i\in\Gamma} \mu_i, r_0) = \bigwedge_{i\in\Gamma} \mu_i.$$

By the definition of $\mathcal{C}_{CO_{12}}(\bigwedge_{i\in\Gamma} \mu_i)$, we obtain $\mathcal{C}_{CO_{12}}(\bigwedge_{i\in\Gamma} \mu_i) \geq r_0 \geq b$. Hence

$$\mathcal{C}_{CO_{12}}(\bigwedge_{i\in\Gamma} \mu_i) \geq \bigwedge_{i\in\Gamma} \mathcal{C}_{CO_{12}}(\mu_i).$$

(LMC3) Let $\{\mu_i : i \in \Gamma\} \subseteq L^X$ is nonempty and totally ordered by inclusion. Suppose that $b \in J(M)$ and $b \triangleleft \bigwedge_{i\in\Gamma} \mathcal{C}_{CO_{12}}(\mu_i)$. Then $b \triangleleft \mathcal{C}_{CO_{12}}(\mu_i)$ for all $i \in \Gamma$. There exists $r_0^i \in M_{\perp_M}$ such that $\mu_i = CO_{12}(\mu_i, r_0^i)$ and $b \triangleleft r_0^i$ (thus $b \leq r_0^i$). Put $r_0 = \bigwedge_{i\in\Gamma} r_0^i$, then $b \leq r_0$. By Theorem 2.3 (6), we have

$$\bigvee_{i\in\Gamma} \mu_i \leq CO_{12}(\bigvee_{i\in\Gamma} \mu_i, r_0) = \bigvee_{i\in\Gamma} CO_{12}(\mu_i, r_0) \leq \bigvee_{i\in\Gamma} CO_{12}(\mu_i, r_0^i) = \bigvee_{i\in\Gamma} \mu_i$$

So, we obtain

$$CO_{12}(\bigvee_{i\in\Gamma} \mu_i, r_0) = \bigvee_{i\in\Gamma} \mu_i.$$

By the definition of $\mathcal{C}_{CO_{12}}(\bigvee_{i\in\Gamma} \mu_i)$, we obtain $\mathcal{C}_{CO_{12}}(\bigvee_{i\in\Gamma} \mu_i) \geq r_0 \geq b$. Hence $\mathcal{C}_{CO_{12}}(\bigvee_{i\in\Gamma} \mu_i) \geq \bigwedge_{i\in\Gamma} \mathcal{C}_{CO_{12}}(\mu_i)$. □

Proposition 3.3. ([4, Proposition 2.8(1)]) *Let $(X, \mathcal{C})$ and $(Y, \mathcal{D})$ be (L, M)-fuzzy convex structures. Then $f : X \longrightarrow Y$ is an (L, M)-fuzzy convexity preserving function if and only if $f^{\rightarrow}(CO_{\mathcal{C}}(\mu, r)) \leq CO_{\mathcal{D}}(f^{\rightarrow}(\mu), r)$ for all $\mu \in L^X$ and $r \in M_{\perp_M}$.*

Proof. ($\Longrightarrow$) Since $f : X \longrightarrow Y$ is an (L, M)-fuzzy convexity preserving function, we obtain $\mathcal{C}(f^{\leftarrow}(\varpi)) \geq \mathcal{D}(\varpi)$ for all $\varpi \in L^Y$. So, for each $r \in M_{\perp_M}$ and $\mu \in L^X$, we obtain

$$\begin{aligned} f^{\leftarrow}[CO_{\mathcal{D}}(f^{\rightarrow}(\mu), r)] &= f^{\leftarrow}\left[\bigwedge \left\{\varpi \in L^Y : f^{\rightarrow}(\mu) \leq \varpi,\ \mathcal{D}(\varpi) \geq r\right\}\right] \\ &= \bigwedge \left\{f^{\leftarrow}(\varpi) \in L^X : f^{\rightarrow}(\mu) \leq \varpi,\ \mathcal{D}(\varpi) \geq r\right\} \\ &\geq \bigwedge \left\{f^{\leftarrow}(\varpi) \in L^X : \mu \leq f^{\leftarrow}(\varpi),\ \mathcal{C}(f^{\leftarrow}(\varpi)) \geq r\right\} \\ &\geq \bigwedge \left\{\nu \in L^X : \mu \leq \nu,\ \mathcal{C}(\nu) \geq r\right\} = CO_{\mathcal{C}}(\mu, r). \end{aligned}$$

Hence

$$f^{\longrightarrow}(CO_{\mathcal{C}}(\mu, r)) \leq f^{\longrightarrow} f^{\leftarrow}[CO_{\mathcal{D}}(f^{\rightarrow}(\mu), r)] \leq CO_{\mathcal{D}}(f^{\rightarrow}(\mu), r).$$

($\Longleftarrow$) Suppose that $b \in J(M)$ and $b \triangleleft \mathcal{D}(\varpi)$ for all $\varpi \in L^Y$, then $b \leq \mathcal{D}(\varpi)$. So,

$$f^{\rightarrow}(CO_{\mathcal{C}}(f^{\leftarrow}(\varpi), b)) \leq CO_{\mathcal{D}}(f^{\rightarrow}(f^{\leftarrow}(\varpi)), b) \leq CO_{\mathcal{D}}(\varpi, b) = \varpi.$$

It follows that

$$f^{\leftarrow}(\varpi) \leq CO_{\mathcal{C}}(f^{\leftarrow}(\varpi), b) \leq f^{\leftarrow}(\varpi).$$

Therefore, $CO_{\mathcal{C}}(f^{\leftarrow}(\varpi), b) = f^{\leftarrow}(\varpi)$. Furthermore,

$$\mathcal{C}(f^{\leftarrow}(\varpi)) = \mathcal{C}(CO_{\mathcal{C}}(f^{\leftarrow}(\varpi), b)) = \mathcal{C}\left(\bigwedge \left\{\nu \in L^X : f^{\leftarrow}(\varpi) \leq \nu,\ \mathcal{C}(\nu) \geq b\right\}\right) \geq \bigwedge_{f^{\leftarrow}(\varpi) \leq \nu,\ \mathcal{C}(\nu) \geq b} \mathcal{C}(\nu) \geq b.$$

Hence $\mathcal{C}(f^{\leftarrow}(\varpi)) \geq \mathcal{D}(\varpi)$ and $f : X \longrightarrow Y$ is an (L, M)-fuzzy convexity preserving function. $\square$

4. Conclusion

In this paper, we point out that the proof of Theorem 2.4(5),Proposition 2.6(1) and Proposition 2.8(1) in [4] are incorrect, and then, we present the modified versions.

Acknowledgement

Authors would like to express their sincere thanks to the referees and the editors for giving valuable comments which helped to improve the presentation of this paper.

References

[1] J.A. Goguen, L-fuzzy sets, J. Math. Anal. Appl. 18 (1967) 145–174.
[2] Y.-M. Liu, M.-K. Luo, Fuzzy Topology, World Scientific Publishing, Singapore, 1997.
[3] S.E. Rodabaugh, Powerset operator based foundation for point-set latticetheoretic (poslat) fuzzy set theories and topologies, Quaest. Math. 20 (1997) 463–530.
[4] O.R. Sayed, E. El-Sanousy, Y.H. Raghp Sayed, On (L, M)-fuzzy convex structures, Filomat 33 (2019) 4151–4163.
[5] F.-G. Shi, Z.-Y. Xiu, (L, M)-Fuzzy convex structures, J. Nonlinear Sci. Appl. 10 (2017) 3655–3669.
[6] F.-G. Shi, Z.-Y. Xiu, A new approach to the fuzzification of convex structures, J. Appl. Math. 2014 (2014) Art. No. 249183.