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Radical Transversal SCR-Lightlike Submanifolds of Indefinite Sasakian Manifolds

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Abstract. In this paper, we introduce the notion of radical transversal screen Cauchy-Riemann (SCR)-lightlike submanifolds of indefinite Sasakian manifolds giving characterization theorem with some non-trivial examples of such submanifolds. Integrability conditions of distributions D_1, D_2, D and $D^{\perp}$ on radical transversal SCR-lightlike submanifolds of an indefinite Sasakian manifold have been obtained. Further, we obtain necessary and sufficient conditions for foliations determined by above distributions to be totally geodesic.

1. Introduction

The theory of lightlike submanifolds of a semi-Riemannian manifold was introduced by Duggal and Bejancu [2]. A Sasakian manifold with pseudo-Riemannian metric is called an indefinite Sasakian manifold and was introduced by T. Takahashi [12]. Various classes of lightlike submanifolds of an indefinite Sasakian manifold are defined according to the behaviour of distributions on these submanifolds with respect to the action of (1, 1) tensor field ϕ in Sasakian structure of the ambient manifolds. Such submanifolds were studied by Duggal-Sahin in [5]. Further, they defined and studied invariant, screen real lightlike, contact CR-lightlike and screen CR-lightlike submanifolds of an indefinite Sasakian manifold [3, 6]. In [4], authors introduced the notion of generalized CR-lightlike submanifolds which includes contact CR-lightlike and SCR-lightlike submanifolds as its particular cases. All these submanifolds of an indefinite Sasakian manifold mentioned above have invariant radical distribution on their tangent bundles. In [17], Yildırim and Sahin introduced radical transversal and transversal lightlike submanifolds of an indefinite Sasakian manifold satisfying the condition that the action of the structure tensor field on radical distribution of such submanifolds does not belong to the tangent bundle.
Later on, Wang and Liu [14] introduced the notion of generalized transversal lightlike submanifolds which contains the classes of radical transversal and transversal lightlike submanifolds of an indefinite Sasakian manifold. Thus motivated sufficiently, we introduce the notion of radical transversal screen Cauchy-Riemann (SCR)-lightlike submanifolds of an indefinite Sasakian manifold. This new class of lightlike submanifolds includes invariant, screen real, contact screen Cauchy-Riemann, transversal, radical transversal and generalized transversal lightlike submanifolds as its sub-cases.

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The paper is arranged as follows. There are some basic results in section 2 . In section 3, we study radical transversal screen Cauchy-Riemann (SCR)-lightlike submanifolds of an indefinite Sasakian manifold, giving some examples. Section 4 is devoted to the study of foliations determined by distributions D_1, D_2, D and $D^{\perp}$ involved in the definition of the above submanifolds of an indefinite Sasakian manifold.

2. Preliminaries

A submanifold (M^m, g) immersed in a semi-Riemannian manifold $(\overline{M}^{m+n}, \overline{g})$ is called a lightlike submanifold [2] if the metric g induced from $\overline{g}$ is degenerate and the radical distribution $RadTM$ is of rank r, where $1 \leq r \leq m$. Let $S(TM)$ be a screen distribution which is a semi-Riemannian complementary distribution of $RadTM$ in TM, that is

$$TM = RadTM \oplus_{orth} S(TM). \tag{1}$$

Now consider a screen transversal vector bundle $S(TM^{\perp})$, which is a semi- Riemannian complementary vector bundle of $RadTM$ in $TM^{\perp}$. Since for any local basis $\{\xi_i\}$ of $RadTM$, there exists a local null frame $\{N_i\}$ of sections with values in the orthogonal complement of $S(TM^{\perp})$ in $[S(TM)]^{\perp}$ such that $\overline{g}(\xi_i, N_j) = \delta_{ij}$ and $\overline{g}(N_i, N_j) = 0$, it follows that there exists a lightlike transversal vector bundle $ltr(TM)$ locally spanned by $\{N_i\}$. Let $tr(TM)$ be complementary (but not orthogonal) vector bundle to TM in $T\overline{M}|_M$. Then

$$tr(TM) = ltr(TM) \oplus_{orth} S(TM^{\perp}), \tag{2}$$

$$T\overline{M}|_M = TM \oplus tr(TM), \tag{3}$$

$$T\overline{M}|_M = S(TM) \oplus_{orth} [RadTM \oplus ltr(TM)] \oplus_{orth} S(TM^{\perp}). \tag{4}$$

Following are four cases of a lightlike submanifold $(M, g, S(TM), S(TM^{\perp}))$:
Case.1 r-lightlike if $r < min\,(m, n)$,
Case.2 co-isotropic if $r = n < m$, $S\,(TM^{\perp}) = \{0\}$,
Case.3 isotropic if $r = m < n$, $S\,(TM) = \{0\}$,
Case.4 totally lightlike if $r = m = n$, $S(TM) = S(TM^{\perp}) = \{0\}$.
The Gauss and Weingarten formulae are given as

$$\overline{\nabla}_X Y = \nabla_X Y + h(X, Y), \quad \forall X, Y \in \Gamma(TM), \tag{5}$$

$$\overline{\nabla}_X V = -A_V X + \nabla^t_X V, \quad \forall V \in \Gamma(tr(TM)), \tag{6}$$

where $\{\nabla_X Y, A_V X\}$ and $\left\{h(X, Y), \nabla^t_X V\right\}$ belong to $\Gamma(TM)$ and $\Gamma(tr(TM))$, respectively. ∇ and ∇^t are linear connections on M and the vector bundle $tr(TM)$, respectively. The second fundamental form h is a symmetric $F(M)$-bilinear form on $\Gamma(TM)$ with values in $\Gamma(tr(TM))$ and the shape operator A_V is a linear endomorphism of $\Gamma(TM)$. From (5) and (6), we have

$$\overline{\nabla}_X Y = \nabla_X Y + h^l\,(X, Y) + h^s\,(X, Y)\,, \quad \forall X, Y \in \Gamma(TM), \tag{7}$$

$$\overline{\nabla}_X N = -A_N X + \nabla^l_X\,(N) + D^s\,(X, N)\,, \quad \forall N \in \Gamma(ltr(TM)), \tag{8}$$

$$\overline{\nabla}_X W = -A_W X + \nabla^s_X\,(W) + D^l\,(X, W)\,, \quad \forall W \in \Gamma(S(TM^{\perp})), \tag{9}$$

where $h^l(X, Y) = L\,(h(X, Y))$, $h^s(X, Y) = S\,(h(X, Y))$, $D^l(X, W) = L(\nabla^t_X W)$, $D^s(X, N) = S(\nabla^t_X N)$. L and S are the projection morphisms of $tr(TM)$ on $ltr(TM)$ and $S(TM^{\perp})$, respectively. $\nabla^l$and ∇^s are linear connections on $ltr(TM)$ and $S(TM^{\perp})$ called the lightlike connection and screen transversal connection on M, respectively. For any vector field X tangent to M , we put

$$\phi X = PX + FX, \tag{10}$$

where PX and FX are tangential and transversal parts of ϕX respectively.
Now by using (5), (7)-(9) and metric connection $\overline{\nabla}$, we obtain

$$\overline{g}(h^s(X,Y),W) + \overline{g}(Y, D^l(X,W)) = g(A_W X, Y), \tag{11}$$

$$\overline{g}(D^s(X,N),W) = \overline{g}(N, A_W X). \tag{12}$$

Denote the projection of TM on $S(TM)$ by $\overline{P}$. Then from the decomposition of the tangent bundle of a lightlike submanifold, we have

$$\nabla_X \overline{P}Y = \nabla^*_X \overline{P}Y + h^*(X, \overline{P}Y), \qquad \forall X, Y \in \Gamma(TM), \tag{13}$$

$$\nabla_X \xi = -A^*_\xi X + \nabla^{*t}_X \xi, \qquad \xi \in \Gamma(RadTM). \tag{14}$$

By using above equations, we obtain

$$\overline{g}(h^l(X, \overline{P}Y), \xi) = g(A^*_\xi X, \overline{P}Y), \tag{15}$$

$$\overline{g}(h^*(X, \overline{P}Y), N) = g(A_N X, \overline{P}Y), \tag{16}$$

$$\overline{g}(h^l(X,\xi),\xi) = 0, \qquad A^*_\xi \xi = 0. \tag{17}$$

It is important to note that in general ∇ is not a metric connection. Since $\overline{\nabla}$ is metric connection, by using (7), we get

$$(\nabla_X g)(Y,Z) = \overline{g}(h^l(X,Y),Z) + \overline{g}(h^l(X,Z),Y). \tag{18}$$

A semi-Riemannian manifold $(\overline{M}, \overline{g})$ is called an ϵ-almost contact metric manifold [5] if there exists a (1, 1) tensor field ϕ, a vector field V called the characteristic vector field and a 1-form η, satisfying

$$\phi^2 X = -X + \eta(X)V, \qquad \eta(V) = \epsilon, \qquad \eta \circ \phi = 0, \qquad \phi V = 0, \tag{19}$$

$$\overline{g}(\phi X, \phi Y) = \overline{g}(X,Y) - \epsilon\eta(X)\eta(Y), \qquad \forall X, Y \in \Gamma(T\overline{M}), \tag{20}$$

where $\epsilon = 1$ or -1. It follows that

$$\overline{g}(V,V) = \epsilon, \tag{21}$$

$$\overline{g}(X,V) = \eta(X), \tag{22}$$

$$\overline{g}(X, \phi Y) = -\overline{g}(\phi X, Y), \qquad \forall X, Y \in \Gamma(T\overline{M}). \tag{23}$$

Then $(\phi, V, \eta, \overline{g})$ is called an ϵ-almost contact metric structure on $\overline{M}$.
An ϵ-almost contact metric structure $(\phi, V, \eta, \overline{g})$ is called an indefinite Sasakian structure [12] if and only if

$$(\overline{\nabla}_X \phi)Y = \overline{g}(X,Y)V - \epsilon\eta(Y)X, \qquad \forall X, Y \in \Gamma(T\overline{M}), \tag{24}$$

where $\overline{\nabla}$ is the Levi-Civita connection with respect to $\overline{g}$.
A semi-Riemannian manifold endowed with an indefinite Sasakian structure is called an indefinite Sasakian manifold. From (6), we get

$$(\overline{\nabla}_X V) = -\phi X, \qquad \forall X \in \Gamma(T\overline{M}). \tag{25}$$

Let $(\overline{M}, \overline{g}, \phi, V, \eta)$ be an ϵ-almost contact metric manifold. If $\epsilon = 1$, then $\overline{M}$ is said to be a spacelike ϵ-almost contact metric manifold and if $\epsilon = -1$, then $\overline{M}$ is called a timelike ϵ-almost contact metric manifold. In this paper, we consider indefinite Sasakian manifolds with spacelike characteristic vector field V.

A plane section S in tangent space $T_x\overline{M}$ at a point x of a Sasakian manifold $\overline{M}$ is called a ϕ-section if it is spanned by a unit vector X orthogonal to V and ϕX, where X is a non-zero vector field on $\overline{M}$. The sectional curvature $K(X, \phi X)$ of a ϕ-section is called a ϕ-sectional curvature. If $\overline{M}$ has a ϕ-sectional curvature c which does not depend on the ϕ-section at each point, then c is constant on $\overline{M}$ and $\overline{M}$ is called a Sasakian space form, which we denote by $\overline{M}(c)$. The curvature tensor $\overline{R}$ of Sasakian space form $\overline{M}(c)$ is given by ([8])

$$\begin{aligned}\overline{R}(X,Y)Z =&\frac{(c+3)}{4}(\overline{g}(Y,Z)X - \overline{g}(X,Z)Y) + \frac{(c-1)}{4}(\eta(X)\eta(Z)Y \\ &- \eta(Y)\eta(Z)X + \overline{g}(X,Z)\eta(Y)V - \overline{g}(Y,Z)\eta(X)V \\ &+ \overline{g}(\phi Y,Z)\phi X + \overline{g}(\phi Z,X)\phi Y - 2\overline{g}(\phi X,Y)\phi Z),\end{aligned} \tag{26}$$

for any smooth vector fields X, Y and Z on $\overline{M}$. This result is also true for an indefinite Sasakian manifold $\overline{M}$.

3. Radical Transversal SCR-Lightlike Submanifolds

In this section, we introduce the notion of radical transversal SCR-lightlike submanifolds of an indefinite Sasakian manifold.

Definition 3.1. Let $(M, g, S(TM), S(TM^{\perp}))$ be a lightlike submanifold of an indefinite Sasakian manifold $\overline{M}$ with structure vector field tangent to M. Then we say that M is radical transversal SCR-lightlike submanifold of $\overline{M}$ if following conditions are satisfied:
(i) there exist orthogonal distributions D_1, D_2, D and $D^{\perp}$ on M such that $RadTM = D_1 \oplus_{orth} D_2$ and $S(TM) = D \oplus_{orth} D^{\perp} \perp \{V\}$,
(ii) the distributions D_1 and D are invariant distributions, i.e. $\phi D_1 = D_1$ and $\phi D = D$,
(iii) the distributions D_2 and $D^{\perp}$ are anti-invariant distributions, i.e. $\phi D_2 \subset \Gamma(ltr(TM))$ and $\phi D^{\perp} \subset \Gamma S(TM^{\perp})$.
From the above definition, we have the following decomposition

$$TM = D_1 \oplus_{orth} D_2 \oplus_{orth} D \oplus_{orth} D^{\perp} \perp \{V\}. \tag{27}$$

In particular, we have
(i) if $D_1 = 0$, then M is a generalized transversal lightlike submanifold,
(ii) if $D_1 = 0$ and $D = 0$, then M is a transversal lightlike submanifold,
(iii) if $D_1 = 0$ and $D^{\perp} = 0$, then M is a radical transversal lightlike submanifold,
(iv) if $D_2 = 0$, then M is a contact screen CR-lightlike submanifold,
(v) if $D_2 = 0$ and $D = 0$, then M is a screen real lightlike submanifold,
(vi) if $D_2 = 0$ and $D^{\perp} = 0$, then M is an invariant lightlike submanifold.
Thus this new class of lightlike submanifolds of an indefinite Sasakian manifold includes radical transversal, transversal, generalized transversal, invariant, screen real, contact screen Cauchy-Riemann lightlike submanifolds which have been studied in ([3], [5], [6], [14], [17]) as its sub-cases.
Let $(\mathbb{R}_q^{2m+1}, \overline{g}, \phi, \eta, V)$ denote the manifold $\mathbb{R}_q^{2m+1}$ with its usual Sasakian structure given by

$\eta = \frac{1}{2}(dz - \sum_{i=1}^{m} y^i dx^i), \qquad V = 2\partial z,$

$\overline{g} = \eta \otimes \eta + \frac{1}{4}(-\sum_{i=1}^{\frac{q}{2}} dx^i \otimes dx^i + dy^i \otimes dy^i + \sum_{i=\frac{q}{2}+1}^{m} dx^i \otimes dx^i + dy^i \otimes dy^i),$

$\phi(\sum_{i=1}^{m}(X_i\partial x_i + Y_i\partial y_i) + Z\partial z) = \sum_{i=1}^{m}(Y_i\partial x_i - X_i\partial y_i) + \sum_{i=1}^{m} Y_i y^i \partial z,$

where (x^i, y^i, z) are the cartesian coordinates on $\mathbb{R}_q^{2m+1}$.
Now, we construct some examples of radical transversal SCR-lightlike submanifolds of an indefinite Sasakian manifold.

Example 1. Let $(\mathbb{R}_4^{17}, \overline{g}, \phi, \eta, V)$ be an indefinite Sasakian manifold, where $\overline{g}$ is of signature $(-,-,+,+,+,+,+,+,-,-,+,+,+,+,+,+,+)$ with respect to $\{\partial x_1, \partial x_2, \partial x_3, \partial x_4, \partial x_5, \partial x_6, \partial x_7, \partial x_8, \partial y_1, \partial y_2, \partial y_3, \partial y_4, \partial y_5, \partial y_6, \partial y_7, \partial y_8, \partial z\}$.
Suppose M is a submanifold of $\mathbb{R}_4^{17}$ given by
$x^1 = -y^3 = u_1$, $x^3 = y^1 = u_2$, $x^2 = u_3$, $y^2 = -u_4$, $x^4 = u_3 \cos\alpha - u_4 \sin\alpha$, $y^4 = u_3 \sin\alpha + u_4 \cos\alpha$, $x^5 = -y^6 = u_5$, $x^6 = y^5 = u_6$, $x^7 = u_8 \cos\beta$, $y^8 = u_8 \sin\beta$, $x^8 = u_7 \cos\beta$, $y^7 = u_7 \sin\beta$, $z = u_9$.

The local frame of TM is given by $\{Z_1, Z_2, Z_3, Z_4, Z_5, Z_6, Z_7, Z_8, Z_9\}$, where
$Z_1 = 2(\partial x_1 - \partial y_3 + y^1\partial z)$, $Z_2 = 2(\partial x_3 + \partial y_1 + y^3\partial z)$,
$Z_3 = 2(\partial x_2 + \cos\alpha\partial x_4 + \sin\alpha\partial y_4 + y^2\partial z + \cos\alpha y^4\partial z)$,
$Z_4 = 2(-\partial y_2 - \sin\alpha\partial x_4 + \cos\alpha\partial y_4 - \sin\alpha y^4\partial z)$,
$Z_5 = 2(\partial x_5 - \partial y_6 + y^5\partial z)$, $Z_6 = 2(\partial x_6 + \partial y_5 + y^6\partial z)$,
$Z_7 = 2(\cos\beta\partial x_8 + \sin\beta\partial y_7 + \cos\beta y^8\partial z)$,
$Z_8 = 2(\cos\beta\partial x_7 + \sin\beta\partial y_8 + \cos\beta y^7\partial z)$,
$Z_9 = V = 2\partial z$.
Hence $RadTM = span\{Z_1, Z_2, Z_3, Z_4\}$ and $S(TM) = span\{Z_5, Z_6, Z_7, Z_8, V\}$.
Now $ltr(TM)$ is spanned by $N_1 = \partial x_1 + \partial y_3 + y^1\partial z$, $N_2 = \partial x_3 - \partial y_1 + y^3\partial z$, $N_3 = 2(-\partial x_2 + \cos\alpha\partial x_4 + \sin\alpha\partial y_4 - y^2\partial z + \cos\alpha y^4\partial z)$, $N_4 = 2(-\partial y_2 + \sin\alpha\partial x_4 - \cos\alpha\partial y_4 + \sin\alpha y^4\partial z)$, and $S(TM^\perp)$ is spanned by
$W_1 = 2(\partial x_5 + \partial y_6 + y^5\partial z)$, $W_2 = 2(\partial x_6 - \partial y_5 + y^6\partial z)$,
$W_3 = 2(\sin\beta\partial x_8 - \cos\beta\partial y_7 + \sin\beta y^8\partial z)$,
$W_4 = 2(\sin\beta\partial x_7 - \cos\beta\partial y_8 + \sin\beta y^7\partial z)$.
It follows that $D_1 = span\{Z_1, Z_2\}$ such that $\phi Z_1 = -Z_2$, $\phi Z_2 = Z_1$, which implies that D_1 is invariant with respect to ϕ and $D_2 = span\{Z_3, Z_4\}$ such that $\phi Z_3 = N_4$, $\phi Z_4 = N_3$, which implies that $\phi D_2 \subset ltr(TM)$. On the other hand, we can see that $D = span\{Z_5, Z_6\}$ such that $\phi Z_5 = -Z_6$, $\phi Z_6 = Z_5$, which implies that D is invariant with respect to ϕ and $D^\perp = span\{Z_7, Z_8\}$ such that $\phi Z_7 = W_4$, $\phi Z_8 = W_3$, which implies that $D^\perp$ is anti-invariant with respect to ϕ. Hence M is a radical transversal SCR-lightlike submanifold of $\mathbb{R}^{17}_4$.

Example 2. Let $(\mathbb{R}^{17}_4, \overline{g}, \phi, \eta, V)$ be an indefinite Sasakian manifold, where $\overline{g}$ is of signature $(-,-,+,+,+,+,+,+,-,-,+,+,+,+,+,+,+)$ with respect to $\{\partial x_1, \partial x_2, \partial x_3, \partial x_4, \partial x_5, \partial x_6, \partial x_7, \partial x_8, \partial y_1, \partial y_2, \partial y_3, \partial y_4, \partial y_5, \partial y_6, \partial y_7, \partial y_8, \partial z\}$. Suppose M is a submanifold of $\mathbb{R}^{17}_4$ given by
$x^3 = u_1$, $y^3 = u_2$, $x^2 = u_1\cos\alpha - u_2\sin\alpha$, $y^2 = u_1\sin\alpha + u_2\cos\alpha$, $x^1 = -y^4 = u_3$, $x^4 = -y^1 = u_4$, $x^5 = u_5\cos\beta$, $y^6 = u_5\sin\beta$, $x^6 = u_6\sin\beta$, $y^5 = -u_6\cos\beta$, $x^7 = y^8 = u_7$, $x^8 = y^7 = u_8$, $z = u_9$.
The local frame of TM is given by $\{Z_1, Z_2, Z_3, Z_4, Z_5, Z_6, Z_7, Z_8, Z_9\}$, where
$Z_1 = 2(\partial x_3 + \cos\alpha\partial x_2 + \sin\alpha\partial y_2 + y^3\partial z + \cos\alpha y^2\partial z)$,
$Z_2 = 2(\partial y_3 - \sin\alpha\partial x_2 + \cos\alpha\partial y_2 - \sin\alpha y^2\partial z)$,
$Z_3 = 2(\partial x_1 - \partial y_4 + y^1\partial z)$, $Z_4 = 2(\partial x_4 - \partial y_1 + y^4\partial z)$,
$Z_5 = 2(\cos\beta\partial x_5 + \sin\beta\partial y_6 + \cos\beta y^5\partial z)$,
$Z_6 = 2(\sin\beta\partial x_6 - \cos\beta\partial y_5 + \sin\beta y^6\partial z)$,
$Z_7 = 2(\partial x_7 + \partial y_8 + y^7\partial z)$, $Z_8 = 2(\partial x_8 + \partial y_7 + y^8\partial z)$,
$Z_9 = V = 2\partial z$.
Hence $RadTM = span\{Z_1, Z_2, Z_3, Z_4\}$ and $S(TM) = span\{Z_5, Z_6, Z_7, Z_8, V\}$.
Now $ltr(TM)$ is spanned by $N_1 = -\partial x_3 + \cos\alpha\partial x_2 + \sin\alpha\partial y_2 - y^3\partial z + \cos\alpha y^2\partial z$, $N_2 = -\partial y_3 - \sin\alpha\partial x_2 + \cos\alpha\partial y_2 - \sin\alpha y^2\partial z)$, $N_3 = 2(\partial x_1 + \partial y_4 + y^1\partial z)$, $N_4 = 2(\partial x_4 + \partial y_1 + y^4\partial z)$ and $S(TM^\perp)$ is spanned by
$W_1 = 2(\sin\beta\partial x_5 - \cos\beta\partial y_6 + \sin\beta y^5\partial z)$,
$W_2 = 2(\cos\beta\partial x_6 + \sin\beta\partial y_5 + \cos\beta y^6\partial z)$,
$W_3 = 2(\partial x_7 - \partial y_8 + y^7\partial z)$, $W_4 = 2(\partial x_8 - \partial y_7 + y^8\partial z)$.
It follows that $D_1 = span\{Z_1, Z_2\}$ such that $\phi Z_1 = -Z_2$, $\phi Z_2 = Z_1$, which implies that D_1 is invariant with respect to ϕ and $D_2 = span\{Z_3, Z_4\}$ such that $\phi Z_3 = -N_4$, $\phi Z_4 = -N_3$, which implies that $\phi D_2 \subset ltr(TM)$. On the other hand, we can see that $D = span\{Z_5, Z_6\}$ such that $\phi Z_5 = Z_6$, $\phi Z_6 = -Z_5$, which implies that D is invariant with respect to ϕ and $D^\perp = span\{Z_7, Z_8\}$ such that $\phi Z_7 = W_4$, $\phi Z_8 = W_3$, which implies that $D^\perp$ is anti-invariant with respect to ϕ. Hence M is a radical transversal SCR-lightlike submanifold of $\mathbb{R}^{17}_4$.
Now, we denote the projection morphisms on D_1, D_2, D and $D^\perp$ in TM by P_1, P_2, P_3 and P_4 respectively. Similarly, we denote the projection morphisms of $tr(TM)$ on ν, ϕD_2, μ and $\phi D^\perp$ by Q_1, Q_2, Q_3 and Q_4 respectively, where ν and μ are orthogonal complementry distributions of ϕD_2 and $\phi D^\perp$ in $ltr(TM)$ and $S(TM^\perp)$ respectively. Then, we get

$$X = P_1X + P_2X + P_3X + P_4X + \eta(X)V, \qquad \forall X \in \Gamma(TM). \tag{28}$$

Now applying ϕ to (28), we have

$$\phi X = \phi P_1X + \phi P_2X + \phi P_3X + \phi P_4X. \tag{29}$$

Thus we get $\phi P_1X \in D_1 \subset RadTM$, $\phi P_2X \in \phi D_2 \subset ltr(TM)$, $\phi P_3X \in D \subset S(TM)$, $\phi P_4X \in \phi D^{\perp} \subset S(TM^{\perp})$. Also, we have

$$W = Q_1W + Q_2W + Q_3W + Q_4W, \qquad \forall W \in \Gamma(tr(TM)). \tag{30}$$

Applying ϕ to (30), we obtain

$$\phi W = \phi Q_1W + \phi Q_2W + \phi Q_3W + \phi Q_4W. \tag{31}$$

Thus we get $\phi Q_1W \in \nu \subset ltr(TM)$, $\phi Q_2W \in D_2 \subset RadTM$, $\phi Q_3W \in \mu \subset S(TM^{\perp})$ and $\phi Q_4W \in D^{\perp} \subset S(TM)$. Now, by using (24), (29), (31) and (7)-(9) and identifying the components on D_1, D_2, D, $D^{\perp}$, ν, ϕD_2, μ and $\phi D^{\perp}$, we obtain

$$\begin{aligned} P_1(\nabla_X\phi P_1Y) + P_1(\nabla_X\phi P_3Y) - P_1(A_{\phi P_2Y}X) - P_1(A_{\phi P_4Y}X) \\ = \phi P_1\nabla_XY - \eta(Y)P_1X, \end{aligned} \tag{32}$$

$$\begin{aligned} P_2(\nabla_X\phi P_1Y) + P_2(\nabla_X\phi P_3Y) - P_2(A_{\phi P_2Y}X) - P_2(A_{\phi P_4Y}X) \\ = \phi Q_2h^l(X,Y) - \eta(Y)P_2X, \end{aligned} \tag{33}$$

$$\begin{aligned} P_3(\nabla_X\phi P_1Y) + P_3(\nabla_X\phi P_3Y) - P_3(A_{\phi P_2Y}X) - P_3(A_{\phi P_4Y}X) \\ = \phi P_3\nabla_XY - \eta(Y)P_3X, \end{aligned} \tag{34}$$

$$\begin{aligned} P_4(\nabla_X\phi P_1Y) + P_4(\nabla_X\phi P_3Y) - P_4(A_{\phi P_2Y}X) - P_4(A_{\phi P_4Y}X) \\ = \phi Q_4h^s(X,Y) - \eta(Y)P_4X, \end{aligned} \tag{35}$$

$$\begin{aligned} Q_1h^l(X,\phi P_1Y) + Q_1h^l(X,\phi P_3Y) + Q_1\nabla^l_X\phi P_2Y + Q_1D^l(X,\phi P_4Y) \\ = \phi Q_1h^l(X,Y), \end{aligned} \tag{36}$$

$$\begin{aligned} Q_2h^l(X,\phi P_1Y) + Q_2h^l(X,\phi P_3Y) + Q_2\nabla^l_X\phi P_2Y + Q_2D^l(X,\phi P_4Y) \\ = \phi P_2\nabla_XY, \end{aligned} \tag{37}$$

$$\begin{aligned} Q_3h^s(X,\phi P_1Y) + Q_3h^s(X,\phi P_3Y) + Q_3\nabla^s_X\phi P_4Y + Q_3D^s(X,\phi P_2Y) \\ = \phi Q_3h^s(X,Y), \end{aligned} \tag{38}$$

$$\begin{aligned} Q_4h^s(X,\phi P_1Y) + Q_4h^s(X,\phi P_3Y) + Q_4\nabla^s_X\phi P_4Y + Q_4D^s(X,\phi P_2Y) \\ = \phi P_4\nabla_XY. \end{aligned} \tag{39}$$

Theorem 3.1. *Let M be a radical transversal SCR-lightlike submanifold of an indefinite Sasakian manifold $\overline{M}$ with structure vector field tangent to M. Then μ is an invariant distribution with respect to ϕ.*

Proof. Let M be a radical transversal SCR-lightlike submanifold of an indefinite Sasakian manifold $\overline{M}$. For any $X \in \Gamma(\mu)$, $\xi \in \Gamma(RadTM)$ and $N \in \Gamma(ltr(TM))$, from (23), we have $\overline{g}(\phi X, \xi) = -\overline{g}(X, \phi\xi) = 0$ and $\overline{g}(\phi X, N) = -\overline{g}(X, \phi N) = 0$. Thus ϕX has no components in $RadTM$ and $ltr(TM)$.
Now, for $X \in \Gamma(\mu)$ and $Y \in \Gamma(D^{\perp})$, we have $\overline{g}(\phi X, Y) = -\overline{g}(X, \phi Y) = 0$, as $\phi Y \in \Gamma(\phi D^{\perp})$, which implies that ϕX has no components in $D^{\perp}$.
Finally, suppose that $\phi X = \alpha V$, where α is a smooth function on $\overline{M}$, then from (19), we get $\phi^2(\phi X) = -\phi X$. Thus $\phi^2(\phi(\alpha V)) = -\phi X$, which implies $X = 0$. Hence $\phi(\mu) \subset \Gamma(\mu)$, which complete the proof.
Now, we give a characterization theorem for radical transversal SCR-lightlike submanifolds.

Theorem 3.2. *Let M be a lightlike submanifold of an indefinite Sasakian space form $(\overline{M}(c), \overline{g})$, $c \neq 1$. Then M is a radical transversal SCR-lightlike submanifold if and only if*

(i) the maximal invariant subspace of T_pM, $p \in M$ defines a distribution $\overline{D} = D_1 \oplus D$, where $RadTM = D_1 \oplus D_2$ and D is a non-degenerate invariant distribution on M,

(ii) $\overline{g}(\overline{R}(\xi, N)\xi_1, \xi_2) \neq 0$, for all $\xi \in \Gamma(D_1)$, $N \in \Gamma(ltr(TM))$ and $\xi_1, \xi_2 \in \Gamma(D_2)$,

(iii) $\overline{g}(\overline{R}(X,Y)Z, W) = 0$, for all $X, Y \in \Gamma(D)$ and $Z, W \in \Gamma(D^{\perp})$, where $D^{\perp}$ is the complementry distribution of D in $S(TM)$.

Proof. Let M be a radical transversal SCR-lightlike submanifold of an indefinite Sasakian space form $(\overline{M}(c), \overline{g})$, $c \neq 1$. Then proof of (i) follows from the definition of a radical transversal SCR-lightlike submanifold. From (26), for $\xi \in \Gamma(D_1), N \in \Gamma(ltr(TM))$ and $\xi_1, \xi_2 \in \Gamma(D_2)$, we have

$$\overline{g}(\overline{R}(\xi, N)\xi_1, \xi_2) = \frac{1-c}{2}\overline{g}(\phi\xi, N)\overline{g}(\phi\xi_1, \xi_2). \tag{40}$$

Since D_1 is invariant distribution, we have $\overline{g}(\phi\xi, N) \neq 0$, $\forall \xi \in \Gamma(D_1), N \in \Gamma ltr(TM)$. Also $\phi D_2 \subset ltr(TM)$, so we get $\overline{g}(\phi\xi_1, \xi_2) \neq 0$, $\forall \xi_1, \xi_2 \in \Gamma(D_2)$. Hence $\overline{g}(\overline{R}(\xi, N)\xi_1, \xi_2) \neq 0$, for all $\xi \in \Gamma(D_1)$, $N \in \Gamma(ltr(TM))$ and $\xi_1, \xi_2 \in \Gamma(D_2)$, which proves (ii). From (26), for $X, Y \in \Gamma(D)$ and $Z, W \in \Gamma(D^\perp)$, we have

$$\overline{g}(\overline{R}(X, Y)Z, W) = \frac{1-c}{2}g(\phi X, Y)\overline{g}(\phi Z, W). \tag{41}$$

In view of $\phi Z \in S(TM^\perp)$, we get $\overline{g}(\phi Z, W) = 0$, $\forall Z, W \in \Gamma(D^\perp)$. Hence $\overline{g}(\overline{R}(X, Y)Z, W) = 0$, which proves (iii). Now, conversely suppose that conditions (i), (ii), (iii) are satisfied. Since D_1 is invariant distribution, $\overline{g}(\phi\xi, N) \neq 0$, $\forall \xi \in \Gamma(D_1)$ and $N \in \Gamma(ltr(TM))$. Thus from (ii) and (40), we have $\overline{g}(\phi\xi_1, \xi_2) \neq 0$, $\forall \xi_1, \xi_2 \in \Gamma(D_2)$, which implies $\phi D_2 \subset ltr(TM)$.
Further, since D is non-degenerate invariant distribution, we may choose $X, Y \in \Gamma(D)$ such that $g(\phi X, Y) \neq 0$. Thus from (iii) and (41), we have $\overline{g}(\phi Z, W) = 0$, $\forall Z, W \in \Gamma(D^\perp)$, which implies that ϕZ have no components in $(D^\perp)$. For any $X \in \Gamma(D)$, $\overline{g}(\phi Z, X) = -\overline{g}(Z, \phi X) = 0$, which implies that ϕZ have no components in D.
Now, form (i) and (ii), we also have $\overline{g}(\phi Z, \xi) = -\overline{g}(Z, \phi\xi) = 0$ and $\overline{g}(\phi Z, N) = -\overline{g}(Z, \phi N) = 0$, $\forall \xi \in \Gamma(RadTM)$ and $N \in \Gamma(ltr(TM))$, which implies that ϕZ have no components in $RadTM$ and $ltr(TM)$. Thus, we get $\phi D^\perp \subseteq S(TM^\perp)$, which completes the proof.

Theorem 3.3. *Let M be a radical transversal SCR-lightlike submanifold of an indefinite Sasakian manifold $\overline{M}$ with structure vector field tangent to M. Then the induced connection ∇ is a metric connection if and only if*
(i) $P_3\nabla_X\phi P_1\xi = P_3A_{\phi P_2\xi}X$ and $g(A^*_\xi X, V) = 0$,
(ii) $Q_4h^s(X, \phi P_1\xi) = 0$ and $Q_4D^s(X, \phi P_2\xi) = 0$,
$\forall X \in \Gamma(TM)$ and $\xi \in \Gamma(RadTM)$.

Proof. Let M be a radical transversal SCR-lightlike submanifold of an indefinite Sasakian manifold $\overline{M}$. Then the induced connection ∇ on M is a metric connection if and only if $RadTM$ is parallel distribution with respect to ∇ ([6]), i.e. $\nabla_X\xi \in \Gamma(RadTM)$, $\forall X \in \Gamma(TM), \forall \xi \in \Gamma(RadTM)$. From (24), we have

$$\overline{\nabla}_X\phi\xi = \phi\overline{\nabla}_X\xi \qquad \forall X \in \Gamma(TM), \forall \xi \in \Gamma(RadTM). \tag{42}$$

From (7), (8), (19) and (42), we obtain

$$\begin{aligned}\phi\nabla_X\phi P_1\xi + \phi h^l(X, \phi P_1\xi) + \phi h^s(X, \phi P_1\xi) - \phi A_{\phi P_2\xi}X + \phi\nabla^l_X\phi P_2\xi \\ +\phi D^s(X, \phi P_2\xi) + \nabla_X\xi + h^l(X, \xi) + h^s(X, \xi) = \eta(\nabla_X\xi)V.\end{aligned} \tag{43}$$

In view of equations (14) and (43) and taking tangential components, we get

$$\begin{aligned}\phi P_1\nabla_X\phi P_1\xi + \phi P_3\nabla_X\phi P_1\xi + \phi Q_2h^l(X, \phi P_1\xi) + \phi Q_4h^s(X, \phi P_1\xi) \\ -\phi P_1A_{\phi P_2\xi}X - \phi P_3A_{\phi P_2\xi}X + \phi Q_2\nabla^l_X\phi P_2\xi + \phi Q_4D^s(X, \phi P_2\xi) \\ +\nabla_X\xi + g(A^*_\xi X, V)V = 0,\end{aligned} \tag{44}$$

which completes the proof.

Lemma 3.4. *Let M be a radical transversal SCR-lightlike submanifold of an indefinite Sasakian manifold $\overline{M}$. Then for any $X, Y \in \Gamma(TM - \{V\})$, we have*
(i) $g(\nabla_XY, V) = \overline{g}(Y, \phi X)$,
(ii) $g([X, Y], V) = 2\overline{g}(X, \phi Y)$.

Proof. Let M be a radical transversal SCR-lightlike submanifold of an indefinite Sasakian manifold $\overline{M}$. Since $\overline{\nabla}$ is a metric connection, from (7) and (25), for any $X, Y \in \Gamma(TM - \{V\})$, we have

$$g(\nabla_X Y, V) = \overline{g}(Y, \phi X). \tag{45}$$

From (23) and (45), we have $g([X,Y], V) = 2\overline{g}(X, \phi Y)$.

Theorem 3.5. *Let M be a radical transversal SCR-lightlike submanifold of an indefinite Sasakian manifold $\overline{M}$ with structure vector field tangent to M. Then D_1 is integrable if and only if*
(i) $Q_2 h^l(Y, \phi P_1 X) = Q_2 h^l(X, \phi P_1 Y)$ and $Q_4 h^s(Y, \phi P_1 X) = Q_4 h^s(X, \phi P_1 Y)$,
(ii) $P_3(\nabla_X \phi P_1 Y) = P_3(\nabla_Y \phi P_1 X)$, $\quad \forall X, Y \in \Gamma(D_1)$.

Proof. Let M be a radical transversal SCR-lightlike submanifold of an indefinite Sasakian manifold $\overline{M}$. Let $X, Y \in \Gamma(D_1)$. From (34), we get $P_3(\nabla_X \phi P_1 Y) = \phi P_3 \nabla_X Y$, which gives $P_3(\nabla_X \phi P_1 Y) - P_3(\nabla_Y \phi P_1 X) = \phi P_3[X,Y]$. In view of (37), we have $Q_2 h^l(X, \phi P_1 Y) = \phi P_2 \nabla_X Y$, which implies $Q_2 h^l(X, \phi P_1 Y) - Q_2 h^l(Y, \phi P_1 X) = \phi P_2[X,Y]$. Also from (39), we obtain $Q_4 h^s(X, \phi P_1 Y) = \phi P_4 \nabla_X Y$, which gives $Q_4 h^s(X, \phi P_1 Y) - Q_4 h^s(Y, \phi P_1 X) = \phi P_4[X,Y]$. This concludes the theorem.

Theorem 3.6. *Let M be a radical transversal SCR-lightlike submanifold of an indefinite Sasakian manifold $\overline{M}$ with structure vector field tangent to M. Then $D_2 \oplus \{V\}$ is integrable if and only if*
(i) $P_1(A_{\phi P_2 Y} X) = P_1(A_{Y \phi P_2 X} Y)$ and $P_3(A_{\phi P_2 Y} X) = P_3(A_{\phi P_2 X} Y)$,
(ii) $Q_4 D^s(Y, \phi P_2 X) = Q_4 D^s(X, \phi P_2 Y)$, $\quad \forall X, Y \in \Gamma(D_2 \oplus \{V\})$.

Proof. Let M be a radical transversal SCR-lightlike submanifold of an indefinite Sasakian manifold $\overline{M}$. Let $X, Y \in \Gamma(D_2 \oplus \{V\})$. From (32), we get $P_1(A_{\phi P_2 Y} X) = -\phi P_1 \nabla_X Y$, which gives $P_1(A_{\phi P_2 X} Y) - P_1(A_{\phi P_2 Y} X) = \phi P_1[X,Y]$. In view of (34), we obtain $P_3(A_{\phi P_2 Y} X) = -\phi P_3 \nabla_X Y$, which implies $P_3(A_{\phi P_2 X} Y) - P_3(A_{\phi P_2 Y} X) = \phi P_3[X,Y]$. Also from (39), we have $Q_4 D^s(X, \phi P_2 Y) = \phi P_4 \nabla_X Y$, which gives $Q_4 D^s(X, \phi P_2 Y) - Q_4 D^s(Y, \phi P_2 X) = \phi P_4[X,Y]$. This completes the proof.

Theorem 3.7. *Let M be a radical transversal SCR-lightlike submanifold of an indefinite Sasakian manifold $\overline{M}$ with structure vector field tangent to M. Then $D \oplus \{V\}$ is integrable if and only if*
(i) $Q_2 h^l(Y, \phi P_3 X) = Q_2 h^l(X, \phi P_3 Y)$ and $Q_4 h^s(Y, \phi P_3 X) = Q_4 h^s(X, \phi P_3 Y)$,
(ii) $P_1(\nabla_X \phi P_3 Y) = P_1(\nabla_Y \phi P_3 X)$, $\quad \forall X, Y \in \Gamma(D \oplus \{V\})$.

Proof. Let M be a radical transversal SCR-lightlike submanifold of an indefinite Sasakian manifold $\overline{M}$. Let $X, Y \in \Gamma(D \oplus \{V\})$. From (32), we get $P_1(\nabla_X \phi P_3 Y) = \phi P_1 \nabla_X Y$, which gives $P_1(\nabla_X \phi P_3 Y) - P_1(\nabla_Y \phi P_3 X) = \phi P_1[X,Y]$. In view of (37), we have $Q_2 h^l(X, \phi P_3 Y) = \phi P_2 \nabla_X Y$, which implies $Q_2 h^l(X, \phi P_3 Y) - Q_2 h^l(Y, \phi P_3 X) = \phi P_2[X,Y]$. Also from (39), we obtain $Q_4 h^s(X, \phi P_3 Y) = \phi P_4 \nabla_X Y$, which gives $Q_4 h^s(X, \phi P_3 Y) - Q_4 h^s(Y, \phi P_3 X) = \phi P_4[X,Y]$. Thus, we obtain the required results.

Theorem 3.8. *Let M be a radical transversal SCR-lightlike submanifold of an indefinite Sasakian manifold $\overline{M}$ with structure vector field tangent to M. Then $D^{\perp}$ is integrable if and only if*
(i) $P_1(A_{\phi P_4 Y} X) = P_1(A_{Y \phi P_4 X} Y)$ and $P_3(A_{\phi P_4 Y} X) = P_3(A_{\phi P_4 X} Y)$,
(ii) $Q_2 D^l(Y, \phi P_4 X) = Q_2 D^l(X, \phi P_4 Y)$, $\quad \forall X, Y \in \Gamma(D^{\perp})$.

Proof. Let M be a radical transversal SCR-lightlike submanifold of an indefinite Sasakian manifold $\overline{M}$. Let $X, Y \in \Gamma(D^{\perp})$. From (32), we get $P_1(A_{\phi P_4 Y} X) = -\phi P_1 \nabla_X Y$, which gives $P_1(A_{\phi P_4 X} Y) - P_1(A_{\phi P_4 Y} X) = \phi P_1[X,Y]$. In view of (34), we have $P_3(A_{\phi P_4 Y} X) = -\phi P_3 \nabla_X Y$, which implies $P_3(A_{\phi P_4 X} Y) - P_3(A_{\phi P_4 Y} X) = \phi P_3[X,Y]$. Also from (37), we obtain $Q_2 D^l(X, \phi P_4 Y) = \phi P_2 \nabla_X Y$, which gives $Q_2 D^l(X, \phi P_4 Y) - Q_2 D^l(Y, \phi P_4 X) = \phi P_2[X,Y]$. This proves the theorem.

4. Foliations Determined by Distributions

In this section, we obtain necessary and sufficient conditions for foliations determined by distributions to be totally geodesic on a radical transversal SCR-lightlike submanifold of an indefinite Sasakian manifold.

Theorem 4.1. *Let M be a radical transversal SCR-lightlike submanifold of an indefinite Sasakian manifold $\overline{M}$ with structure vector field tangent to M. Then $RadTM \oplus \{V\}$ defines a totally geodesic foliation if and only if*
(i) $h^l(X, \phi Z) = 0$ and $D^l(X, \phi W) = 0$,
(ii) $\nabla_X \phi Z$ and $A_{\phi W} X$ have no components in Rad TM,
$\forall X \in \Gamma(RadTM \oplus \{V\})$, $Z \in \Gamma(D)$ and $W \in \Gamma(D^\perp)$.

Proof. Let M be a radical transversal SCR-lightlike submanifold of an indefinite Sasakian manifold $\overline{M}$. The distribution Rad$TM \oplus \{V\}$ defines a totally geodesic foliation if and only if $\nabla_X Y \in \text{Rad}TM \oplus \{V\}$, $\forall X, Y \in \Gamma(RadTM \oplus \{V\})$. Since $\overline{\nabla}$ is a metric connection, from (7), (20) and (24), for any $X, Y \in \Gamma(RadTM \oplus \{V\})$ and $Z \in \Gamma(D)$, we have $\overline{g}(\nabla_X Y, Z) = \overline{g}(\overline{\nabla}_X \phi Y, \phi Z)$, which gives $\overline{g}(\nabla_X Y, Z) = -\overline{g}(\overline{\nabla}_X \phi Z, \phi Y) = -\overline{g}(\nabla_X \phi Z, \phi P_2 Y) - \overline{g}(h^l(X, \phi Z), \phi P_1 Y)$. In view of (7), (20) and (24), for any $X, Y \in \Gamma(RadTM \oplus \{V\})$ and $W \in \Gamma(D^\perp)$, we obtain $\overline{g}(\nabla_X Y, W) = \overline{g}(\overline{\nabla}_X \phi Y, \phi W)$, which implies $\overline{g}(\nabla_X Y, W) = -\overline{g}(\overline{\nabla}_X \phi W, \phi Y) = \overline{g}(A_{\phi W} X, \phi P_2 Y) - \overline{g}(D^l(X, \phi W), \phi P_1 Y)$. This completes the proof.

Theorem 4.2. *Let M be a radical transversal SCR-lightlike submanifold of an indefinite Sasakian manifold $\overline{M}$ with structure vector field tangent to M. Then $D \oplus \{V\}$ defines a totally geodesic foliation if and only if $A_{\phi W} X$, $A_{\phi Q_1 N} X$ and $A^*_{\phi Q_2 N} X$ have no components in $D \oplus \{V\}$, $\forall X \in \Gamma(D \oplus \{V\})$, $\forall N \in \Gamma(ltr(TM))$ and $\forall W \in \Gamma(D^\perp)$.*

Proof. Let M be a radical transversal SCR-lightlike submanifold of an indefinite Sasakian manifold $\overline{M}$. The distribution $D \oplus \{V\}$ defines a totally geodesic foliation if and only if $\nabla_X Y \in D \oplus \{V\}$, $\forall X, Y \in \Gamma(D \oplus \{V\})$. Since $\overline{\nabla}$ is a metric connection,from (7), (20) and (24), for any $X, Y \in \Gamma(D \oplus \{V\})$ and $W \in \Gamma(D^\perp)$, we have $\overline{g}(\nabla_X Y, W) = \overline{g}(\overline{\nabla}_X \phi Y, \phi W)$, which gives $\overline{g}(\nabla_X Y, W) = -\overline{g}(\overline{\nabla}_X \phi W, \phi Y) = \overline{g}(A_{\phi W} X, \phi Y)$. In view of (7), (20) and (24), for any $X, Y \in \Gamma(D \oplus \{V\})$ and $N \in \Gamma(ltr(TM))$, we obtain $\overline{g}(\nabla_X Y, N) = \overline{g}(\overline{\nabla}_X \phi Y, \phi N)$, which implies $\overline{g}(\nabla_X Y, N) = -\overline{g}(\phi Y, \overline{\nabla}_X(\phi Q_1 N + \phi Q_2 N))$. This concludes the theorem.

Theorem 4.3. *Let M be a radical transversal SCR-lightlike submanifold of an indefinite Sasakian manifold $\overline{M}$ with structure vector field tangent to M. Then $D^\perp$ defines a totally geodesic foliation if and only if*
(i) $D^s(X, \phi Q_1 N) = 0$ and $h^s(X, \phi Q_2 N) = 0$, $\forall N \in \Gamma(ltr(TM))$,
(ii)$h^s(X, \phi Z) = 0$, $\forall X \in \Gamma(D^\perp)$ and $\forall Z \in \Gamma(D)$.

Proof. Let M be a radical transversal SCR-lightlike submanifold of an indefinite Sasakian manifold $\overline{M}$. The distribution $D^\perp$ defines a totally geodesic foliation if and only if $\nabla_X Y \in D^\perp$, $\forall X, Y \in \Gamma(D^\perp)$. Since $\overline{\nabla}$ is a metric connection, in view of (7), (20) and (24), for any $X, Y \in \Gamma(D^\perp)$ and $Z \in \Gamma(D)$, we have $\overline{g}(\nabla_X Y, Z) = \overline{g}(\overline{\nabla}_X \phi Y, \phi Z)$, which gives $\overline{g}(\nabla_X Y, Z) = -\overline{g}(\overline{\nabla}_X \phi Z, \phi Y) = \overline{g}(h^s(X, \phi Z), \phi Y)$. From (7), (20) and (24), for any $X, Y \in \Gamma(D^\perp)$ and $N \in \Gamma(ltr(TM))$, we obtain $\overline{g}(\nabla_X Y, N) = \overline{g}(\overline{\nabla}_X \phi Y, \phi N)$, which implies $\overline{g}(\nabla_X Y, N) = -\overline{g}(\overline{\nabla}_X(\phi Q_1 N + \phi Q_2 N), \phi Y) = -\overline{g}(h^s(X, \phi Q_2 N) + D^s(X, \phi Q_1 N), \phi Y)$. Thus, we obtain the required results.

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