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Erratum to "c_0 Can Be Renormed to Have the Fixed Point Property for Affine Nonexpansive Mappings"

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We have noticed that in our joint paper [2], Lemma 3.12 is not valid and so proof of theorems using the lemma cannot be valid either. Therefore, Theorem 3.13 and Theorem 3.14 with further results are not true. It can be said that Lemma 3.12 is true for only some subsequences of the sequences in the hypothesis but Theorem 3.13 requires the main sequences, then since Theorem 3.14 and further results use Theorem 3.13, they all turn out to be invalid.

Here, we explicitly give some details: Álvaro, Cembranos and Mendoza introduced a beautiful property for c_0 in their study [1]. Unfortunately, we were unaware of their property because our paper was under review before [1] was published.

In [1], the authors call their property N1 property. After recalling the definition of c_0-sequence, they introduced N1 property. Now, we also note the definition of c_0-sequence and then we recall the definition of N1 property.

Definition 0.1. *A Banach space $(X, \|.\|)$ is said to contain a c_0-sequence $(x_n)_{n\in\mathbb{N}}$ if there exist scalars $0 < m < M < \infty$ such that for any finite sequence of scalars $(t_n)_{n\in\mathbb{N}}$,*

$$m \sup_{n\in\mathbb{N}} |t_n| \leq \left\| \sum_{n=1}^{\infty} t_n x_n \right\| \leq M \sup_{n\in\mathbb{N}} |t_n| \, .$$

Definition 0.2. *We say that a c_0-sequence $(x_n)_{n\in\mathbb{N}}$ in a Banach space $(X, \|.\|)$ has N1 property if there exists a sequence $(\alpha_n)_{n\in\mathbb{N}}$ in $[0,1)$ convergent to 1 such that for any finite sequence of scalars $(t_n)_{n\in\mathbb{N}}$,*

$$\left\| \sum_{n=1}^{\infty} \alpha_n t_n x_n \right\| \leq \left\| \sum_{n=1}^{\infty} t_n x_n \right\| \, .$$

Then, Álvaro, Cembranos and Mendoza concludes in Theorem 2 of [1] that every Banach space with N1 property fails the fixed point property for nonexpansive mappings. Next, in Remark 3, they note that their conclusion is true even for affine nonexpansive mappings.

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Now, recall definition of our equivalent norm $|||\cdot|||$ on c_0 in [2]. For $x = (\xi_k)_k \in c_0$,

$$|||x||| := \lim_{p\to\infty} \sup_{k\in\mathbb{N}} \gamma_k \left(\sum_{j=k}^{\infty} \frac{\left|\xi_j\right|^p}{2^j} \right)^{\frac{1}{p}} \text{ where } \gamma_k \uparrow_k 3,\ \gamma_k \text{ is strictly increasing with } \gamma_k > 2,\ \forall k \in \mathbb{N}\,.$$

Then, the usual unit vector basis $(e_n)_{n\in\mathbb{N}}$ is a c_0-sequence with coefficients $m = 2$ and $M = 3$. Moreover, one can also see that for any $(\alpha_n)_{n\in\mathbb{N}}$ in $[0, 1)$ convergent to 1 and for any finite sequence of scalars $(t_n)_{n\in\mathbb{N}}$,

$$\left|\left|\left|\sum_{n=1}^{\infty} \alpha_n t_n e_n\right|\right|\right| \le \left|\left|\left|\sum_{n=1}^{\infty} t_n e_n\right|\right|\right|.$$

Thus, by Remark 3 in [1], our renorming $(c_0, |||\cdot|||)$ fails the fixed point property for affine nonexpansive mappings. This contradicts with the result of Theorem 3.14. Therefore, Theorem 3.14 and further results cannot be valid in our joint paper [2].

The error occurs at the end of the proof of Theorem 3.14 on page 5658 where Theorem 3.13 is used such that proof of Theorem 3.13 uses Lemma 3.12. The statement of Lemma 3.12 is wrong because it was actually proved that there exist subsequences $(x_{n_k})_k$ of $(x_n)_n$ and $(y_{n_k})_k$ of $(y_n)_n$ such that

$$\limsup_m \left\| \frac{1}{m} \sum_{k=1}^{m} y_{n_k} - y \right\|_\infty \le d - \lim_{m\to\infty} \left\| \frac{1}{m} \sum_{k=1}^{m} x_{n_k} - u \right\|_\infty .$$

References

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[2] V. Nezir, N. Mustafa, c_0 can be renormed to have the fixed point property for affine nonexpansive mappings, Filomat 32 (2018) 5645-5663.