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On Cellular-Countably Compact Spaces

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Abstract. A space X is said to be cellular-countably compact if for each cellular family $\mathcal{U}$ in X, there is a countably compact subspace K of X such that $U \cap K \neq \emptyset$ for each $U \in \mathcal{U}$. The class of cellular-countably compact spaces contain the classes of countably compact spaces and cellular-compact spaces and contained in a class of pseudocompact spaces. We give an example of Tychonoff DCCC space which is not cellular-countably compact. By using Erdős and Radó's theorem, we establish the cardinal inequalities for cellular-countably compact spaces. We show that the cardinality of a normal cellular-countably compact space with a G_δ-diagonal is at most $\mathfrak{c}$. Finally, we study the topological behavior of cellular-countably compact spaces on subspaces and products.

1. Introduction

In the last few years, there has been a great deal of activity regarding properties defined using cellular families. Given a topological property $\mathcal{P}$, a space X is said to be cellular-$\mathcal{P}$ if for every cellular family $\mathcal{U}$ there is a subspace $Y \subset X$ having property $\mathcal{P}$ such that $U \cap Y \neq \emptyset$, for every $U \in \mathcal{U}$. This program was started by Bella and Spadaro, who in their article [3] defined *cellular-Lindelöf spaces* and asked whether every first-countable cellular-Lindelöf space has cardinality continuum. Their original motivation to introduce cellular-Lindelöf spaces was to look for a common generalization to Arhangel'skii's Theorem and the Hajnal-Juhász inequality stating that every CCC first-countable space has cardinality at most continuum (see [4]). Indeed in [3] the authors showed that the cardinality of a cellular-Lindelöf first-countable space does not exceed $2^{\mathfrak{c}}$ and asked whether it is always bounded by the continuum. Bella and Spadaro managed to find such a common generalization by other means (see [5]) but the original question is still open despite several attacks by various authors (see, for example, [2, 5, 12, 14]). Moreover, the introduction of the cellular-Lindelöf property led several authors to study cellular-$\mathcal{P}$-spaces, for various other properties $\mathcal{P}$ (see, for example, [1, 14]).

In this paper, we study the case $\mathcal{P}$ = countably compact of the above definition and investigate some topological properties of cellular-countably compact spaces. Evidently, every countably compact space is cellular-countably compact and every cellular-compact space is cellular-countably compact.

It is proved that cellularity of cellular-countably compact space with a G_δ-diagonal is at most $\mathfrak{c}$. It is also shown that the cardinality of a normal cellular-countably compact space with a G_δ-diagonal is at most $\mathfrak{c}$. We establish the cardinal inequalities of cellular-countably compact spaces by using Erdős and Radó's theorem. We prove that if X is a cellular-countably compact space with a symmetric g-functions such that

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$\cap\{g^2(n,x) : n \in \omega\} = \{x\}$ for each $x \in X$, then $|X| \leq 2^{\mathfrak{c}}$. We also prove that if X is a cellular-countably compact space with a symmetric g-function such that $\cap\{g(n,x) : n \in \omega\} = \{x\}$ for each $x \in X$, then every weakly separated subset $Y \subset X$ has cardinality at most $\mathfrak{c}$.

2. Preliminaries

Throughout the paper, all spaces are assumed to be Hausdorff topological spaces unless otherwise is stated. Given a space X, the collection $\tau(X)$ is a topology on X and $\tau(x, X) = \{U \in \tau(X) : x \in U\}$ for any $x \in X$.

Throughout the paper, the cardinality of a set is denoted by $|A|$ and $[X]^2$ denote the set of two-element subsets of X. Let ω denote the first infinite cardinal, ω_1 the first uncountable cardinal, $\mathfrak{c}$ the cardinality of the set of all real numbers. For each pair of ordinals α, β with $\alpha < \beta$, we write $[\alpha,\beta) = \{\gamma : \alpha \leq \gamma < \beta\}$, $(\alpha,\beta] = \{\gamma : \alpha < \gamma \leq \beta\}$, $(\alpha,\beta) = \{\gamma : \alpha < \gamma < \beta\}$, $[\alpha,\beta] = \{\gamma : \alpha \leq \gamma \leq \beta\}$. As usual, a cardinal is an initial ordinal and an ordinal is the set of smaller ordinals. A cardinal is often viewed as a space with the usual order topology.

As usual, $\psi(X)$ and $d(X)$ denote respectively the pseudocharacter and the density of X.

Definition 2.1. A cellular family is a family of pairwise disjoint nonempty open sets. The cellularity of a space X is the supremum of the cardinalities of the cellular families in X and is denoted by $c(X)$.

Definition 2.2. A space X satisfies the countable chain condition (in short, X is CCC) if any disjoint family of nonempty open subsets in X is countable, that is, the Souslin number (or cellularity) of X is at most ω.

Definition 2.3. A space X satisfies the discrete countable chain condition (in short, X is DCCC) if every discrete family of nonempty open subsets of X is countable.

Definition 2.4. A set $S \subset X$ is weakly separated if there exists a subset $A \subset S$ such that $|A| = |S|$ and A has a disjoint open expansion.

Definition 2.5. ([10]) A g-function for a space X is a map $g : \omega \times X \to \tau(X)$ such that for every $x \in X$, $x \in g(n,x)$ and $g(n+1,x) \subset g(n,x)$ for all $n \in \omega$.

Definition 2.6. ([10]) A g-function g is said to be symmetric if for any $n \in \omega$ and $x, y \in X$, $y \in g(n,x)$ whenever $x \in g(n,y)$.

Definition 2.7. ([16]) A space X has a regular G_δ-diagonal if there is a countable family $\{U_n : n \in \omega\}$ of open neighborhoods of the diagonal Δ_X in the square $X \times X$ such that $\Delta_X = \cap\{\overline{U_n} : n \in \omega\}$, where $\Delta_X = \{(x,x) : x \in X\}$.

Definition 2.8. A space X has a rank 2-diagonal if there exists a sequence $(\mathcal{U}_n : n \in \omega)$ of open covers of X such that for each $x \in X$, $\{x\} = \bigcap\{St^2(x, \mathcal{U}_n) : n \in \omega\}$.

All notations and terminology not explained in the paper are given in [8].

3. Cellular-countably compact spaces

The following lemma follows from the definitions.

Lemma 3.1. *The following statements hold:*

1. *Every countably compact space is cellular-countably compact.*
2. *Every cellular-compact space is cellular-countably compact.*

Lemma 3.2. ([1, Corollary 3.2]) *If a space X has a countably compact dense subspace D, then X is cellular-countably compact.*

Using the above lemma we see that the Tychonoff Plank is an example of cellular-countably compact non-countably compact.

We have the following observation from the [1, Proposition 3.5].

Observation 3.3. *The following statements hold:*

1. *Every cellular-countably compact space is feebly compact.*
2. *Every regular cellular-countably compact Lindelöf space is compact.*
3. *Every normal cellular-countably compact space is countably compact.*
4. *Every cellular-countably compact space is DCCC.*

The following example shows that the converse of Observation 3.3(4) is not true.

Example 3.4. There exists a Tychonoff DCCC space which is not cellular-countably compact.

Proof. Let $D(\mathfrak{c}) = \{d_\alpha : \alpha < \mathfrak{c}\}$ be a discrete space of cardinality $\mathfrak{c}$ and let $Y = D(\mathfrak{c}) \cup \{d^*\}$, where $d^* \notin D(\mathfrak{c})$ is the one-point Lindelöfication. Then Y is Lindelöf and every countably compact subset of Y is finite. Let

$$X = (Y \times [0, \omega]) \setminus \{\langle d^*, \omega\rangle\}$$

be the subspace of the product space $Y \times [0, \omega]$. Then X is DCCC space, since $Y \times \omega$ is a Lindelöf dense subset of X.

To show X is not cellular-countably compact. For each $\alpha < \mathfrak{c}$, let $U_\alpha = \{d_\alpha\} \times [0, \omega]$. Then each U_α is open in X. Let

$$\mathcal{U} = \{U_\alpha : \alpha < \mathfrak{c}\}.$$

Then $\mathcal{U}$ is a cellular family in X. It is enough to show that there exists a $U_\beta \in \mathcal{U}$ such that $U_\beta \cap K = \emptyset$, for any countably compact subset K of X. Let K be any countably compact subset of X. Since $\{\langle d_\alpha, \omega\rangle : \alpha < \mathfrak{c}\}$ is a discrete closed subset of X, the set

$K \cap \{\langle d_\alpha, \omega\rangle : \alpha < \mathfrak{c}\}$ is finite. Then there exists $\alpha' < \mathfrak{c}$ such that

$$K \cap \{\langle d_\alpha, \omega\rangle : \alpha > \alpha'\} = \emptyset.$$

Pick $\beta > \alpha'$. Then $U_\beta \cap K = \emptyset$. Therefore X is not cellular-countably compact. □

4. Cardinal inequalities

Theorem 4.1. *If X is a cellular-countably compact space with a G_δ-diagonal, then $c(X) \leq \mathfrak{c}$.*

Proof. Let $\mathcal{U}$ be a cellular family in X. Since X is cellular-countably compact, there is a countably compact subset of $K \subset X$ such that $K \cap U \neq \emptyset$ for every $U \in \mathcal{U}$. Since every countably compact space with a G_δ-diagonal is metrizable, then $|K| \leq \mathfrak{c}$. Thus $|\mathcal{U}| \leq \mathfrak{c}$. □

Theorem 4.2. *Every cellular-countably compact Moore space X has cardinality at most $\mathfrak{c}$.*

Proof. Every Moore space is perfect, so, by [13, Proposition 2.3], the space X is CCC. Since every Moore space is first-countable the result follows from the Hajnal-Juhász inequality $|X| \leq 2^{\chi(X) \cdot c(X)}$. □

It is interesting to note that, in the above theorem "cellular-countably compact" cannot be replaced with "cellularity at most continuum (see [6, Theorem 2.3]).

Bella and Spadaro proved that every normal cellular-Lindelöf space X with a G_δ-diagonal of rank 2 has cardinality at most $\mathfrak{c}$ (see [5, Theorem 13]). We have a related result for cellular-countably compact spaces.

Theorem 4.3. *Every normal cellular-countably compact space X with a G_δ-diagonal has cardinality at most $\mathfrak{c}$.*

Proof. Every cellular-countably compact is feebly compact and every normal feebly compact space is countably compact, then X has countable extent. By the Ginsburg-Woods inequality [9], every space with a G_δ-diagonal and countable extent has cardinality at most $\mathfrak{c}$. $\square$

Bella and Spadaro proved that every cellular-Lindelöf space with a regular G_δ-diagonal has cardinality at most $2^{\mathfrak{c}}$ (see [5]).

Theorem 4.4. *Every Tychonoff cellular-countably compact space with a regular G_δ-diagonal has cardinality at most $\mathfrak{c}$.*

Proof. Since every cellular-countably compact space is feebly compact, and thus pseudocompact. Every Tychonoff pesudocompact space with a regular G_δ-diagonal is compact and metrizable and thus it has cardinality at most $\mathfrak{c}$. $\square$

Problem 4.5. *Does every cellular-countably compact space with a G_δ-diagonal (of rank 2) have cardinality at most $\mathfrak{c}$?*

For the next results, we need the following lemma due to Erdős and Radó.

Lemma 4.6. ([11, p. 8]) *Let κ be an infinite cardinal, let X be a set with $|X| > 2^\kappa$ and suppose $[X]^2 = \cup\{P_\alpha : \alpha < \kappa\}$. Then there exist $\alpha < \kappa$ and a subset $S \subset X$ with $|S| > \kappa$ such that $[S]^2 \subset P_\alpha$.*

Theorem 4.7. *If X is a cellular-countably compact space with a symmetric g-function such that $\cap\{g^2(n,x) : n \in \omega\} = \{x\}$ for each $x \in X$, then $|X| \leq 2^{\mathfrak{c}}$.*

Proof. Let $\mathcal{U}_n = \{g(n,x) : x \in X\}$ for each $n \in \omega$. Then each $\mathcal{U}_n$ is an open cover of X and

$$St(x,\mathcal{U}_n) = \cup\{g(n,\xi) : x \in g(n,\xi)\} = \cup\{g(n,\xi) : \xi \in g(n,x)\} = g^2(n,x).$$

Since $\cap\{g^2(n,x) : n \in \omega\} = \{x\}$ for each $x \in X$, thus $\cap\{St(x,\mathcal{U}_n) : n \in \omega\} = \{x\}$ and hence X has a G_δ-diagonal. Thus by Theorem 4.1, $c(X) \leq \mathfrak{c}$.

Now we prove that $|X| \leq 2^{\mathfrak{c}}$. Suppose $|X| > 2^{\mathfrak{c}}$. For each $n \in \omega$, let

$$P_n = \Big\{\{x,y\} \in [X]^2 : x \notin g^2(n,y)\Big\}.$$

If g-function is symmetric, then g^2 is also symmetric, which make the sets P_n well-defined. Thus $[X]^2 = \cup\{P_n : n \in \omega\}$. Then by Lemma 4.6, there exists a subset $S \subset X$ with $|S| > \mathfrak{c}$ and $[S]^2 \subset P_n$ for some $n \in \omega$. Thus for any two distinct points $x, y \in S$, $x \notin g^2(n,y)$, which implies that $g(n,x) \cap g(n,y) = \emptyset$, since g is symmetric. Thus $\{g(n,x) : x \in S\}$ is a cellular family of X with cardinality greater than $\mathfrak{c}$, contradict the fact that $c(X) \leq \mathfrak{c}$. $\square$

Theorem 4.8. *If X is a cellular-countably compact space with a symmetric g-function such that $\cap\{g^3(n,x) : n \in \omega\} = \{x\}$ for each $x \in X$, then $|X| \leq \mathfrak{c}$.*

Proof. Xuan [15] proved that if X is a DCCC space with a symmetric g-function such that $\cap\{g^3(n,x) : n \in \omega\} = \{x\}$ for each $x \in X$, then $|X| \leq \mathfrak{c}$. Hence by Observation 3.3(4), the result follows. $\square$

Theorem 4.9. *If X is a cellular-countably compact space with a symmetric g-function such that $\cap\{g(n,x) : n \in \omega\} = \{x\}$ for each $x \in X$, then every weakly separated subset $Y \subset X$ has cardinality at most $\mathfrak{c}$.*

Proof. We first show that every countably compact subspace $K \subset X$ has cardinality at most $\mathfrak{c}$. Suppose $|K| > \mathfrak{c}$. For each $n \in \omega$, define a subset P_n of $[K]^2$ by

$$P_n = \Big\{\{x,y\} \in [K]^2 : x \notin g(n,y)\Big\}.$$

Thus the sets P_n are well-defined. Thus $[K]^2 = \cup\{P_n : n \in \omega\}$, by Lemma 4.6, there exists a subset $S \subset X$ with $|S| > \omega$ and $[S]^2 \subset P_k$ for some $k \in \omega$. Thus by the definition of P_k and for any two distinct points $x, y \in S$, $x \notin g(k, y)$.

We claim that the set S is a closed discrete in X. If not, let $\xi \in X$ is a accumulation point of X. Since X is T_1, the neighborhood $g(k, \xi)$ of ξ meets infinitely many members of S. Pick any $x \in g(k, \xi) \cap S$. By symmetry $\xi \in g(k, x)$, and hence, there exists $y \in (S \setminus \{x\}) \cap g(k, x)$, a contradiction. But K is countably compact, thus K cannot have an uncountable closed discrete subset, which contradicts the fact that $S \subset K$. Thus $|K| \le \mathfrak{c}$.

If $Y \subset X$ is weakly separated, then there is a subset $A \subset Y$ such that $|A| = |Y|$ and A has a disjoint expansion $\mathcal{U} = \{U_x : x \in A\}$. By the cellular-countably compactness of X, there is a countably compact subspace $K' \subset X$ such that $K' \cap U_x \neq \emptyset$ for each $x \in A$. Since $|K'| \le \mathfrak{c}$, thus $|\mathcal{U}| \le \mathfrak{c}$, Therefore $|Y| = |A| \le \mathfrak{c}$. □

For a space X, we define $hccc(X) = sup\{c(Y) : \text{Y is a countably compact subspace of } X\}$.

The following result shows that $c(X) \le hccc(X)$ for any cellular-countably compact space X and its proof follows immediately from the definitions.

Proposition 4.10. *If X is a cellular-countably compact space and $c(Y) \le \kappa$ for every countably compact subspace Y of X, then $c(X) \le \kappa$.*

Since $c(X) \le |X|$ for any space X, the following corollary follows.

Corollary 4.11. *If X is a cellular-countably compact space and every countably compact subspace of X has cardinality not exceeding κ, then $c(X) \le \kappa$.*

We note that $c(X) \le hccc(X)$ for a space X, need not hold in general, which can be seen in the following example.

Example 4.12. There exists a Tychonoff Lindelöf space X such that $c(X) > hccc(X)$.

Proof. Let $D(\mathfrak{c}) = \{d_\alpha : \alpha < \mathfrak{c}\}$ be a discrete space of cardinality $\mathfrak{c}$ and let $X = D(\mathfrak{c}) \cup \{d^*\}$, where $d^* \notin D(\mathfrak{c})$ is the one-point Lindelöfication. Then X is Tychonoff Lindelöf space. Since $D(\mathfrak{c})$ is the discrete subspace of X with cardinality $\mathfrak{c}$, thus $c(X) = \mathfrak{c}$.

On the other hand, it is not difficult to see that every subspace K of X is countably compact if and only if K is finite, thus $hccc(X) = \omega < c(X)$, which completes the proof. □

Since the extent of an infinite countably compact space is ω and every normal cellular-countably compact space is countably compact. The following corollary follows.

Corollary 4.13. *If X is a normal cellular-countably compact space, then the extent of X is ω.*

5. Topological properties of cellular-countably compact spaces

Theorem 5.1. *(i) Any space X with discrete topology and cardinality at least ω is not cellular-countably compact.*
(ii) Every clopen subset of a cellular-countably compact space is cellular-countably compact.

Proof. The proof is straightforward. □

Example 5.2. There exists a Tychonoff cellular-countably compact space having a closed subset which is not cellular-countably compact.

Proof. Let $D(\mathfrak{c}) = \{d_\lambda : \lambda < \mathfrak{c}\}$ be a discrete space of cardinality $\mathfrak{c}$ and let

$$X = (\beta D(\mathfrak{c}) \times [0, \mathfrak{c})) \cup (D(\mathfrak{c}) \times \{\mathfrak{c}\})$$

viewed as subspace of the product space $\beta D(\mathfrak{c}) \times [0, \mathfrak{c}]$. Since $\beta D(\mathfrak{c}) \times [0, \mathfrak{c})$ is a dense countably compact subset of X, thus X is cellular-countably compact. Since $D \times \{\mathfrak{c}\}$ is a closed discrete subset of X with cardinality $\mathfrak{c}$. Therefore, $D \times \{\mathfrak{c}\}$ is not cellular-countably compact. □

The following result follows from [1, Lemma 3.3].

Theorem 5.3. *A regular closed subset of a cellular-countably compact space is cellular-countably compact.*

In [7], Dow and Stephenson gave several examples of spaces related to the preservation of cellular-$\mathcal{P}$ property in products. The well-known example [8, Example 3.10.19], shows that the product of two cellular-countably compact Tychonoff spaces need not be pseudocompact (hence, not cellular-countably compact).

Example 5.4. There exist a Tychonoff countably compact space X and Tychonoff Lindelöf space Y such that $X \times Y$ is not cellular-countably compact.

Proof. Let $X = [0, \omega_1)$ with the usual order topology. Then X is countably compact. Let $D(\omega_1) = \{d_\alpha : \alpha < \omega_1\}$ be a discrete space of cardinality ω_1, let $Y = D \cup \{d^*\}$ be one-point Lindelöfication of $D(\omega_1)$, where $d^* \notin D(\omega_1)$.

Now we show that $X \times Y$ is not cellular-countably compact. Let $\mathcal{U} = \{(\alpha, \omega_1) \times \{d_\alpha\} : \alpha < \omega_1\}$. Then $\mathcal{U}$ is a disjoint family of open subsets of $X \times Y$. Let K be any countably compact subset of $X \times Y$. Then $\pi(K)$ is a countably compact subset of Y, where $\pi : X \times Y \to Y$ is the projection. Hence $\pi(K)$ is a finite subset of Y. Thus there exists $\alpha < \omega_1$ such that $K \cap ((\alpha, \omega_1) \times \{d_\alpha\}) = \emptyset$.

This shows that $X \times Y$ is not cellular-countably compact. □

Dow and Stephenson showed that product of cellular-compact space and a compact space is not necessarily cellular-Lindelöf (see [7, Theorem 2.4]). The following question seems natural.

Problem 5.5. *Is the product of cellular-countably compact space and a compact space cellular-countably compact?*

The proof of the following theorem is straightforward.

Theorem 5.6. *A continuous image of a cellular-countably compact space is cellular-countably compact.*

It is well-known that the Alexandorff duplicate $\mathsf{AD(X)}$ of a space X is countably compact if X is countably compact. We show that a similar result is not hold for cellular-countably compact spaces. The Alexandorff duplicate $\mathsf{AD(X)} = X \times \{0, 1\}$ of a space X. The basic neighborhood of a point $\langle x, 0\rangle \in X \times \{0\}$ is of the form $(U \times \{0\}) \bigcup (U \times \{1\} \setminus \{\langle x, 1\rangle\})$, where U is a neighborhood of x in X and each points $\langle x, 1\rangle \in X \times \{1\}$ are isolated points.

Example 5.7. There exists a Tychonoff cellular-countably compact space X such that $\mathsf{AD(X)}$ is not cellular-countably compact.

Proof. Let X be the same space X in the proof of Example 5.2. Thus X is cellular-countably compact. Let $A = \{\langle\langle d_\lambda, \mathfrak{c}\rangle, 1\rangle : \lambda < \mathfrak{c}\}$. Then A is a clopen subset of $\mathsf{AD(X)}$ with $|A| = \mathfrak{c}$ and each point $\langle\langle d_\lambda, \mathfrak{c}\rangle, 1\rangle$ is isolated. Hence $\mathsf{AD(X)}$ is not cellular-countably compact, since every clopen subset of a cellular-countably compact space is cellular-countably compact and A is not cellular-countably compact. □

Remark 5.8. Let X be the same space of Example 5.2. Then by Example 5.7, X is cellular-countably compact, but $\mathsf{AD(X)}$ is not. Define $f : \mathsf{AD(X)} \to X$ by $f(\langle x, 0\rangle) = f(\langle x, 1\rangle) = x$ for each $x \in X$. Then f is a closed 2-to-1 continuous map. Thus the preimage of cellular-countably compact space under closed 2-to-1 continuous map is not cellular-countably compact.

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