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Accuracy measure of rough membership multiset functions with application

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Abstract. In this paper, the notion of rough membership function has been introduced in the multiset context. As a consequence, we introduced the lower and upper approximations in terms of the rough membership multiset functions. Finally, an application about how to select the best possible suppliers of chemicals has been presented.

1. Introduction

In classical set theory, a set is a well-defined grouping of unique items. If an object can appear more than once in a set, a mathematical structure known as a multiset ([1], [2], [11]) will result. A multiset is therefore distinct from a set in that each element has a multiplicity natural number rather than one that specifically denotes how many times it is a part of the multiset. The multiset of prime factors for a positive integer n is one of the most straightforward and natural examples.

A fundamental idea used to depict various circumstances in mathematical notation where it is forbidden for elements to occur more than once is known as classical set theory. However, under some conditions, the system must repeat certain elements. For instance, in a graph containing loops, there are many hydrogen atoms, many water molecules, many identical DNA strands, etc.

The concept of multisets as proposed by Yager [11], Blizard [1, 2], and Jena et al. [5] has been briefly reviewed in this section. In addition, Girish and John introduced several varieties of collections of multisets, rough multisets, and fundamental definitions and conceptions of relations in multiset context.

In what follows, a brief survey of the notion of multisets as introduced by Yager [11], Blizard [1], [2] and Jena et al. [5] have been collected. Furthermore, the different types of collections of multisets, Rough multisets, and the basic definitions and notions of relations in multiset context introduced by Girish and John [3], [4] and Zakaria et al. [12]. For further reading in rough sets check [6], [7], [8], [9], [10], [13].

Definition 1.1. A collection of elements containing duplicates is called an multiset. Formally, if X is a set of elements, a multiset M drawn from the set X is represented by a function count M or C_M defined as $C_M : X \to \mathbb{N}$, where $\mathbb{N}$ represents the set of nonnegative integers.

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Let M be a multiset from the set $X = \{x_1, x_2, \ldots, x_n\}$ with x appearing n times in M. It is denoted by $x \in^n M$. The multiset M drawn from the set X is given by

$$M = \{k_1/x_1, k_2/x_2, \ldots, k_n/x_n\},$$

where M is a multiset with x_1 appearing k_1 times, x_2 appearing k_2 times and so on. In Definition 1.1, $C_M(x)$ is the number of occurrences of the element x in the multiset M. However those elements which are not included in the multiset M have zero count. A multiset M is a set if $C_M(x) = 0$ or 1 for all $x \in X$.

Definition 1.2. A domain X, is defined as a set of elements from which multisets are constructed. The multiset space $[X]^\omega$ is the set of all multisets whose elements are in X such that no element in the multiset occurs more than m times. The set $[X]^\infty$ is the set of all multisets over a domain X such that there is no limit on the number of occurrences of an element in a multiset.

Let $M, N \in [X]^\omega$. Then, the following are defined:

1. M is a submultiset of N denoted by ($M \subseteq N$) if $C_M(x) \leq C_N(x)$ $\forall$ $x \in X$.
2. $M = N$ if $M \subseteq N$ and $N \subseteq M$.
3. M is a proper submultiset of N denoted by ($M \subset N$) if $C_M(x) \leq C_N(x)$ $\forall$ $x \in X$ and there exists at least one element $x \in X$ such that $C_M(x) < C_N(x)$.
4. $P = M \cup N$ if $C_P(x) = \max\{C_M(x), C_N(x)\}$ for all $x \in X$.
5. $P = M \cap N$ if $C_P(x) = \min\{C_M(x), C_N(x)\}$ for all $x \in X$.
6. Subtraction of M and N results in a new multiset $P = M \ominus N$ such that $C_P(x) = \max\{C_M(x) - C_N(x), 0\}$ for all $x \in X$, where $\oplus$ and $\ominus$ represent multiset addition and multiset subtraction, respectively.
7. A multiset M is empty if $C_M(x) = 0$ $\forall$ $x \in X$.
8. The support set of M denoted by M^* is a subset of X and $M^* = \{x \in X \mid C_M(x) > 0\}$; that is, M^* is an ordinary set and it is also called root set.
9. The cardinality of a multiset M drawn from a set X is Card(M)= $\sum_{x \in X} C_M(x)$.

Definition 1.3. Let $M \in [X]^\omega$. Then the complement M^c of M in $[X]^\omega$ is an element of $[X]^\omega$ such that

$$C_{M^c}(x) = m - C_M(x) \qquad \text{for all } x \in X.$$

Definition 1.4. Let $M \in [X]^\omega$. The power multiset $P(M)$ of M is the set of all submultisets of M.

The power set of a multiset is the support set of the power multiset and is denoted by $P^*(M)$. The following theorem shows the cardinality of the power set of a multiset.

Definition 1.5. Let $M \in [X]^\omega$ and $\tau \subseteq P^*(M)$. Then τ is called a multiset topology if τ satisfies the following properties.

1. ϕ and M are in τ.
2. The union of the elements of any sub collection of τ is in τ.
3. The intersection of the elements of any finite sub collection of τ is in τ.

A multiset topological space is an ordered pair (M, τ) consisting of a multiset M and a multiset topology $\tau \subseteq P^*(M)$. Note that τ is an ordinary set whose elements are multisets and the multiset topology is abbreviated as a M–topology. Also, a submultiset U of M is an open multiset of M if U belongs to the collection τ. Moreover, a submultiset N of M is closed multiset if $M \ominus N$ is an open multiset.

Definition 1.6. Let M_1 and M_2 be two multisets drawn from a set X, then the Cartesian product of M_1 and M_2 is defined as

$$M_1 \times M_2 = \{(m/x, n/y)/mn \mid x \in^m M_1,\ y \in^n M_2\}.$$

Here the entry $(m/x, n/y)/mn$ in $M_1 \times M_2$ denotes x is repeated m times in M_1, y is repeated n times in M_2 and the pair (x, y) is repeated mn times in $M_1 \times M_2$.

The Cartesian product of three or more nonempty multisets can be defined by generalizing the definition of the Cartesian product of two multisets.

Theorem 1.7. *Let M_1 and M_2 be two nonempty multisets. Then*

$$C_{M_1\times M_2}[(x,y)] = C_{M_1}(x)\cdot C_{M_2}(y) \qquad \textit{and} \qquad |M_1\times M_2| = |M_1|\cdot|M_2|.$$

In general, we have

$$|M_1\times M_2\times\cdots\times M_n| = |M_1|\cdot|M_2|\cdots|M_n|.$$

Definition 1.8. Let $M\in[X]^k$. Then the following are defined:

1. A multiset relation R on a multiset M is reflexive if $(m/x)R(m/x)$ for all m/x in M, where
$$\Delta = \{(m/x, m/x)/m^2 \mid x\in^m M\}$$
is the identity multiset relation on M.
2. A multiset relation R on a multiset M is symmetric if $(m/x)R(n/y)$ implies $(n/y)R(m/x)$, antisymmetric if $(m/x)R(n/y)$ and $(n/y)R(m/x)$ implies m/x and n/y are equal.
3. A multiset relation R on a multiset M is transitive if $(m/x)R(n/y)$ and $(n/y)R(k/z)$, then $(m/x)R(k/z)$.
4. A multiset relation R on a multiset M is called an equivalence multiset relation if it is reflexive, symmetric and transitive.

Definition 1.9. Let R be a multiset relation on M. The post-multiset of $x\in^m M$ is defined as

$$(m/x)R = \{n/y \mid \exists \text{ some } k \text{ with } (k/x)R(n/y)\}.$$

Definition 1.10. Let R be any binary multiset relation on M in $[X]^\omega$. Then the multiset $\langle n/y\rangle_R$ is defined as the intersection of all post-multisets containing y with nonzero multiplicity; that is,

$$\langle n/y\rangle_R = \cap\{(m/x)R \mid y\in^n (m/x)R\}. \tag{1}$$

Definition 1.11. Let R be an equivalence multiset relation on a nonempty multiset M, $[m/x]$ be the equivalence class containing m/x. For $N\subseteq M$, a pair of lower and upper multiset approximations, $\underline{R}(N)$ and $\overline{R}(N)$, are defined respectively as

$$\underline{R}(N) = \{m/x \mid [m/x]\subseteq N\},$$
$$\overline{R}(N) = \{m/x \mid [m/x]\cap N\neq\phi\}.$$

The pair $(\underline{R}(N),\overline{R}(N))$ is referred to as the rough multiset of N.

Definition 1.12. Let R be a binary multiset relation on M. For $N\subseteq M$, a pair of lower and upper multiset approximations, $\underline{R}(N)$ and $\overline{R}(N)$, are defined respectively as

$$\underline{R}(N) = \{m/x \mid (m/x)R\subseteq N\},$$
$$\overline{R}(N) = \{m/x \mid (m/x)R\cap N\neq\phi\}.$$

The pair $(\underline{R}(N),\overline{R}(N))$ is referred to as the rough multiset of N. It's clear that if R is an equivalence multiset relation, then $(m/x)R = [m/x]$. In addition, this definition is equivalent to Definition 1.11.

Definition 1.13. Let R be a binary multiset relation on M. For $N\subseteq M$, a pair of lower and upper multiset approximations, $\underline{R}_\ell(N)$ and $\overline{R}_U(N)$, are defined respectively as

$$\underline{R}_\ell(N) = \{m/x \mid \langle m/x\rangle_R\subseteq N\}, \tag{2}$$
$$\overline{R}_U(N) = \{m/x \mid \langle m/x\rangle_R\cap N\neq\phi\}. \tag{3}$$

The pair $(\underline{R}_\ell(N),\overline{R}_U(N))$ is referred to as the rough multiset of N. It's clear that if R is a reflexive and transitive multiset relation, then $\langle m/x\rangle_R = (m/x)R$. In addition, this definition is equivalent to Definition 1.12.

Definition 1.14. Let $M \in [X]^\omega$, R be a binary multiset relation on M, and let N be a nonempty submultiset of M. Then the *boundary*, *positive region*, and *negative region* of N are defined, respicitvely, as follows

$$BND_R(N) = \overline{R}_U(N) \ominus \underline{R}_\ell(N),$$
$$POS_R(N) = \underline{R}_\ell(N),$$
$$NEG_R(N) = M \ominus \overline{R}_U(N).$$

Theorem 1.15. *Let R be a reflexive multiset relation on M. Then the operator $\overline{R}_U$ on $P^*(M)$ defined by equation* (3) *satisfies the Kuratowski's axioms and induces an M–topology on M called τ_R given by*

$$\tau_R = \{N \subseteq M \mid \overline{R}_U(N^c) = N^c\}.$$

2. Rough membership multiset functions

Definition 2.1. Let $M \in [X]^\omega$, R be a reflexive multiset relation on M, and let N be a nonempty submultiset of M. The rough multiset membership function $\mu : M \to [0,1]$ is defined as

$$\mu_N(k/x) = \frac{\mid \langle k/x \rangle_R \cap N \mid}{\mid \langle k/x \rangle_R \mid}. \tag{4}$$

The rough membership function in the multiset mode represents the conditional probability that x belongs to N k–times given a multiset relation R. It also shows the degree of membership of k/x to N in light of information about k/x given by R. It is easily seen that $0 \leq \mu_N(k/x) \leq 1$.

The following example is to clarify the computations of the rough multiset membership function in practice.

Example 2.2. Let $M = \{3/a, 2/b, 4/c, 8/d\}$, let $N = \{2/a, 5/d\}$ be a submultiset of M, and let

$$R = \Delta \cup \{(3/a, 2/b)/6, (3/a, 4/c)/12, (4/c, 8/d)/32, (2/b, 4/c)/8, (2/b, 8/d)/16\}$$

be a reflexive multiset relation on M. Then

$$(3/a)R = \{3/a, 2/b, 4/c\},$$
$$(2/b)R = \{2/b, 4/c, 8/d\},$$
$$(4/c)R = \{4/c, 8/d\},$$
$$(8/d)R = \{8/d\},$$

and hence

$$\langle 3/a \rangle_R = \{3/a, 2/b, 4/c\},$$
$$\langle 2/b \rangle_R = \{2/b, 4/c\},$$
$$\langle 4/c \rangle_R = \{4/c\},$$
$$\langle 8/d \rangle_R = \{8/d\}.$$

Therefore, the degree of membership of the elements of M is given as

$$\mu_N(3/a) = \frac{2}{9}, \qquad \mu_N(2/b) = 0, \qquad \mu_N(4/c) = 0, \qquad \text{and} \qquad \mu_N(8/d) = \frac{5}{8}.$$

Theorem 2.3. *Let $M \in [X]^\omega$, R be a reflexive multiset relation on M, and let N, N_1, and N_2 be nonempty submultisets of M. Then the following assertions are hold:*

(i) $\mu_N(k/x) = 1$ *if and only if* $k/x \in \underline{R}_\ell(N)$,
(ii) $\mu_N(k/x) = 0$ *if and only if* $k/x \in NEG(N)$,
(iii) $0 < \mu_N(k/x) < 1$ *if and only if* $k/x \in BND(N)$,
(iv) $\mu_\emptyset(k/x) = 0$ *and* $\mu_M(k/x) = 1$ *for all* $k/x \in M$,
(v) $\mu_{N^c}(k/x) = 1 - \mu_N(k/x)$ *for all* $k/x \in M$,
(vi) *If* $N_1 \subseteq N_2$, *then* $\mu_{N_1}(k/x) \leq \mu_{N_2}(k/x)$ *for all* $k/x \in M$,
(vii) $\mu_{N_1\cup N_2}(k/x) \geq \max\{\mu_{N_1}(k/x), \mu_{N_2}(k/x)\}$ *for all* $k/x \in M$, *the equality holds if* $N_1 \subseteq N_2$ *or* $N_2 \subseteq N_1$,
(viii) $\mu_{N_1\cap N_2}(k/x) \leq \min\{\mu_{N_1}(k/x), \mu_{N_2}(k/x)\}$ *for all* $k/x \in M$, *the equality holds if* $N_1 \subseteq N_2$ *or* $N_2 \subseteq N_1$.

Proof. Assertions (i) and (ii) are direct consequences of equation (4), respectively, as follows:

$$\mu_N(k/x) = 1 \Leftrightarrow \langle k/x\rangle_R \subseteq N \Leftrightarrow k/x \in \underline{R}_\ell(N),$$

and

$$\begin{aligned}\mu_N(k/x) = 0 &\Leftrightarrow \langle k/x\rangle_R \cap N = \emptyset\\ &\Leftrightarrow k/x \notin \overline{R}_U(N)\\ &\Leftrightarrow k/x \in M - \overline{R}_U(N)\\ &\Leftrightarrow k/x \in NEG(N).\end{aligned}$$

Using part (i), part (ii), and the inequality $0 \leq \mu_N(k/x) \leq 1$, then assertion (iii) follows directly. To prove assertion (iv), we follow the following computations

$$\begin{aligned}\mu_\emptyset(k/x) &= \frac{|\langle k/x\rangle_R \cap \emptyset|}{|\langle k/x\rangle_R|} = \frac{|\emptyset|}{|\langle k/x\rangle_R|} = 0,\\ \mu_M(k/x) &= \frac{|\langle k/x\rangle_R \cap M|}{|\langle k/x\rangle_R|} = \frac{|\langle k/x\rangle_R|}{|\langle k/x\rangle_R|} = 1.\end{aligned}$$

Now we have,

$$\mu_N(k/x) + \mu_{N^c}(k/x) = \frac{|\langle k/x\rangle \cap N| + |\langle k/x\rangle \cap N^c|}{|\langle k/x\rangle|} = \frac{|\langle k/x\rangle|}{|\langle k/x\rangle|} = 1,$$

which implies assertion (v).

To prove (vi), let $k/x \in M$ and $N_1 \subseteq N_2$. Then we have

$$|\langle k/x\rangle \cap N_1| \leq |\langle k/x\rangle \cap N_2|.$$

Hence $\frac{|\langle k/x\rangle \cap N_1|}{|\langle k/x\rangle|} \leq \frac{|\langle k/x\rangle \cap N_2|}{|\langle k/x\rangle|}$. That is to say $\mu_{N_1}(k/x) \leq \mu_{N_2}(k/x)$. Thus assertion (vi) is completed.

Now, for all $k/x \in M$, we get

$$\begin{aligned}\mu_{N_1\cup N_2}(k/x) &= \frac{|\langle k/x\rangle \cap (N_1 \cup N_2)|}{|N(u)|}\\ &= \frac{|(\langle k/x\rangle \cap N_1) \cup (\langle k/x\rangle \cap N_2)|}{|\langle k/x\rangle|}\\ &\geq \frac{\max\{|\langle k/x\rangle \cap N_1|, |\langle k/x\rangle \cap N_2|\}}{|\langle k/x\rangle|}\\ &= \max\left\{\frac{|\langle k/x\rangle \cap N_1|}{|\langle k/x\rangle|}, \frac{|\langle k/x\rangle \cap N_2|}{|\langle k/x\rangle|}\right\}\\ &= \max\{\mu_{N_1}(k/x), \mu_{N_2}(k/x)\}.\end{aligned}$$

On the other hand, it is easy to see that if $N_1 \subseteq N_2$ or $N_2 \subseteq N_1$, then we get that

$$\max\{\mu_{N_1}(k/x), \mu_{N_2}(k/x)\} = \mu_{N_2}(k/x) \quad \text{or} \quad \mu_{N_1}(k/x).$$

This completes the proof of assertion (vii).

Finally, we prove assertion (viii). For all $k/x \in M$, we have

$$\begin{aligned}
\mu_{N_1 \cap N_2}(k/x) &= \frac{|\langle k/x\rangle \cap (N_1 \cap N_2)|}{|\langle k/x\rangle|} \\
&= \frac{|(\langle k/x\rangle \cap N_1) \cap (\langle k/x\rangle \cap N_2)|}{|\langle k/x\rangle|} \\
&\leq \frac{\min\{|\langle k/x\rangle \cap N_1|, |\langle k/x\rangle \cap N_2|\}}{|\langle k/x\rangle|} \\
&= \min\left\{\frac{|\langle k/x\rangle_R \cap N_1|}{|\langle k/x\rangle_R|}, \frac{|\langle k/x\rangle_R \cap N_2|}{|\langle k/x\rangle_R|}\right\} \\
&= \min\{\mu_{N_1}(k/x), \mu_{N_2}(k/x)\}.
\end{aligned}$$

Obviously, If $N_1 \subseteq N_2$ or $N_2 \subseteq N_1$, then

$$\min\{\mu_{N_1}(k/x), \mu_{N_2}(k/x)\} = \mu_{N_1}(k/x) \quad \text{or} \quad \mu_{N_2}(k/x),$$

which completes the proof of (viii). □

The following example shows that the equality does not hold in (vii) and (viii) of Theorem 2.3, in general.

Example 2.4. Consider Example 2.2. Let $N_1 = \{3/a, 2/b\}$ and $N_2 = \{3/a, 4/c\}$. Hence

$$\mu_{N_1}(3/a) = \frac{2}{3}, \qquad \mu_{N_2}(3/a) = \frac{2}{3}, \qquad \mu_{N_1 \cup N_2}(3/a) = 1, \qquad \text{and} \qquad \mu_{N_1 \cap N_2}(3/a) = \frac{1}{3},$$

which clarifies that the equality in assertions (vii) and (viii) of Theorem 2.3 is not true, in general.

3. Rough multiset via membership functions

This section introduces the rough multiset technique using the rough membership function.

Definition 3.1. Let $M \in [X]^{\omega}$, R be a reflexive multiset relation on M, and let N be a nonempty submultiset of M. Then the *lower multiset*, the *upper multiset* approximations, and the accuracy of N are defined, respectively, as:

$$\underline{N} = \{k/x \mid \mu_N(k/x) = 1\}, \tag{5}$$

$$\overline{N} = \{k/x \mid \mu_N(k/x) > 0\}, \tag{6}$$

$$\alpha_R(N) = \frac{|\underline{N}|}{|\overline{N}|}. \tag{7}$$

The pair $(\underline{N}, \overline{N})$ is referred to as the *rough multiset* of N.

Theorem 3.2. *Let $M \in [X]^{\omega}$, R be a reflexive multiset relation on M, and let N and L be nonempty submultisets of M. Then the following assertions are hold:*

(i) $\underline{N} = (\overline{N^c})^c$,
(ii) $\underline{N} \subseteq N$,
(iii) $\underline{\emptyset} = \emptyset$,
(iv) $\underline{M} = M$,

(v) $L \subseteq N \Rightarrow \underline{L} \subseteq \underline{N}$,
(vi) $\underline{L \cap N} = \underline{L} \cap \underline{N}$,
(vii) $\underline{L} \cup \underline{N} \subseteq \underline{L \cup N}$,
(viii) $\underline{\underline{N}} = \underline{N}$.

Proof. The assertions (iii) and (iv) are consequences of part (iv) of Theorem 2.3 and equation (5). To prove assertion (i), we follow the following calculations:

$$\begin{aligned}
(\overline{N^c})^c &= \{k/x \mid \mu_{N^c}(k/x) > 0\}^c \\
&= \{k/x \mid 1 - \mu_N(k/x) > 0\}^c, \qquad \text{by part (v) of Theorem 2.3} \\
&= \{k/x \mid \mu_N(k/x) < 1\}^c \\
&= \{k/x \mid \mu_N(k/x) \geq 1\} \\
&= \{k/x \mid \mu_N(k/x) = 1\}, \qquad \text{as } 0 \leq \mu_N(k/x) \leq 1 \\
&= \underline{N}.
\end{aligned}$$

Then assertion (i) proved.

Let $k/x \in \underline{N}$. Then $\mu_N(k/x) = 1$. Using part (i) of Theorem 2.3, then we get $k/x \in \underline{R}_\ell(N)$. That is, by equation (2), we have that $k/x \in \langle k/x \rangle \subseteq N$, which implies assertion (ii).

Now we proof part (v). Let $L \subseteq N$ and $k/x \in \underline{L}$. Then equation (5) and part (vi) of Theorem 2.3 imply that

$$\mu_N(k/x) \geq \mu_L(k/x) = 1.$$

Thus, using $0 \leq \mu_N(k/x) \leq 1$, we get $\mu_N(k/x) = 1$. This completes assertion (v).

For assertion (vi), the inclusion $\underline{L \cap N} \subseteq \underline{L} \cap \underline{N}$ is a consequence of part (v) of this theorem. Now, let $k/x \in \underline{L} \cap \underline{N}$. Then we have

$$\mu_L(k/x) = 1 \qquad \text{and} \qquad \mu_N(k/x) = 1.$$

Thus (i) of Theorem 2.3 implies

$$k/x \in \underline{R}_\ell(L) \cap \underline{R}_\ell(N) = \underline{R}_\ell(L \cap N).$$

Again, part (i) of Theorem 2.3 implies $\mu_{L \cap N}(u) = 1$. Thus, $k/x \in \underline{L \cap N}$. This result concludes part (vi) of this theorem.

The result of (vii) is a direct consequence of part (v) of this theorem. Thus we omit the proof of this assertion.

Finally, we prove assertion (viii). The inclusion $\underline{\underline{N}} \subseteq \underline{N}$ is a consequence of assertions (ii) and (v) of this theorem. Thus, it is sufficient to prove that $\underline{N} \subseteq \underline{\underline{N}}$. Let $k/x \in \underline{N}$. Then $\mu_N(k/x) = 1$. This result, together with part (i) of Theorem 2.3, implies $k/x \in \underline{\underline{R}}_\ell(N)$. Again, part (i) of Theorem 2.2 implies $\mu_{\underline{N}}(k/x) = 1$. Thus $k/x \in \underline{\underline{N}}$, which completes the proof of assertion (viii). □

Theorem 3.3. *Let $M \in [X]^\omega$, R be a reflexive multiset relation on M, and let N and L be nonempty submultisets of M. Then the following assertions are hold:*

(i) $\overline{N} = (\underline{N^c})^c$,
(ii) $N \subseteq \overline{N}$,
(iii) $\overline{\emptyset} = \emptyset$,
(iv) $\overline{M} = M$,
(v) $\overline{L \cup N} = \overline{L} \cup \overline{N}$,
(vi) $L \subseteq N \Rightarrow \overline{L} \subseteq \overline{N}$,

(vii) $\overline{L \cap N} \subseteq \overline{L} \cap \overline{N}$,

(viii) $\overline{\overline{N}} = \overline{N}$.

Proof. The proof is similar to that of Theorem 3.2. □

Corollary 3.4. *Let $M \in [X]^\omega$, R be a reflexive multiset relation on M. Then the lower multiset approximation, defined in* (5), *satisfies Kuratowski's axioms and induces a topology on M called τ_R given by*

$$\tau_R = \{N \subseteq M \mid \underline{N} = N\}.$$

Proof. The proof is a direct consequence of Theorem 3.2. □

4. Application

In this section we discuss how to select the best possible chemical suppliers among several providers.

Let $X = \{s_1, \dots, s_{10}\}$ be the set of suppliers and let a_1, a_2, and a_3 be the cost, the experience, and the green suppliers of chemicals, respectively. This is shown in Table 1. You can imagine if we have a large number

Table 1: Information table

X	a_1	a_2	a_3
s_1	expensive	beginner	Yes
s_2	expensive	advanced	Yes
s_3	expensive	beginner	Yes
s_4	moderate	professional	No
s_5	moderate	professional	No
s_6	expensive	advanced	Yes
s_7	cheap	advanced	No
s_8	cheap	advanced	No
s_9	moderate	professional	No
s_{10}	expensive	professional	Yes

of data and there are similarities in the attributes. Here is the essential role of the multiset to collect these similar suppliers of attributes together as follows:

$$s_1 = s_3 = x$$
$$s_2 = s_6 = y$$
$$s_4 = s_5 = s_9 = z$$
$$s_7 = s_8 = r$$
$$s_{10} = t.$$

Thus our data can be simplified in this multiset $M = \{2/x, 2/y, 3/z, 2/r, 1/t\}$. That is, we can prettify Table 1 into Table 2.

Now, we define the multiset relation as follows:

$$(m/x)R(n/y) \Leftrightarrow x(a_k) = y(a_k) \qquad \text{for some} \qquad k \in \{1, 2, 3\}.$$

Therefore, we can express the multiset relation R in the following form

$$R = \Delta \cup \{(2/x, 2/y), (2/y, 2/x), (2/x, 1/t), (1/t, 2/x), (2/y, 2/r), (2/r, 2/y),$$
$$(2/y, 1/t), (1/t, 2/y), (3/z, 2/r), (2/r, 3/z), (3/z, 1/t), (1/t, 3/z)\}.$$

Table 2: Multiset information table

X	a_1	a_2	a_3
$2/x$	expensive	beginner	Yes
$2/y$	expensive	advanced	Yes
$3/z$	moderate	professional	No
$2/r$	cheap	advanced	No
$1/t$	expensive	professional	Yes

Consequently, for all elements $m/a \in M$, we can compute $(m/a)R$ as follows:

$$(2/x)R = \{2/x, 2/y, 1/t\}$$
$$(2/y)R = \{2/x, 2/y, 2/r, 1/t\}$$
$$(3/z)R = \{3/z, 2/r, 1/t\}$$
$$(2/r)R = \{2/y, 3/z, 2/r\}$$
$$(1/t)R = \{2/x, 2/y, 3/z, 1/t\}.$$

Now, using (1), we can calculate $\langle m/a\rangle_R$ as follows:

$$\langle 2/x\rangle_R = \{2/x, 2/y, 1/t\}$$
$$\langle 2/y\rangle_R = \{2/y\}$$
$$\langle 3/z\rangle_R = \{2/y, 3/z\}$$
$$\langle 2/r\rangle_R = \{2/r\}$$
$$\langle 1/t\rangle_R = \{1/t\}.$$

Let $N = \{2/y, 1/t\}$ be a submultiset of M. Then we find the rough multiset membership function with respect to N.

$$\mu_N(2/x) = \frac{3}{5}, \qquad \mu_N(2/y) = 1, \qquad \mu_N(3/z) = \frac{2}{5}, \qquad \mu_N(2/r) = 0, \qquad \mu_N(1/t) = 1.$$

Using this result, then we can compute the lower multiset, uper multiset, and the accuracy of N as follows

$$\underline{N} = \{2/y, 1/t\}, \qquad \overline{N} = \{2/x, 2/y, 3/z, 1/t\}, \qquad \text{and} \qquad \alpha_R(N) = \frac{3}{8}.$$

Therefore, the accuracy of choosing supplier between category $2/y$ or $1/t$ is a bout 40%.

5. Conclusion

A set is a well-defined collection of singular items according to classical set theory. A mathematical structure known as a multiset exists when an object can occur more than once in a set. In this paper, the idea of a rough membership function has been proposed in the environment of multisets. We introduced counter-examples to clarify some identities and properties of this concept. As a result of this notion, we introduced the lower and upper multiset approximations via this approach of rough membership multiset functions. The properties of these approximations have been studied. Also, we generated a topology via this approach. Finally, the application for choosing the best chemical suppliers has finally been presented.

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