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# Integral inequalities for differentiable $s$-convex functions in the second sense via Atangana-Baleanu fractional integral operators

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**Abstract.** Fractional integral operators, which form strong links between fractional analysis and integral inequalities, make unique contributions to the field of inequality theory due to their properties and strong kernel structures. In this context, the novelty brought to the field by the study can be expressed as the new and first findings of Ostrowski type that contain Atangana-Baleanu fractional integral operators for differentiable s-convex functions in the second sense. In the study, two new integral identities were established for Atangana-Baleanu fractional integral operators and by using these two new integral identities, Ostrowski type integral inequalities were obtained. In the findings, it was aimed to contribute to the field due to the structural properties of Atangana-Baleanu fractional integral operators.

## 1. Introduction

Convex analysis is a field with a very broad spectrum, where many concepts that include highly effective applications in space classification, programming, statistics and numerical analysis are introduced and offered to the service of mathematics. In particular, the field of inequality theory to which convex analysis is related reveals new inequalities by using convex function types.

We will start by introducing the convex function and its general variant, $s-$convex functions in the second sense, whose algebraic definitions are presented as inequality, which stand out in terms of their applications and areas of use in function types.

In [35], Orlicz defined $s-$convex functions as following:

**Definition 1.1.** *A function $f:\mathbb{R}^+\to\mathbb{R}$, where $\mathbb{R}^+=[0,\infty)$, is said to be $s-$convex in the first sense if*

$$f(\alpha\omega_1+\beta\omega_2)\leq\alpha^s f(\omega_1)+\beta^s f(\omega_2)$$

*for all $\omega_1,\omega_2\in[0,\infty)$, $\alpha,\beta\geq 0$ with $\alpha^s+\beta^s=1$ and for some fixed $s\in(0,1]$.*

We denote by $K_s^1$ the class of all $s-$convex functions in the first sense.

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**Definition 1.2.** *A function $f : \mathbb{R}^+ \to \mathbb{R}$, where $\mathbb{R}^+ = [0, \infty)$, is said to be $s-$convex in the second sense if*

$$f(\alpha\omega_1 + \beta\omega_2) \leq \alpha^s f(\omega_1) + \beta^s f(\omega_2)$$

*for all $\omega_1, \omega_2 \in [0, \infty)$, $\alpha, \beta \geq 0$ with $\alpha + \beta = 1$ and for some fixed $s \in (0, 1]$.*

We denote by $K_s^2$ the class of all $s-$convex functions in the second sense.

Obviously, one can see that in case of $s = 1$, both definitions overlap with the standard concept of convexity.

Various integral inequalities have been proved on convex functions. Hermite-Hadamard inequality, which produces lower and upper bounds for the mean value of a convex function, is of particular importance among these inequalities. Hadamard's inequality has an aesthetic structure that can be used in numerical integration to calculate errors with the help of mid-point and trapezoidal formulas. Let's introduce this celebrated inequality.

Suppose that $f : I \subset \mathbb{R} \to \mathbb{R}$ is convex mapping on $I \subseteq \mathbb{R}$ where $\omega_1, \omega_2 \in I$, with $\omega_1 < \omega_2$. The following double inequality is called Hermite-Hadamard's inequality for convex functions:

$$f\left(\frac{\omega_1 + \omega_2}{2}\right) \leq \frac{1}{\omega_2 - \omega_1} \int_{\omega_1}^{\omega_2} f(\omega)\, d\omega \leq \frac{f(\omega_1) + f(\omega_2)}{2} \tag{1}$$

Ostrowski's inequality is an aesthetic and useful inequality as well as Hadamard's inequality and is valid for differentiable and bounded functions. In [24], Ostrowski proved this inequality as follows.

**Theorem 1.3.** *Let $f$ be a differentiable mapping on $(\omega_1, \omega_2)$ and let, on $(\omega_1, \omega_2)$, $\left|f'(\omega)\right| \leq K$. Then, for every $\omega \in (\omega_1, \omega_2)$, one has*

$$\left| f(\omega) - \frac{1}{\omega_2 - \omega_1} \int_{\omega_1}^{\omega_2} f(t)dt \right| \leq \left[ \frac{1}{4} + \frac{\left(\omega - \frac{\omega_1+\omega_2}{2}\right)^2}{(\omega_2 - \omega_1)^2} \right] (\omega_2 - \omega_1) K. \tag{2}$$

To investigate different kinds of convex functions and generalizations, new variants and different forms of these two important inequalities, we recommend to see the papers [16], [21]-[23], [25]-[34],[36]-[38] and [40].

Although the origins of fractional analysis are as old as classical analysis, its real value has not been understood for a long time and its usage areas have remained quite limited. In recent years, the development process of fractional analysis has regained momentum and has become the focus of many researchers. Of course, this development has been due to the impact of fractional analysis on many disciplines and its effectiveness in usage. The fact that researchers have turned to new fractional derivatives and integral operators and that the defined new operators have strong kernel structures increases the interest in the subject day by day. Fractional derivative operators and associated integral operators have brought a new perspective to real world problems, as in many areas of mathematics, with their kernel structure-based properties such as locality, singularity, and aspects such as innovation, the effect of reaching general forms, stability of solutions, and time memory effect. This positive effect of fractional analysis on processes and fields is undoubtedly seen in inequality theory.

To collect more findings about applications, structures and further features of fractional operators, we recommend to the interested readers the following papers [2], [5]-[15], [17]-[20] and [39],[41].

Now, we are in a position to remember some of the derivative and integral operators that come to the fore in fractional analysis.

**Definition 1.4.** *(See [4]) Let $f \in H^1(0, \omega_2)$, $\omega_2 > \omega_1$, $\alpha \in [0, 1]$ then, the definition of the new Caputo fractional derivative can be given as:*

$${}^{CF}D^{\alpha} f(t) = \frac{M(\alpha)}{1 - \alpha} \int_{\omega_1}^{t} f'(s) exp\left[ -\frac{\alpha}{(1 - \alpha)}(t - s) \right] ds \tag{3}$$

*where $M(\alpha)$ is normalization function.*

The integral operator associated to this fractional derivative has been given with a non-singular kernel structure as follows.

**Definition 1.5.** *(See [19]) Let $f \in H^1(0,\omega_2)$, $\omega_2 > \omega_1$, $\alpha \in [0,1]$ then, the definition of the left and right side of Caputo-Fabrizio fractional integral can be given as:*

$$\left({}^{CF}_{\omega_1}I^{\alpha}\right)(t) = \frac{1-\alpha}{B(\alpha)}f(t) + \frac{\alpha}{B(\alpha)}\int_{\omega_1}^{t} f(y)dy,$$

*and*

$$\left({}^{CF}I^{\alpha}_{\omega_2}\right)(t) = \frac{1-\alpha}{B(\alpha)}f(t) + \frac{\alpha}{B(\alpha)}\int_{t}^{\omega_2} f(y)dy$$

*where $B(\alpha)$ is normalization function.*

The lack of this fractional operator, which is given as a very useful definition, is that the original function does not appear for any of the special values of the parameter. Based on this deficiency, Atangana and Baleanu have defined a new fractional derivative and integral operator that containing a similar normalization function with the same properties.

**Definition 1.6.** *(See [3]) Let $f \in H^1(\omega_1,\omega_2)$, $\omega_2 > \omega_1$, $\alpha \in [0,1]$ then, the definition of the new fractional derivative is given:*

$${}^{ABC}_{\omega_1}D^{\alpha}_{t}\left[f(t)\right] = \frac{B(\alpha)}{1-\alpha}\int_{\omega_1}^{t} f'(x)E_{\alpha}\left[-\alpha\frac{(t-x)^{\alpha}}{(1-\alpha)}\right]dx. \tag{4}$$

**Definition 1.7.** *(See [3]) Let $f \in H^1(\omega_1,\omega_2)$, $\omega_2 > \omega_1$, $\alpha \in [0,1]$ then, the definition of the new fractional derivative is given as:*

$${}^{ABR}_{\omega_1}D^{\alpha}_{t}\left[f(t)\right] = \frac{B(\alpha)}{1-\alpha}\frac{d}{dt}\int_{\omega_1}^{t} f(x)E_{\alpha}\left[-\alpha\frac{(t-x)^{\alpha}}{(1-\alpha)}\right]dx. \tag{5}$$

This interesting fractional derivative operator, which derives its non-locality and non-singularity properties thanks to the Mittag-Leffler function at its kernel, has become an effective tool in engineering, physics, statistics and mathematical biology. The associated integral operator is presented as follows.

**Definition 1.8.** *[3] The fractional integral associate to the new fractional derivative with non-local kernel of a function $f \in H^1(\omega_1,\omega_2)$ is defined as:*

$${}^{AB}_{\omega_1}I^{\alpha}_{t}\left\{f(t)\right\} = \frac{1-\alpha}{B(\alpha)}f(t) + \frac{\alpha}{B(\alpha)\Gamma(\alpha)}\int_{\omega_1}^{t} f(u)(t-u)^{\alpha-1}du$$

*where $\omega_2 > \omega_1, \alpha \in [0,1]$.*

In [1], the authors have given the right hand side of integral operator as following;

$${}^{AB}I^{\alpha}_{\omega_2}\left\{f(t)\right\} = \frac{1-\alpha}{B(\alpha)}f(t) + \frac{\alpha}{B(\alpha)\Gamma(\alpha)}\int_{t}^{\omega_2} f(u)(u-t)^{\alpha-1}du.$$

Here, $\Gamma(\alpha)$ is the Gamma function. Since the normalization function $B(\alpha) > 0$ is positive, it immediately follows that the fractional Atangana-Baleanu integral of a positive function is positive. It should be noted that, when the order $\alpha \longrightarrow 1$, we recover the classical integral. Also, the initial function is recovered whenever the fractional order $\alpha \longrightarrow 0$.
Since Atangana-Baleanu fractional integral operators are derived from the non-singular and non-local

derivative operator with a strong kernel, they are an effective tool especially in real world problems. Due to these features, it is highly preferred in many applied fields such as engineering, physics and mathematical biology. In inequality theory emerges as an efficient operator that is preferred to generalize the results that are exist in the literature and to obtain new approaches.

The general motivation points of the studies on integral inequalities in the literature are to obtain new boundaries and approaches, to introduce generalizations, to improve the known boundaries and to reach modifications in different spaces. Our main motivation point in this study is to present Ostrowski type inequalities with the help of Atangana-Baleanu integral operators and to prove generalizations. For this purpose, firstly, two new integral equations were created and Ostrowski type inequalities were obtained for functions whose first and second order derivatives are $s-$convex in the second sense based on these two identities.

## 2. New results for *s*-convex functions of first order differentiable

We will start with our main findings by giving the proof of the following integral identity that involves Atangana-Baleanu fractional integral operators below (See [42]).

**Lemma 2.1.** *Let $\omega_1 < \omega_2$, $\omega_1, \omega_2 \in J^\circ$ and $\varphi : J \subset \mathbb{R} \longrightarrow \mathbb{R}$ be a differentiable mapping on $J^\circ$. If $\varphi' \in L[\omega_1, \omega_2]$, the following identity for Atangana-Baleanu fractional integral operators is valid for all $\omega \in [\omega_1, \omega_2]$, $\xi \in (0, 1]$ and $\varpi \in [0, 1]$ :*

$$\begin{aligned}
&\frac{\varphi(\omega)}{(\omega_2-\omega_1)B(\xi)\Gamma(\xi)}\left[(\omega_2-\omega)^{\xi}+(\omega-\omega_1)^{\xi}\right] \\
&-\frac{1}{(\omega_2-\omega_1)}\left[{}^{AB}I_{\omega}^{\xi}\{\varphi(\omega_1)\}+{}_{\omega}^{AB}I_{\omega_2}^{\xi}\{\varphi(\omega_2)\}\right] \\
&+\frac{1-\xi}{(\omega_2-\omega_1)B(\xi)}\left[\varphi(\omega_1)+\varphi(\omega_2)\right] \\
=\;&\frac{(\omega-\omega_1)^{\xi+1}}{(\omega_2-\omega_1)B(\xi)\Gamma(\xi)}\int_0^1\varpi^{\xi}\varphi'(\varpi\omega+(1-\varpi)\omega_1)d\varpi \\
&-\frac{(\omega_2-\omega)^{\xi+1}}{(\omega_2-\omega_1)B(\xi)\Gamma(\xi)}\int_0^1\varpi^{\xi}\varphi'(\varpi\omega+(1-\varpi)\omega_2)d\varpi.
\end{aligned} \tag{6}$$

*Here $B(\xi) > 0$ and $\Gamma(\xi)$ are normalization function and Euler gamma function respectively.*

*Proof.* The method of integration by parts was used to prove Lemma 2.1. By using this method, we can write

$$\begin{aligned}
&\frac{(\omega-\omega_1)^{\xi}}{B(\xi)\Gamma(\xi)}\int_0^1\varpi^{\xi-1}\varphi(\varpi\omega+(1-\varpi)\omega_1)d\varpi \\
=\;&\frac{(\omega-\omega_1)^{\xi}}{B(\xi)\Gamma(\xi)}\left[\varphi(\varpi\omega+(1-\varpi)\omega_1)\frac{\varpi^{\xi}}{\xi}\bigg|_0^1\right. \\
&\left.-\int_0^1\frac{\varpi^{\xi}}{\xi}\varphi'(\varpi\omega+(1-\varpi)\omega_1)(\omega-\omega_1)d\varpi\right] \\
=\;&\frac{(\omega-\omega_1)^{\xi}}{\xi B(\xi)\Gamma(\xi)}\varphi(\omega)-\frac{(\omega-\omega_1)^{\xi+1}}{\xi B(\xi)\Gamma(\xi)}\int_0^1\varpi^{\xi}\varphi'(\varpi\omega+(1-\varpi)\omega_1)d\varpi
\end{aligned} \tag{7}$$

and

$$\begin{aligned}
& \frac{(\omega_2-\omega)^{\xi}}{B(\xi)\Gamma(\xi)}\int_0^1 \varpi^{\xi-1}\varphi(\varpi\omega+(1-\varpi)\omega_2)d\varpi \\
= & \frac{(\omega_2-\omega)^{\xi}}{B(\xi)\Gamma(\xi)}\left[\varphi(\varpi\omega+(1-\varpi)\omega_2)\frac{\varpi^{\xi}}{\xi}\Big|_0^1\right. \\
& \left.-\int_0^1 \frac{\varpi^{\xi}}{\xi}\varphi'(\varpi\omega+(1-\varpi)\omega_2)(\omega-\omega_2)d\varpi\right] \\
= & \frac{(\omega_2-\omega)^{\xi}}{\xi B(\xi)\Gamma(\xi)}\varphi(\omega)-\frac{(\omega_2-\omega)^{\xi+1}}{\xi B(\xi)\Gamma(\xi)}\int_0^1 \varpi^{\xi}\varphi'(\varpi\omega+(1-\varpi)\omega_2)d\varpi.
\end{aligned} \tag{8}$$

If we multiply the equations in (7) and (8) by $-\frac{\xi}{\omega_2-\omega_1}$ and then by adding the resulting equations with change the variables for left hand side the last equation, we complete the proof of Lemma 2.1. □

This lemma is important in the theory of inequality in terms of being the first lemma of Ostrowski type that includes Atangana-Baleanu fractional integral operators.

Now, we will express the first theorem by using this lemma, which is the main motivation of the study, for the concept of $s$-convexity in the second sense.

**Theorem 2.2.** *Let $\omega_1 < \omega_2$, $\omega_1, \omega_2 \in J^\circ$ and $\varphi : J \subset [0,\infty) \longrightarrow \mathbb{R}$ be a differentiable mapping on $J^\circ$ and $\varphi' \in L[\omega_1,\omega_2]$. If $|\varphi'|$ is an s-convex mapping in the second sense on $[\omega_1,\omega_2]$ and $|\varphi'| \le K, K>0$ for all $\omega \in [\omega_1,\omega_2]$, $\xi, s \in (0,1]$. Then we obtain the inequality below that includes Atangana-Baleanu fractional integral operators:*

$$\begin{aligned}
& \left|\frac{\varphi(\omega)}{(\omega_2-\omega_1)B(\xi)\Gamma(\xi)}\left[(\omega_2-\omega)^{\xi}+(\omega-\omega_1)^{\xi}\right]\right. \\
& -\frac{1}{(\omega_2-\omega_1)}\left[{}^{AB}I^{\xi}_{\omega}\{\varphi(\omega_1)\}+{}^{AB}_{\omega}I^{\xi}_{\omega_2}\{\varphi(\omega_2)\}\right] \\
& \left.+\frac{1-\xi}{(\omega_2-\omega_1)B(\xi)}\left[\varphi(\omega_1)+\varphi(\omega_2)\right]\right| \\
\le & \frac{K}{B(\xi)\Gamma(\xi)}\left(\frac{(\omega-\omega_1)^{\xi+1}+(\omega_2-\omega)^{\xi+1}}{\omega_2-\omega_1}\right)\left(\frac{1}{\xi+s+1}+\beta(\xi+1,s+1)\right).
\end{aligned} \tag{9}$$

*Here $B(\xi)>0$ and $\beta$ is Euler Beta function.*

*Proof.* By using the equality in (6), we have

$$\begin{aligned}
& \left|\frac{\varphi(\omega)}{(\omega_2-\omega_1)B(\xi)\Gamma(\xi)}\left[(\omega_2-\omega)^{\xi}+(\omega-\omega_1)^{\xi}\right]\right. \\
& -\frac{1}{(\omega_2-\omega_1)}\left[{}^{AB}I^{\xi}_{\omega}\{\varphi(\omega_1)\}+{}^{AB}_{\omega}I^{\xi}_{\omega_2}\{\varphi(\omega_2)\}\right] \\
& \left.+\frac{1-\xi}{(\omega_2-\omega_1)B(\xi)}\left[\varphi(\omega_1)+\varphi(\omega_2)\right]\right| \\
\le & \frac{(\omega-\omega_1)^{\xi+1}}{(\omega_2-\omega_1)B(\xi)\Gamma(\xi)}\int_0^1 \varpi^{\xi}\left|\varphi(\varpi\omega+(1-\varpi)\omega_1)\right|d\varpi \\
& +\frac{(\omega_2-\omega)^{\xi+1}}{(\omega_2-\omega_1)B(\xi)\Gamma(\xi)}\int_0^1 \varpi^{\xi}\left|\varphi(\varpi\omega+(1-\varpi)\omega_2)\right|d\varpi.
\end{aligned} \tag{10}$$

If we use the $s$-convexity of $|\varphi'|$ and the fact that $|\varphi'| \leq K$ in (10), we can deduce

$$\begin{aligned}
&\Bigg|\frac{\varphi(\omega)}{(\omega_2-\omega_1)B(\xi)\Gamma(\xi)}\left[(\omega_2-\omega)^{\xi}+(\omega-\omega_1)^{\xi}\right] \\
&-\frac{1}{\omega_2-\omega_1}\left[{}^{AB}I_{\omega}^{\xi}\{\varphi(\omega_1)\}+{}_{\omega}^{AB}I_{\omega_2}^{\xi}\{\varphi(\omega_2)\}\right] \\
&+\frac{1-\xi}{(\omega_2-\omega_1)B(\xi)}\left[\varphi(\omega_1)+\varphi(\omega_2)\right]\Bigg| \\
\leq\ & \frac{(\omega-\omega_1)^{\xi+1}}{(\omega_2-\omega_1)B(\xi)\Gamma(\xi)}\int_0^1 \varpi^{\xi}\left[\varpi^{s}\left|\varphi'(\omega)\right|+(1-\varpi)^{s}\left|\varphi'(\omega_1)\right|\right]d\varpi \\
&+\frac{(\omega_2-\omega)^{\xi+1}}{(\omega_2-\omega_1)B(\xi)\Gamma(\xi)}\int_0^1 \varpi^{\xi}\left[\varpi^{s}\left|\varphi'(\omega)\right|+(1-\varpi)^{s}\left|\varphi'(\omega_2)\right|\right]d\varpi \\
\leq\ & \frac{K}{B(\xi)\Gamma(\xi)}\left(\frac{(\omega-\omega_1)^{\xi+1}+(\omega_2-\omega)^{\xi+1}}{\omega_2-\omega_1}\right)\left(\frac{1}{\xi+s+1}+\beta(\xi+1,s+1)\right).
\end{aligned}$$

The proof is obtained. □

**Corollary 2.3.** *In Theorem 2.2, if we choose $\omega=\frac{\omega_1+\omega_2}{2}$, we have the following inequality:*

$$\begin{aligned}
&\Bigg|\frac{(\omega_2-\omega_1)^{\xi-1}}{2^{\xi-1}B(\xi)\Gamma(\xi)}\varphi\left(\frac{\omega_1+\omega_2}{2}\right) \\
&-\frac{1}{\omega_2-\omega_1}\left[{}^{AB}I_{\frac{\omega_1+\omega_2}{2}}^{\xi}\{\varphi(\omega_1)\}+{}_{\frac{\omega_1+\omega_2}{2}}^{AB}I_{\omega_2}^{\xi}\{\varphi(\omega_2)\}\right] \\
&+\frac{1-\xi}{(\omega_2-\omega_1)B(\xi)}\left[\varphi(\omega_1)+\varphi(\omega_2)\right]\Bigg| \\
\leq\ & \frac{K}{B(\xi)\Gamma(\xi)}\left(\frac{\omega_2-\omega_1}{2}\right)^{\xi}\left(\frac{1}{\xi+s+1}+\beta(\xi+1,s+1)\right).
\end{aligned}$$

In the rest of the this section, for the simplicity we will use the following notations:

$$\begin{aligned}
M =\ & \frac{\varphi(\omega)}{(\omega_2-\omega_1)B(\xi)\Gamma(\xi)}\left[(\omega_2-\omega)^{\xi}+(\omega-\omega_1)^{\xi}\right] \\
&-\frac{1}{(\omega_2-\omega_1)}\left[{}^{AB}I_{\omega}^{\xi}\{\varphi(\omega_1)\}+{}_{\omega}^{AB}I_{\omega_2}^{\xi}\{\varphi(\omega_2)\}\right] \\
&+\frac{1-\xi}{(\omega_2-\omega_1)B(\xi)}\left[\varphi(\omega_1)+\varphi(\omega_2)\right],
\end{aligned}$$

$$\begin{aligned}
N =\ & \frac{(\omega_2-\omega_1)^{\xi-1}}{2^{\xi-1}B(\xi)\Gamma(\xi)}\varphi\left(\frac{\omega_1+\omega_2}{2}\right) \\
&-\frac{1}{\omega_2-\omega_1}\left[{}^{AB}I_{\frac{\omega_1+\omega_2}{2}}^{\xi}\{\varphi(\omega_1)\}+{}_{\frac{\omega_1+\omega_2}{2}}^{AB}I_{\omega_2}^{\xi}\{\varphi(\omega_2)\}\right] \\
&+\frac{1-\xi}{(\omega_2-\omega_1)B(\xi)}\left[\varphi(\omega_1)+\varphi(\omega_2)\right].
\end{aligned}$$

It will also not be repeated in the rest of the study that $B$ is the normalization function that takes positive values, $\Gamma$ is the Gamma function and $\beta$ is the Beta function.

**Theorem 2.4.** *Let $\omega_1 < \omega_2$, $\omega_1, \omega_2 \in J^\circ$ and $\varphi : J \subset [0,\infty) \longrightarrow \mathbb{R}$ be a differentiable mapping on $J^\circ$ and $\varphi' \in L[\omega_1,\omega_2]$. If $\left|\varphi'\right|^q$ is an s-convex mapping in the second sense on $[\omega_1,\omega_2]$ and $\left|\varphi'\right| \leq K$, for all $\omega \in [\omega_1,\omega_2]$, $\xi, s \in (0,1]$. Then we obtain the inequality below that includes Atangana-Baleanu fractional integral operators:*

$$|M| \leq \frac{K}{B(\xi)\Gamma(\xi)}\left(\frac{1}{\xi p+1}\right)^{\frac{1}{p}}\left(\frac{2}{s+1}\right)^{\frac{1}{q}}\left(\frac{(\omega-\omega_1)^{\xi+1}+(\omega_2-\omega)^{\xi+1}}{\omega_2-\omega_1}\right)$$

*where $q > 1$ and $\frac{1}{p}+\frac{1}{q}=1$.*

*Proof.* To prove Theorem 2.4; Lemma 2.1, property of modulus, Hölder inequality, $s$-convexity of $\left|\varphi'\right|^q$ and the fact that $\left|\varphi'\right| \leq K$ will be used. If we take advantage of these expressions respectively, we have

$$\begin{aligned}
|M| \leq\ & \frac{(\omega-\omega_1)^{\xi+1}}{(\omega_2-\omega_1)B(\xi)\Gamma(\xi)}\left(\int_0^1 \varpi^{\xi p}d\varpi\right)^{\frac{1}{p}}\left(\int_0^1 \left|\varphi'(\varpi\omega+(1-\varpi)\omega_1)\right|^q d\varpi\right)^{\frac{1}{q}} \\
&+\frac{(\omega_2-\omega)^{\xi+1}}{(\omega_2-\omega_1)B(\xi)\Gamma(\xi)}\left(\int_0^1 \varpi^{\xi p}d\varpi\right)^{\frac{1}{p}}\left(\int_0^1 \left|\varphi'(\varpi\omega+(1-\varpi)\omega_2)\right|^q d\varpi\right)^{\frac{1}{q}} \\
\leq\ & \frac{(\omega-\omega_1)^{\xi+1}}{(\omega_2-\omega_1)B(\xi)\Gamma(\xi)}\left(\frac{1}{\xi p+1}\right)^{\frac{1}{p}}\left(\int_0^1 \left[\varpi^s\left|\varphi'(\omega)\right|^q+(1-\varpi)^s\left|\varphi'(\omega_1)\right|^q\right]d\varpi\right)^{\frac{1}{q}} \\
&+\frac{(\omega_2-\omega)^{\xi+1}}{(\omega_2-\omega_1)B(\xi)\Gamma(\xi)}\left(\frac{1}{\xi p+1}\right)^{\frac{1}{p}}\left(\int_0^1 \left[\varpi^s\left|\varphi'(\omega)\right|^q+(1-\varpi)^s\left|\varphi'(\omega_2)\right|^q\right]d\varpi\right)^{\frac{1}{q}} \\
=\ & \frac{(\omega-\omega_1)^{\xi+1}}{(\omega_2-\omega_1)B(\xi)\Gamma(\xi)}\left(\frac{1}{\xi p+1}\right)^{\frac{1}{p}}\left(\frac{\left|\varphi'(\omega)\right|^q+\left|\varphi'(\omega_1)\right|^q}{s+1}\right)^{\frac{1}{q}} \\
&+\frac{(\omega_2-\omega)^{\xi+1}}{(\omega_2-\omega_1)B(\xi)\Gamma(\xi)}\left(\frac{1}{\xi p+1}\right)^{\frac{1}{p}}\left(\frac{\left|\varphi'(\omega)\right|^q+\left|\varphi'(\omega_2)\right|^q}{s+1}\right)^{\frac{1}{q}} \\
\leq\ & \frac{K}{B(\xi)\Gamma(\xi)}\left(\frac{1}{\xi p+1}\right)^{\frac{1}{p}}\left(\frac{2}{s+1}\right)^{\frac{1}{q}}\left(\frac{(\omega-\omega_1)^{\xi+1}+(\omega_2-\omega)^{\xi+1}}{\omega_2-\omega_1}\right).
\end{aligned}$$

So, the proof of Theorem 2.4 is done. □

As we did in Theorem 2.2, we will give some results via making special choices in the inequality we get in Theorem 2.4.

**Corollary 2.5.** *In Theorem 2.4, if we choose $\omega = \frac{\omega_1+\omega_2}{2}$, we have the following inequality:*

$$|N| \leq \frac{K}{B(\xi)\Gamma(\xi)}\left(\frac{1}{\xi p+1}\right)^{\frac{1}{p}}\left(\frac{2}{s+1}\right)^{\frac{1}{q}}\left(\frac{\omega_2-\omega_1}{2}\right)^{\xi}.$$

We will obtain new results in the following two theorems by constructing Hölder's inequality in different ways.

**Theorem 2.6.** *Assume that the assumptions given in the Theorem 2.4 are valid. Then, we have the following inequality:*

$$|M| \leq \frac{K}{B(\xi)\Gamma(\xi)}\left(\frac{1}{\xi+1}\right)^{\frac{1}{p}}\left(\frac{1}{\xi+s+1}+\beta(\xi+1,s+1)\right)^{\frac{1}{q}}\left(\frac{(\omega-\omega_1)^{\xi+1}+(\omega_2-\omega)^{\xi+1}}{\omega_2-\omega_1}\right).$$

*Proof.* In addition to the operations we used in Theorem 2.4, we obtain the following inequality by using the Hölder's inequality in a different way:

$$\begin{aligned}|M| \leq\ & \frac{(\omega-\omega_1)^{\xi+1}}{(\omega_2-\omega_1)B(\xi)\Gamma(\xi)}\left(\int_0^1 \varpi^{\xi} d\varpi\right)^{\frac{1}{p}}\left(\int_0^1 \varpi^{\xi}\left|\varphi'(\varpi\omega+(1-\varpi)\omega_1)\right|^q d\varpi\right)^{\frac{1}{q}} \\ & +\frac{(\omega_2-\omega)^{\xi+1}}{(\omega_2-\omega_1)B(\xi)\Gamma(\xi)}\left(\int_0^1 \varpi^{\xi} d\varpi\right)^{\frac{1}{p}}\left(\int_0^1 \varpi^{\xi}\left|\varphi'(\varpi\omega+(1-\varpi)\omega_2)\right|^q d\varpi\right)^{\frac{1}{q}}.\end{aligned}$$

We complete the proof by making the necessary calculations in obtained new inequality by using the $s$-convexity of $\left|\varphi'\right|^q$ and the fact that $\left|\varphi'\right| \leq K$ above. □

**Corollary 2.7.** *In Theorem 2.6, if we choose $\omega = \frac{\omega_1+\omega_2}{2}$, we have the following inequality:*

$$|N| \leq \frac{K}{B(\xi)\Gamma(\xi)}\left(\frac{1}{\xi+1}\right)^{\frac{1}{p}}\left(\frac{\omega_2-\omega_1}{2}\right)^{\xi}\left(\frac{1}{\xi+s+1}+\beta(\xi+1,s+1)\right)^{\frac{1}{q}}.$$

**Theorem 2.8.** *Assume that the assumptions given in the Theorem 2.4 are valid. Then, we have the following inequality:*

$$\begin{aligned}|M| \leq\ & \frac{K}{B(\xi)\Gamma(\xi)}\left(\frac{(\omega-\omega_1)^{\xi+1}+(\omega_2-\omega)^{\xi+1}}{\omega_2-\omega_1}\right)\left(\frac{q-1}{\xi(q-p)+q-1}\right)^{1-\frac{1}{q}} \\ & \times\left(\frac{1}{\xi p+s+1}+\beta(\xi p+1,s+1)\right)^{\frac{1}{q}}\end{aligned}$$

*where $q \geq p > 1$.*

*Proof.* Again, similar to the proof of the previous theorem, applying Hölder's inequality in a different way, we have

$$\begin{aligned}|M| \leq\ & \frac{(\omega-\omega_1)^{\xi+1}}{(\omega_2-\omega_1)B(\xi)\Gamma(\xi)}\left(\int_0^1 \varpi^{\xi\left(\frac{q-p}{q-1}\right)} d\varpi\right)^{1-\frac{1}{q}}\left(\int_0^1 \varpi^{\xi p}\left|\varphi'(\varpi\omega+(1-\varpi)\omega_1)\right|^q d\varpi\right)^{\frac{1}{q}} \\ & +\frac{(\omega_2-\omega)^{\xi+1}}{(\omega_2-\omega_1)B(\xi)\Gamma(\xi)}\left(\int_0^1 \varpi^{\xi\left(\frac{q-p}{q-1}\right)} d\varpi\right)^{1-\frac{1}{q}}\left(\int_0^1 \varpi^{\xi p}\left|\varphi'(\varpi\omega+(1-\varpi)\omega_2)\right|^q d\varpi\right)^{\frac{1}{q}}.\end{aligned}$$

We complete the proof by making the necessary calculations in obtained new inequality by using the $s$-convexity of $\left|\varphi'\right|^q$ and the fact that $\left|\varphi'\right| \leq K$ above. □

**Corollary 2.9.** *In Theorem 2.8, if we choose $\omega = \frac{\omega_1+\omega_2}{2}$, we have the following inequality:*

$$|N| \leq \frac{K}{B(\xi)\Gamma(\xi)}\left(\frac{\omega_2-\omega_1}{2}\right)^{\xi}\left(\frac{q-1}{\xi(q-p)+q-1}\right)^{1-\frac{1}{q}}\left(\frac{1}{\xi p+s+1}+\beta(\xi p+1,s+1)\right)^{\frac{1}{q}}.$$

Using the $s$-concavity concept, the following theorem is obtained.

**Theorem 2.10.** *Let $\omega_1 < \omega_2$, $\omega_1, \omega_2 \in J^{\circ}$and $\varphi : J \subset [0,\infty) \longrightarrow \mathbb{R}$ be a differentiable mapping on $J^{\circ}$ and $\varphi' \in L[\omega_1,\omega_2]$. If $\left|\varphi'\right|^q$ is an $s$-concave mapping on $[\omega_1,\omega_2]$, for all $\omega \in [\omega_1,\omega_2]$ and $\xi, s \in (0,1]$ we obtain the inequality below:*

$$\begin{aligned}|M| \leq\ & \frac{(\omega-\omega_1)^{\xi+1}}{(\omega_2-\omega_1)B(\xi)\Gamma(\xi)}\left(\frac{1}{\xi p+1}\right)^{\frac{1}{q}} 2^{\frac{s-1}{q}}\left|\varphi'\left(\frac{\omega+\omega_1}{2}\right)\right| \\ & +\frac{(\omega_2-\omega)^{\xi+1}}{(\omega_2-\omega_1)B(\xi)\Gamma(\xi)}\left(\frac{1}{\xi p+1}\right)^{\frac{1}{q}} 2^{\frac{s-1}{q}}\left|\varphi'\left(\frac{\omega+\omega_2}{2}\right)\right|\end{aligned}$$

*where* $q > 1$ *and* $\frac{1}{p} + \frac{1}{q} = 1$.

*Proof.* If we apply Hölder's inequality similar to the proof of Theorem 2.4, we have

$$\begin{aligned}|M| \le\ & \frac{(\omega-\omega_1)^{\xi+1}}{(\omega_2-\omega_1)B(\xi)\Gamma(\xi)}\left(\int_0^1 \varpi^{\xi p}d\varpi\right)^{\frac{1}{p}}\left(\int_0^1 \left|\varphi'(\varpi\omega+(1-\varpi)\omega_1)\right|^q d\varpi\right)^{\frac{1}{q}} \\ &+\frac{(\omega_2-\omega)^{\xi+1}}{(\omega_2-\omega_1)B(\xi)\Gamma(\xi)}\left(\int_0^1 \varpi^{\xi p}d\varpi\right)^{\frac{1}{p}}\left(\int_0^1 \left|\varphi'(\varpi\omega+(1-\varpi)\omega_2)\right|^q d\varpi\right)^{\frac{1}{q}}.\end{aligned}$$

Since $\left|\varphi'\right|^q$ is $s$-concave on $[\omega_1,\omega_2]$, we can write following results by taking into account the variant of the Hermite-Hadamard inequality for $s$-concave functions:

$$\int_0^1 \left|\varphi'(\varpi\omega+(1-\varpi)\omega_1)\right|^q d\varpi \le 2^{s-1}\left|\varphi'\left(\frac{\omega+\omega_1}{2}\right)\right|^q,$$

$$\int_0^1 \left|\varphi'(\varpi\omega+(1-\varpi)\omega_2)\right|^q d\varpi \le 2^{s-1}\left|\varphi'\left(\frac{\omega+\omega_2}{2}\right)\right|^q.$$

If we use these results above, we complete the proof of Theorem 2.10. □

**Corollary 2.11.** *In Theorem 2.10, if we choose* $\omega = \frac{\omega_1+\omega_2}{2}$, *we have the following inequality:*

$$|N| \le \frac{(\omega_2-\omega_1)^{\xi}}{2^{\xi+1}B(\xi)\Gamma(\xi)}\left(\frac{1}{\xi p+1}\right)^{\frac{1}{q}} 2^{\frac{s-1}{q}}\left[\left|\varphi'\left(\frac{3\omega_1+\omega_2}{4}\right)\right|+\left|\varphi'\left(\frac{\omega_1+3\omega_2}{4}\right)\right|\right].$$

## 3. New results for $s$-convex functions of second order differentiable

We will begin to give the results in this section by proving the Ostrowski-like lemma that contains second order derivatives.

**Lemma 3.1.** *Let* $\omega_1 < \omega_2$, $\omega_1,\omega_2 \in J^\circ$ *and* $\varphi : J \subset \mathbb{R} \longrightarrow \mathbb{R}$ *be a differentiable mapping on* $J^\circ$. *If* $\varphi'' \in L[\omega_1,\omega_2]$, *identity for Atangana-Baleanu fractional integral operators in equation (11) is valid for all* $\omega \in [\omega_1,\omega_2]$, $\varpi,\xi \in [0,1]$ :

$$\begin{aligned}& \frac{1}{(\omega_2-\omega_1)}\left[{}^{AB}I_{\omega}^{\xi}\{\varphi(\omega_1)\} + {}_{\omega}^{AB}I_{\omega_2}^{\xi}\{\varphi(\omega_2)\}\right] - \frac{1-\xi}{(\omega_2-\omega_1)B(\xi)}\left[\varphi(\omega_1)+\varphi(\omega_2)\right] \qquad (11)\\ &-\frac{\varphi(\omega)}{(\omega_2-\omega_1)B(\xi)\Gamma(\xi)}\left[(\omega_2-\omega)^{\xi}+(\omega-\omega_1)^{\xi}\right]+\frac{(\omega-\omega_1)^{\xi+1}-(\omega_2-\omega)^{\xi+1}}{(\omega_2-\omega_1)B(\xi)\Gamma(\xi)(\xi+1)}\varphi'(\omega)\\ =\ & \frac{(\omega-\omega_1)^{\xi+2}}{(\omega_2-\omega_1)B(\xi)\Gamma(\xi)(\xi+1)}\int_0^1 \varpi^{\xi+1}\varphi''(\varpi\omega+(1-\varpi)\omega_1)d\varpi\\ &+\frac{(\omega_2-\omega)^{\xi+2}}{(\omega_2-\omega_1)B(\xi)\Gamma(\xi)(\xi+1)}\int_0^1 \varpi^{\xi+1}\varphi''(\varpi\omega+(1-\varpi)\omega_2)d\varpi.\end{aligned}$$

*Proof.* Via integration by parts, we can write

$$\begin{aligned}& \frac{(\omega-\omega_1)^{\xi+1}}{B(\xi)\Gamma(\xi)}\int_0^1 \varpi^{\xi}\varphi'(\varpi\omega+(1-\varpi)\omega_1)d\varpi \qquad (12)\\ =\ & \frac{(\omega-\omega_1)^{\xi+1}}{B(\xi)\Gamma(\xi)}\left[\varphi'(\varpi\omega+(1-\varpi)\omega_1)\frac{\varpi^{\xi+1}}{\xi+1}\Big|_0^1 - \int_0^1 \frac{\varpi^{\xi+1}}{\xi+1}\varphi''(\varpi\omega+(1-\varpi)\omega_1)(\omega-\omega_1)d\varpi\right]\\ =\ & \frac{(\omega-\omega_1)^{\xi+1}}{B(\xi)\Gamma(\xi)(\xi+1)}\varphi'(\omega) - \frac{(\omega-\omega_1)^{\xi+2}}{B(\xi)\Gamma(\xi)(\xi+1)}\int_0^1 \varpi^{\xi+1}\varphi''(\varpi\omega+(1-\varpi)\omega_1)d\varpi\end{aligned}$$

and

$$
\begin{aligned}
& -\frac{(\omega_2-\omega)^{\xi+1}}{B(\xi)\Gamma(\xi)}\int_0^1 \varpi^{\xi}\varphi'(\varpi\omega+(1-\varpi)\omega_2)d\varpi \qquad (13)\\
=\; & -\frac{(\omega_2-\omega)^{\xi+1}}{B(\xi)\Gamma(\xi)}\left[\varphi'(\varpi\omega+(1-\varpi)\omega_2)\frac{\varpi^{\xi+1}}{\xi+1}\Big|_0^1-\int_0^1\frac{\varpi^{\xi+1}}{\xi+1}\varphi''(\varpi\omega+(1-\varpi)\omega_2)(\omega-\omega_2)d\varpi\right]\\
=\; & -\frac{(\omega_2-\omega)^{\xi+1}}{B(\xi)\Gamma(\xi)(\xi+1)}\varphi'(\omega)-\frac{(\omega_2-\omega)^{\xi+2}}{B(\xi)\Gamma(\xi)(\xi+1)}\int_0^1\varpi^{\xi+1}\varphi''(\varpi\omega+(1-\varpi)\omega_2)d\varpi.
\end{aligned}
$$

If we add (12) and (13), and after this operation if we multiply the resulting equality by $\frac{1}{(\omega_2-\omega_1)}$, we complete the proof of Lemma 3.1. □

We will write some new results using this lemma and the concept of $s$-convexity.

**Theorem 3.2.** *Let $\omega_1<\omega_2$, $\omega_1,\omega_2\in J^\circ$ and $\varphi: J\subset[0,\infty)\longrightarrow\mathbb{R}$ be a differentiable mapping on $J^\circ$ and $\varphi''\in L[\omega_1,\omega_2]$. If $|\varphi''|$ is an s-convex mapping in the second sense on $[\omega_1,\omega_2]$, for all $\omega\in[\omega_1,\omega_2]$, $s\in(0,1]$ and $\xi\in[0,1]$. Then, we obtain the inequality below:*

$$
\begin{aligned}
& \left|\frac{1}{(\omega_2-\omega_1)}\left[{}^{AB}I_{\omega}^{\xi}\{\varphi(\omega_1)\}+{}^{AB}_{\omega}I_{\omega_2}^{\xi}\{\varphi(\omega_2)\}\right]-\frac{1-\xi}{(\omega_2-\omega_1)B(\xi)}[\varphi(\omega_1)+\varphi(\omega_2)]\right.\\
& \left.-\frac{\varphi(\omega)}{(\omega_2-\omega_1)B(\xi)\Gamma(\xi)}\left[(\omega_2-\omega)^{\xi}+(\omega-\omega_1)^{\xi}\right]+\frac{(\omega-\omega_1)^{\xi+1}-(\omega_2-\omega)^{\xi+1}}{(\omega_2-\omega_1)B(\xi)\Gamma(\xi)(\xi+1)}\varphi'(\omega)\right|\\
\leq\; & \frac{|\varphi''(\omega)|}{\xi+s+2}\left[\frac{(\omega-\omega_1)^{\xi+2}+(\omega_2-\omega)^{\xi+2}}{B(\xi)\Gamma(\xi)(\xi+1)(\omega_2-\omega_1)}\right]\\
& +\frac{\beta(\xi+2,s+1)}{B(\xi)\Gamma(\xi)(\xi+1)}\left[\frac{(\omega-\omega_1)^{\xi+2}|\varphi''(\omega_1)|+(\omega_2-\omega)^{\xi+2}|\varphi''(\omega_2)|}{\omega_2-\omega_1}\right].
\end{aligned}
$$

*Proof.* By using the equality in (11), property of modulus and $s$-convexity of $|\varphi''|$ we have

$$
\begin{aligned}
& \left|\frac{1}{(\omega_2-\omega_1)}\left[{}^{AB}I_{\omega}^{\xi}\{\varphi(\omega_1)\}+{}^{AB}_{\omega}I_{\omega_2}^{\xi}\{\varphi(\omega_2)\}\right]-\frac{1-\xi}{(\omega_2-\omega_1)B(\xi)}[\varphi(\omega_1)+\varphi(\omega_2)]\right.\\
& \left.-\frac{\varphi(\omega)}{(\omega_2-\omega_1)B(\xi)\Gamma(\xi)}\left[(\omega_2-\omega)^{\xi}+(\omega-\omega_1)^{\xi}\right]+\frac{(\omega-\omega_1)^{\xi+1}-(\omega_2-\omega)^{\xi+1}}{(\omega_2-\omega_1)B(\xi)\Gamma(\xi)(\xi+1)}\varphi'(\omega)\right|\\
\leq\; & \frac{(\omega-\omega_1)^{\xi+2}}{(\omega_2-\omega_1)B(\xi)\Gamma(\xi)(\xi+1)}\int_0^1\varpi^{\xi+1}\left[\varpi^s|\varphi''(\omega)|+(1-\varpi)^s|\varphi''(\omega_1)|\right]d\varpi\\
& +\frac{(\omega_2-\omega)^{\xi+1}}{(\omega_2-\omega_1)B(\xi)\Gamma(\xi)}\int_0^1\varpi^{\xi+1}\left[\varpi^s|\varphi''(\omega)|+(1-\varpi)^s|\varphi''(\omega_2)|\right]d\varpi.
\end{aligned}
$$

We complete the proof by making the necessary calculations in above. □

**Corollary 3.3.** *In addition to the assumptions of Theorem 3.2, if $|\varphi''|\leq K_1$, $K_1>0$, we have the following inequality:*

$$
\begin{aligned}
& \left|\frac{1}{(\omega_2-\omega_1)}\left[{}^{AB}I_{\omega}^{\xi}\{\varphi(\omega_1)\}+{}^{AB}_{\omega}I_{\omega_2}^{\xi}\{\varphi(\omega_2)\}\right]-\frac{1-\xi}{(\omega_2-\omega_1)B(\xi)}[\varphi(\omega_1)+\varphi(\omega_2)]\right.\\
& \left.-\frac{\varphi(\omega)}{(\omega_2-\omega_1)B(\xi)\Gamma(\xi)}\left[(\omega_2-\omega)^{\xi}+(\omega-\omega_1)^{\xi}\right]+\frac{(\omega-\omega_1)^{\xi+1}-(\omega_2-\omega)^{\xi+1}}{(\omega_2-\omega_1)B(\xi)\Gamma(\xi)(\xi+1)}\varphi'(\omega)\right|\\
\leq\; & \frac{(\omega-\omega_1)^{\xi+2}+(\omega_2-\omega)^{\xi+2}}{B(\xi)\Gamma(\xi)(\xi+1)(\omega_2-\omega_1)}K_1\left(\frac{1}{\xi+s+2}+\beta(\xi+2,s+1)\right).
\end{aligned}
$$

**Corollary 3.4.** *In Corollary 3.3, if we choose* $\omega = \frac{\omega_1+\omega_2}{2}$, *we have the following inequality:*

$$|N| \leq \frac{K_1}{B(\xi)\Gamma(\xi)(\xi+1)}\left(\frac{\omega_2-\omega_1}{2}\right)^{\xi+1}\left(\frac{1}{\xi+s+2}+\beta(\xi+2,s+1)\right).$$

**Corollary 3.5.** *In Corollary 3.3 , if we choose* $s = 1$, *we have the following inequality:*

$$\begin{aligned}&\left|\frac{1}{(\omega_2-\omega_1)}\left[{}^{AB}I^{\xi}_{\omega}\{\varphi(\omega_1)\} + {}^{AB}_{\omega}I^{\xi}_{\omega_2}\{\varphi(\omega_2)\}\right] - \frac{1-\xi}{(\omega_2-\omega_1)B(\xi)}[\varphi(\omega_1)+\varphi(\omega_2)]\right.\\ &\left. -\frac{\varphi(\omega)}{(\omega_2-\omega_1)B(\xi)\Gamma(\xi)}\left[(\omega_2-\omega)^{\xi}+(\omega-\omega_1)^{\xi}\right] + \frac{(\omega-\omega_1)^{\xi+1}-(\omega_2-\omega)^{\xi+1}}{(\omega_2-\omega_1)B(\xi)\Gamma(\xi)(\xi+1)}\varphi'(\omega)\right|\\ \leq\ & \frac{K_1}{B(\xi)\Gamma(\xi)(\xi+1)(\xi+2)}\left(\frac{(\omega-\omega_1)^{\xi+2}+(\omega_2-\omega)^{\xi+2}}{\omega_2-\omega_1}\right).\end{aligned}$$

**Corollary 3.6.** *In Corollary 3.5, if we choose* $\omega = \frac{\omega_1+\omega_2}{2}$, *we have the following inequality:*

$$|N| \leq \frac{K_1}{B(\xi)\Gamma(\xi)(\xi+1)(\xi+2)}\left(\frac{\omega_2-\omega_1}{2}\right)^{\xi+1}.$$

In the rest of the this section, for simplicity we will use

$$\begin{aligned}M_1 \;=\;& \frac{1}{(\omega_2-\omega_1)}\left[{}^{AB}I^{\xi}_{\omega}\{\varphi(\omega_1)\} + {}^{AB}_{\omega}I^{\xi}_{\omega_2}\{\varphi(\omega_2)\}\right].\\ & -\frac{1-\xi}{(\omega_2-\omega_1)B(\xi)}[\varphi(\omega_1)+\varphi(\omega_2)]\\ & -\frac{\varphi(\omega)}{(\omega_2-\omega_1)B(\xi)\Gamma(\xi)}\left[(\omega_2-\omega)^{\xi}+(\omega-\omega_1)^{\xi}\right]\\ & +\frac{(\omega-\omega_1)^{\xi+1}-(\omega_2-\omega)^{\xi+1}}{(\omega_2-\omega_1)B(\xi)\Gamma(\xi)(\xi+1)}\varphi'(\omega).\end{aligned}$$

**Theorem 3.7.** *Let* $\omega_1 < \omega_2$, $\omega_1, \omega_2 \in J^{\circ}$*and* $\varphi : J \subset [0,\infty) \longrightarrow \mathbb{R}$ *be a differentiable mapping on* $J^{\circ}$*and* $\varphi'' \in L[\omega_1,\omega_2]$. *If* $\left|\varphi''\right|^q$ *is an s-convex mapping in the second sense on* $[\omega_1,\omega_2]$, *for all* $\omega \in [\omega_1,\omega_2]$, $s \in (0,1]$ *and* $\xi \in [0,1]$. *Then, we obtain the inequality below:*

$$\begin{aligned}|M_1| \;\leq\;& \frac{(\omega-\omega_1)^{\xi+2}}{B(\xi)\Gamma(\xi)(\xi+1)(\omega_2-\omega_1)}\left(\frac{1}{(\xi+1)p+1}\right)^{\frac{1}{p}}\left(\frac{\left|\varphi''(\omega)\right|^q+\left|\varphi''(\omega_1)\right|^q}{s+1}\right)^{\frac{1}{q}}\\ & +\frac{(\omega_2-\omega)^{\xi+2}}{B(\xi)\Gamma(\xi)(\xi+1)(\omega_2-\omega_1)}\left(\frac{1}{(\xi+1)p+1}\right)^{\frac{1}{p}}\left(\frac{\left|\varphi''(\omega)\right|^q+\left|\varphi''(\omega_2)\right|^q}{s+1}\right)^{\frac{1}{q}}\end{aligned}$$

*where* $q > 1$ *and* $\frac{1}{p}+\frac{1}{q} = 1$.

*Proof.* To prove this theorem, we will consider the operations we used when proving Theorem 2.4. So, we have

$$\begin{aligned}|M_1| \;\leq\;& \frac{(\omega-\omega_1)^{\xi+2}}{(\omega_2-\omega_1)B(\xi)\Gamma(\xi)(\xi+1)}\left(\int_0^1 \varpi^{(\xi+1)p}d\varpi\right)^{\frac{1}{p}}\left(\int_0^1\left|\varphi''(\varpi\omega+(1-\varpi)\omega_1)\right|^q d\varpi\right)^{\frac{1}{q}}\\ & +\frac{(\omega_2-\omega)^{\xi+2}}{(\omega_2-\omega_1)B(\xi)\Gamma(\xi)(\xi+1)}\left(\int_0^1 \varpi^{(\xi+1)p}d\varpi\right)^{\frac{1}{p}}\left(\int_0^1\left|\varphi''(\varpi\omega+(1-\varpi)\omega_2)\right|^q d\varpi\right)^{\frac{1}{q}}.\end{aligned}$$

If we calculate the integrals above, we have the desired result. □

**Corollary 3.8.** *In addition to the assumptions of Theorem 3.7, if* $\left|\varphi''\right| \leq K_1, K_1 > 0$, *we have the following inequality:*

$$|M_1| \leq \frac{K_1}{B(\xi)\Gamma(\xi)(\xi+1)}\left(\frac{1}{(\xi+1)p+1}\right)^{\frac{1}{p}}\left(\frac{2}{s+1}\right)^{\frac{1}{q}}\left(\frac{(\omega-\omega_1)^{\xi+2}+(\omega_2-\omega)^{\xi+2}}{\omega_2-\omega_1}\right).$$

**Corollary 3.9.** *In Corollary 3.8, if we choose* $\omega = \frac{\omega_1+\omega_2}{2}$, *we have the following inequality:*

$$|N| \leq \frac{K_1}{B(\xi)\Gamma(\xi)(\xi+1)}\left(\frac{1}{(\xi+1)p+1}\right)^{\frac{1}{p}}\left(\frac{2}{s+1}\right)^{\frac{1}{q}}\left(\frac{\omega_2-\omega_1}{2}\right)^{\xi+1}.$$

**Corollary 3.10.** *In Corollary 3.8, if we choose* $s = 1$, *we have the following inequality:*

$$|M_1| \leq \frac{K_1}{B(\xi)\Gamma(\xi)(\xi+1)}\left(\frac{1}{(\xi+1)p+1}\right)^{\frac{1}{p}}\left(\frac{(\omega-\omega_1)^{\xi+2}+(\omega_2-\omega)^{\xi+2}}{\omega_2-\omega_1}\right).$$

**Corollary 3.11.** *In Corollary 3.10, if we choose* $\omega = \frac{\omega_1+\omega_2}{2}$, *we have the following inequality:*

$$|N| \leq \frac{K_1}{B(\xi)\Gamma(\xi)(\xi+1)}\left(\frac{1}{(\xi+1)p+1}\right)^{\frac{1}{p}}\left(\frac{\omega_2-\omega_1}{2}\right)^{\xi+1}.$$

**Theorem 3.12.** *Assume that the assumptions given in the Theorem 3.7 are valid. Then, we have the following inequality:*

$$\begin{aligned}|M_1| &\leq \frac{(\omega-\omega_1)^{\xi+2}}{(\omega_2-\omega_1)B(\xi)\Gamma(\xi)(\xi+1)}\left(\frac{1}{\xi+2}\right)^{\frac{1}{p}}\left(\frac{\left|\varphi''(\omega)\right|^q}{\xi+s+2}+\left|\varphi''(\omega_1)\right|^q\beta(\xi+2,s+1)\right)^{\frac{1}{q}}\\ &+\frac{(\omega_2-\omega)^{\xi+2}}{(\omega_2-\omega_1)B(\xi)\Gamma(\xi)(\xi+1)}\left(\frac{1}{\xi+2}\right)^{\frac{1}{p}}\left(\frac{\left|\varphi''(\omega)\right|^q}{\xi+s+2}+\left|\varphi''(\omega_2)\right|^q\beta(\xi+2,s+1)\right)^{\frac{1}{q}}.\end{aligned}$$

*Proof.* Via Hölder's inequality in a different way, we can write

$$\begin{aligned}|M_1| &\leq \frac{(\omega-\omega_1)^{\xi+2}}{(\omega_2-\omega_1)B(\xi)\Gamma(\xi)(\xi+1)}\left(\int_0^1\varpi^{\xi+1}d\varpi\right)^{\frac{1}{p}}\left(\int_0^1\varpi^{\xi+1}\left|\varphi''(\varpi\omega+(1-\varpi)\omega_1)\right|^q d\varpi\right)^{\frac{1}{q}}\\ &+\frac{(\omega_2-\omega)^{\xi+2}}{(\omega_2-\omega_1)B(\xi)\Gamma(\xi)(\xi+1)}\left(\int_0^1\varpi^{\xi+1}d\varpi\right)^{\frac{1}{p}}\left(\int_0^1\varpi^{\xi+1}\left|\varphi''(\varpi\omega+(1-\varpi)\omega_2)\right|^q d\varpi\right)^{\frac{1}{q}}.\end{aligned}$$

If we apply $s$-convexity of $\left|\varphi''\right|^q$ and calculate the integrals, we get the desired. □

**Corollary 3.13.** *In addition to the assumptions of Theorem 3.12, if* $\left|\varphi''\right| \leq K_1, K_1 > 0$, *we have the following inequality:*

$$|M_1| \leq \frac{K_1}{B(\xi)\Gamma(\xi)(\xi+1)}\left(\frac{1}{\xi+2}\right)^{\frac{1}{p}}\left(\frac{(\omega-\omega_1)^{\xi+2}+(\omega_2-\omega)^{\xi+2}}{\omega_2-\omega_1}\right)\left(\frac{1}{\xi+s+2}+\beta(\xi+2,s+1)\right)^{\frac{1}{q}}.$$

**Corollary 3.14.** *In Corollary 3.13, if we choose* $\omega = \frac{\omega_1+\omega_2}{2}$, *we have the following inequality:*

$$|N| \leq \frac{K_1}{B(\xi)\Gamma(\xi)(\xi+1)}\left(\frac{1}{\xi+2}\right)^{\frac{1}{p}}\left(\frac{\omega_2-\omega_1}{2}\right)^{\xi+1}\left(\frac{1}{\xi+s+2}+\beta(\xi+2,s+1)\right)^{\frac{1}{q}}.$$

**Corollary 3.15.** *In Corollary 3.13, if we choose $s = 1$, we have the following inequality:*

$$|M_1| \leq \frac{K_1}{B(\xi)\Gamma(\xi)(\xi+1)(\xi+2)}\left(\frac{(\omega-\omega_1)^{\xi+2}+(\omega_2-\omega)^{\xi+2}}{\omega_2-\omega_1}\right).$$

**Corollary 3.16.** *In Corollary 3.15, if we choose $\omega = \frac{\omega_1+\omega_2}{2}$, we have the following inequality:*

$$|N| \leq \frac{K_1}{B(\xi)\Gamma(\xi)(\xi+1)(\xi+2)}\left(\frac{\omega_2-\omega_1}{2}\right)^{\xi+1}.$$

**Theorem 3.17.** *Assume that the assumptions given in the Theorem 3.7 are valid. Then, we have the following inequality:*

$$\begin{aligned}|M_1| &\leq \frac{1}{B(\xi)\Gamma(\xi)(\xi+1)(\omega_2-\omega_1)}\left(\frac{q-1}{(\xi+1)(q-p)+q-1}\right)^{\frac{1}{p}}\\ &\times\Bigg[(\omega-\omega_1)^{\xi+2}\left(\frac{\left|\varphi''(\omega)\right|^q}{(\xi+1)p+s+1}+\beta((\xi+1)p+1,s+1)\left|\varphi''(\omega_1)\right|^q\right)^{\frac{1}{q}}\\ &+(\omega_2-\omega)^{\xi+2}\left(\frac{\left|\varphi''(\omega)\right|^q}{(\xi+1)p+s+1}+\beta((\xi+1)p+1,s+1)\left|\varphi''(\omega_2)\right|^q\right)^{\frac{1}{q}}\Bigg]\end{aligned}$$

*where $q \geq p > 1$.*

*Proof.* We will make use of a version of the Hölder inequality that we have used in the proof of Theorem 2.8. So, we can write

$$\begin{aligned}|M_1| &\leq \frac{(\omega-\omega_1)^{\xi+2}}{(\omega_2-\omega_1)B(\xi)\Gamma(\xi)(\xi+1)}\left(\int_0^1 \varpi^{(\xi+1)\left(\frac{q-p}{q-1}\right)}d\varpi\right)^{1-\frac{1}{q}}\left(\int_0^1 \varpi^{(\xi+1)p}\left|\varphi''(\varpi\omega+(1-\varpi)\omega_1)\right|^q d\varpi\right)^{\frac{1}{q}}\\ &+\frac{(\omega_2-\omega)^{\xi+2}}{(\omega_2-\omega_1)B(\xi)\Gamma(\xi)(\xi+1)}\left(\int_0^1 \varpi^{(\xi+1)\left(\frac{q-p}{q-1}\right)}d\varpi\right)^{1-\frac{1}{q}}\left(\int_0^1 \varpi^{(\xi+1)p}\left|\varphi''(\varpi\omega+(1-\varpi)\omega_2)\right|^q d\varpi\right)^{\frac{1}{q}}.\end{aligned}$$

If we use $s$-convexity of $\left|\varphi''\right|^q$ above, we have

$$\begin{aligned}|M_1| &\leq \frac{(\omega-\omega_1)^{\xi+2}}{(\omega_2-\omega_1)B(\xi)\Gamma(\xi)(\xi+1)}\left(\int_0^1 \varpi^{(\xi+1)\left(\frac{q-p}{q-1}\right)}d\varpi\right)^{1-\frac{1}{q}}\left(\int_0^1 \varpi^{(\xi+1)p}\left[\varpi^s\left|\varphi''(\omega)\right|^q+(1-\varpi)^s\left|\varphi''(\omega_1)\right|^q\right]d\varpi\right)^{\frac{1}{q}}\\ &+\frac{(\omega_2-\omega)^{\xi+2}}{(\omega_2-\omega_1)B(\xi)\Gamma(\xi)(\xi+1)}\left(\int_0^1 \varpi^{(\xi+1)\left(\frac{q-p}{q-1}\right)}d\varpi\right)^{1-\frac{1}{q}}\left(\int_0^1 \varpi^{(\xi+1)p}\left[\varpi^s\left|\varphi''(\omega)\right|^q+(1-\varpi)^s\left|\varphi''(\omega_2)\right|^q\right]d\varpi\right)^{\frac{1}{q}}.\end{aligned}$$

Proof will be obtained if necessary integral calculations are made. □

**Corollary 3.18.** *In addition to the assumptions of Theorem 3.17, if $\left|\varphi''\right| \leq K_1, K_1 > 0$, we have the following inequality:*

$$\begin{aligned}|M_1| &\leq \frac{K_1}{B(\xi)\Gamma(\xi)(\xi+1)}\left(\frac{(\omega-\omega_1)^{\xi+2}+(\omega_2-\omega)^{\xi+2}}{\omega_2-\omega_1}\right)\\ &\times\left(\frac{q-1}{(\xi+1)(q-p)+q-1}\right)^{1-\frac{1}{q}}\left(\frac{1}{(\xi+1)p+s+1}+\beta((\xi+1)p+1,s+1)\right)^{\frac{1}{q}}.\end{aligned}$$

**Corollary 3.19.** *In Corollary 3.18, if we choose* $\omega = \frac{\omega_1+\omega_2}{2}$*, we have the following inequality:*

$$|N| \leq \frac{K_1}{B(\xi)\Gamma(\xi)(\xi+1)}\left(\frac{\omega_2-\omega_1}{2}\right)^{\xi+1}\left(\frac{q-1}{(\xi+1)(q-p)+q-1}\right)^{1-\frac{1}{q}} \times\left(\frac{1}{(\xi+1)p+s+1}+\beta((\xi+1)p+1,s+1)\right)^{\frac{1}{q}}.$$

**Corollary 3.20.** *In Corollary 3.18, if we choose* $s = 1$*, we have the following inequality:*

$$|M_1| \leq \frac{K_1}{B(\xi)\Gamma(\xi)(\xi+1)}\left(\frac{1}{(\xi+1)p+1}\right)^{\frac{1}{q}}\left(\frac{q-1}{(\xi+1)(q-p)+q-1}\right)^{1-\frac{1}{q}}\left(\frac{(\omega-\omega_1)^{\xi+2}+(\omega_2-\omega)^{\xi+2}}{\omega_2-\omega_1}\right).$$

**Corollary 3.21.** *In Corollary 3.20, if we choose* $\omega = \frac{\omega_1+\omega_2}{2}$*, we have the following inequality:*

$$|N| \leq \frac{K_1}{B(\xi)\Gamma(\xi)(\xi+1)}\left(\frac{\omega_2-\omega_1}{2}\right)^{\xi+1}\left(\frac{1}{(\xi+1)p+1}\right)^{\frac{1}{q}}\left(\frac{q-1}{(\xi+1)(q-p)+q-1}\right)^{1-\frac{1}{q}}.$$

We will conclude this section by obtaining the following results for functions whose $q$−th power of absolute value of second derivatives is $s$-concave.

**Theorem 3.22.** *Let* $\omega_1 < \omega_2$, $\omega_1, \omega_2 \in J^\circ$*and* $\varphi : J \subset [0,\infty) \longrightarrow \mathbb{R}$ *be a differentiable mapping on* $J^\circ$ *and* $\varphi'' \in L[\omega_1, \omega_2]$*. If* $|\varphi''|^q$ *is an s-concave mapping on* $[\omega_1, \omega_2]$*, for all* $\omega \in [\omega_1, \omega_2]$, $s \in (0, 1]$ *and* $\xi \in [0, 1]$*. Then, we obtain the inequality below:*

$$|M_1| \leq \frac{(\omega-\omega_1)^{\xi+2}}{(\omega_2-\omega_1)B(\xi)\Gamma(\xi)(\xi+1)}\left(\frac{1}{(\xi+1)p+1}\right)^{\frac{1}{q}} 2^{\frac{s-1}{q}}\left|\varphi''\left(\frac{\omega+\omega_1}{2}\right)\right| +\frac{(\omega_2-\omega)^{\xi+2}}{(\omega_2-\omega_1)B(\xi)\Gamma(\xi)(\xi+1)}\left(\frac{1}{(\xi+1)p+1}\right)^{\frac{1}{q}} 2^{\frac{s-1}{q}}\left|\varphi''\left(\frac{\omega+\omega_2}{2}\right)\right|$$

*where* $q > 1$ *and* $\frac{1}{p}+\frac{1}{q} = 1$.

*Proof.* If we apply Hölder's inequality similar to the proof of Theorem 3.7, we have

$$|M_1| \leq \frac{(\omega-\omega_1)^{\xi+2}}{(\omega_2-\omega_1)B(\xi)\Gamma(\xi)(\xi+1)}\left(\int_0^1 \varpi^{(\xi+1)p}d\varpi\right)^{\frac{1}{p}}\left(\int_0^1 \left|\varphi''(\varpi\omega+(1-\varpi)\omega_1)\right|^q d\varpi\right)^{\frac{1}{q}} +\frac{(\omega_2-\omega)^{\xi+1}}{(\omega_2-\omega_1)B(\xi)\Gamma(\xi)}\left(\int_0^1 \varpi^{(\xi+1)p}d\varpi\right)^{\frac{1}{p}}\left(\int_0^1 \left|\varphi''(\varpi\omega+(1-\varpi)\omega_2)\right|^q d\varpi\right)^{\frac{1}{q}}.$$

Since $|\varphi''|^q$ is $s$-concave on $[\omega_1, \omega_2]$, we can write following results by taking into account the variant of the Hermite-Hadamard inequality for $s$-concave functions:

$$\int_0^1 \left|\varphi''(\varpi\omega+(1-\varpi)\omega_1)\right|^q d\varpi \leq 2^{s-1}\left|\varphi''\left(\frac{\omega+\omega_1}{2}\right)\right|^q,$$

$$\int_0^1 \left|\varphi''(\varpi\omega+(1-\varpi)\omega_2)\right|^q d\varpi \leq 2^{s-1}\left|\varphi''\left(\frac{\omega+\omega_2}{2}\right)\right|^q.$$

By using these results in the above inequality we complete the proof. □

**Corollary 3.23.** *In Theorem 3.22, if we choose* $\omega = \frac{\omega_1+\omega_2}{2}$*, we have the following inequality:*

$$|N| \leq \frac{(\omega_2-\omega_1)^{\xi+1}}{2^{\xi+2}B(\xi)\Gamma(\xi)(\xi+1)}\left(\frac{1}{(\xi+1)p+1}\right)^{\frac{1}{q}} 2^{\frac{s-1}{q}}\left[\left|\varphi''\left(\frac{3\omega_1+\omega_2}{4}\right)\right|+\left|\varphi''\left(\frac{\omega_1+3\omega_2}{4}\right)\right|\right].$$

**Corollary 3.24.** *In Theorem 3.22 , if we choose* $s = 1$*, we have the following inequality:*

$$\begin{aligned}|M_1| &\leq \frac{(\omega-\omega_1)^{\xi+2}}{(\omega_2-\omega_1)B(\xi)\Gamma(\xi)(\xi+1)}\left(\frac{1}{(\xi+1)p+1}\right)^{\frac{1}{q}}\left|\varphi''\left(\frac{\omega+\omega_1}{2}\right)\right| \\ &\quad+\frac{(\omega_2-\omega)^{\xi+2}}{(\omega_2-\omega_1)B(\xi)\Gamma(\xi)(\xi+1)}\left(\frac{1}{(\xi+1)p+1}\right)^{\frac{1}{q}}\left|\varphi''\left(\frac{\omega+\omega_2}{2}\right)\right|.\end{aligned}$$

**Corollary 3.25.** *In Corollary 3.24, if we choose* $\omega = \frac{\omega_1+\omega_2}{2}$*, we have the following inequality:*

$$|N| \leq \frac{(\omega_2-\omega_1)^{\xi+1}}{2^{\xi+2}B(\xi)\Gamma(\xi)(\xi+1)}\left(\frac{1}{(\xi+1)p+1}\right)^{\frac{1}{q}}\left[\left|\varphi''\left(\frac{3\omega_1+\omega_2}{4}\right)\right|+\left|\varphi''\left(\frac{\omega_1+3\omega_2}{4}\right)\right|\right].$$

## 4. Conclusions

Many researchers are working intensively on integral inequalities, and many new and general inequalities have been obtained with the help of different integral operators. The original aspect of this study is that for $s-$convex functions in the second sense, new Ostrowski type inequalities are obtained by using Atangana-Baleanu fractional integral operators. The new integral identity, which contributes to reaching these new and general inequalities, and the new inequality established especially for $s-$concave functions reveal the innovative aspect of the study. In addition, the consistency of the results was tested by giving many reduced results.

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