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Some remarks on $\mathcal{I}(\mathcal{S}_\theta)$-summability via neutrosophic norm

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Abstract. For an admissible ideal $\mathcal{I} \subseteq \mathcal{P}(\mathbb{N})$ and a lacunary sequence $\theta = (k_s)$, the aim of the present paper is to introduce the concept of $\mathcal{I}(\mathcal{S})$-summability and $\mathcal{I}(\mathcal{S}_\theta)$-summability w.r.t neutrosophic norm (G, B, Y). We also investigate some relations among these notions and prove that these are equivalent if and only if $1 < \lim\inf_s q_s \leq \lim\sup_s q_s < \infty$.

1. Introduction

Statistical convergence was originated by Fast[7] in 1951 and linked with the summability theory by Schoenberg [9]. After the work of Maddox[8], Connor[10], Fridy[11] and Šalát[24], statistical convergence appeared as one of the most prominent fields of study in the summability theory.

In 2000, Kostyrko et al.[21] established a generalized concept of statistical convergence, called as $\mathcal{I}$-convergence using the concept of ideals of subsets of $\mathbb{N}$. Some other studies on $\mathcal{I}$-convergence and its applications can be found in [3], [15], [26], [27], [28], [29] etc.

Fridy and Orhan [12] united the ideas of lacunary sequence and statistical convergence to define the concept of lacunary statistical convergence. "By a lacunary sequence we mean an increasing integer sequence $\theta = (k_s)$ with $k_0 = 0$ and $h_s = k_s - k_{s-1} \to \infty$ as $s \to \infty$. The intervals determined by θ will be denoted by $I_s = (k_{s-1}, k_s]$ and the ratio $\frac{k_s}{k_{s-1}}$ will be abbreviated as q_s. Let $K \subseteq \mathbb{N}$. The number $\delta_\theta(K) = \dfrac{1}{h_s}|\{k \in I_s : k \in K\}|$ is called θ-density of K, provided the limit exists.

A sequence $x = (x_k)$ of numbers is said to be lacunary statistically convergent (briefly S_θ-convergent) to x_0 if for every $\epsilon > 0$, $\lim_s \frac{1}{h_s}|\{k \in I_s : |x_k - x_0| \geq \epsilon\}| = 0$ or equivalently, the set $K(\epsilon)$ has θ-density zero, where $K(\epsilon) = \{k \in \mathbb{N} : |x_k - x_0| \geq \epsilon\}$. In this case, we write $(\mathcal{S}_\theta) - \lim_{k\to\infty} x_k = x_0$."Some further interesting works on lacunary statistical convergence can be found in [4], [18], [30] etc.

In recent years these notions have been considered in fuzzy environment as well. Initially fuzzy sets was introduced by Zadeh [17] in 1965 as a more convenient tool for handling issues which cannot be modelled within the framework of crisp sets. Subsequently, numerous researchers have explored different components of the theory and the uses of fuzzy sets. This work is significant for real-life situations, yet some issues are not adequately addressed, leading to new quests. Atanassov [14] developed intuitionistic

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fuzzy sets for such situations. After the introduction of intuitionistic fuzzy sets, a progressive development is made in this field. For instance, intuitionistic fuzzy metric spaces(*IFMS*) were introduced by Park [13], intuitionistic fuzzy topological spaces(*IFTS*) by Saadati and Park [23] etc.

Smarandache [6] proposed the notion of a neutrosophic set as a generalisation of a fuzzy set(*FS*) and an intuitionistic fuzzy set(*IFS*) to avoid the complexity arising from uncertainty in settling many practical challenges in real-world activities more precisely. For ongoing development on neutrosophic set(*NS*) and its applications, we refer to [5], [20], [22] etc.

Kirişçi and Şimşek[19] defined neutrosophic norm and studied statistical convergence in neutrosophic normed spaces(*NNS*). For a broad view in this direction, we recommend to the reader [1], [2], [25]. In this article, we developed and studied the concepts of $\mathcal{I}$-statistical convergence, $\mathcal{I}$-lacunary statistical convergence, strongly $\mathcal{I}$-lacunary convergence in *NNS* and investigate some of their properties in *NNS*.

2. Preliminaries

We start this section with some basic definitions and results required for the present study.

Throughout this work, $\mathbb{N}$, $\mathbb{R}$ and $\mathbb{R}^+$ will respectively denote the set natural numbers, the set of real numbers and the set of positive real numbers.

Definition 2.1 [16] "A binary operation $\circ : [0,1]\times[0,1] \to [0,1]$ is continuous t-norm if $\circ$ satisfies the following conditions:
(i) $d\circ e = e\circ d$ and $d\circ(e\circ f) = (d\circ e)\circ f$.
(ii) $\circ$ is continuous.
(iii) $d\circ 1 = 1\circ d = d$ for all $d\in[0,1]$.
(iv) $d\circ e \leq f\circ g$ if $d\leq f, e\leq g$ with $d,e,f,g\in[0,1]$."

Definition 2.2 [16] "A binary operation $\diamond : [0,1]\times[0,1] \to [0,1]$ is continuous t-conorm(s-norm) if $\diamond$ satisfies the following conditions:
(i) $d\diamond e = e\diamond d$ and $d\diamond(e\diamond f) = (d\diamond e)\diamond f$.
(ii) $\diamond$ is continuous.
(iii) $d\diamond 0 = 0\diamond d =$ for all $d\in[0,1]$.
(iv) $d\diamond e \leq f\diamond g$ if $d\leq f, e\leq g$ with $d,e,f,g\in[0,1]$."

Definition 2.3 [19] "Let F be a vector space, $\mathcal{N} = \{\langle\vartheta, G(\vartheta), B(\vartheta), Y(\vartheta)\rangle : \vartheta\in F\}$ be a normed space such that $\mathcal{N} : F\times\mathbb{R}^+ \to [0,\ 1]$ and $\circ, \diamond$ respectively are $t-$norm and $t-$conorm. Then a four tuple $V = (F, \mathcal{N}, \circ, \diamond)$ is called a neutrosophic normed spaces (briefly *NNS*) if the following conditions are satisfied. For every $t, u\in F$, $\varrho,\ \kappa > 0$ and for every $\varsigma\neq 0$ we have
(i) $0\leq G(t,\ \varrho)\leq 1, 0\leq B(t,\ \varrho)\leq 1, 0\leq Y(t,\ \varrho)\leq 1$ for every $\varrho\in\mathbb{R}^+$;
(ii) $G(t,\ \varrho) + B(t,\ \varrho) + Y(t,\ \varrho)\leq 3$ for $\varrho\in\mathbb{R}^+$;
(iii) $G(t,\ \varrho) = 1$ (for $\varrho > 0$) if and only if $t = 0$;
(iv) $G(\varsigma t, \varrho) = G(t,\ \frac{\varrho}{|\varsigma|})$;
(v) $G(t,\ \kappa)\circ G(u,\ \varrho)\leq G(t+u, \kappa+\varrho)$;
(vi) $G(t,\cdot)$ is continuous non-decreasing function;
(vii) $\lim\limits_{\varrho\to\infty} G(t,\varrho) = 1$;
(viii) $B(t,\ \varrho) = 0$ (for $\varrho > 0$) if and only if $t = 0$;
(ix) $B(\varsigma t, \varrho) = G(t,\ \frac{\varrho}{|\varsigma|})$;
(x)$B(t,\ \kappa)\diamond B(u,\ \varrho)\geq B(t+u, \kappa+\varrho)$;
(xi) $B(t,\cdot)$ is continuous non-increasing function;
(xii) $\lim\limits_{\varrho\to\infty} B(t,\varrho) = 0$;
(xiii) $Y(t,\ \varrho) = 0$ (for $\varrho > 0$) if and only if $t = 0$;
(xiv) $Y(\varsigma t, \varrho) = Y(t,\ \frac{\varrho}{|\varsigma|})$;
(xv) $Y(t,\ \kappa)\diamond Y(u,\ \varrho)\geq Y(t+u, \kappa+\varrho)$;
(xvi) $Y(t,\cdot)$ is continuous non-increasing function;
(xvii) $\lim\limits_{\varrho\to\infty} Y(t,\varrho) = 0$;

(xviii) If $\varrho \leq 0$, then $G(t, \varrho) = 0, B(t, \varrho) = Y(t, \varrho) = 1$.
$\mathscr{N} = (G, B, Y)$ is called the neutrosophic norm."
Definition 2.4 [19] "A sequence $w = (w_k)$ is said to be convergent to w_0 in *NNS V* if for each $\epsilon > 0$ and $\varrho > 0$ $\exists\, n_0 \in \mathbb{N}$ s.t $G(w_k - w_0, \varrho) > 1 - \epsilon$ and $B(w_k - w_0, \varrho) < \epsilon, Y((w_k - w_0, \varrho) < \epsilon\ \forall\ k \geq n_0$."
Definition 2.5 [19] "A sequence $w = (w_k)$ is said to be Cauchy sequence in *NNS V* if for each $\epsilon > 0$ and $\varrho > 0$ $\exists\, n_0 \in \mathbb{N}$ s.t $G(w_k - w_p, \varrho) > 1 - \epsilon,$ and $B(w_k - w_p, \varrho) < \epsilon, Y((w_k - w_p, \varrho) < \epsilon\ \forall\ k, p \geq n_0$."

For any set X, let $\mathscr{P}(X)$ denotes the power set of X.
Definition 2.6 [21] "A family of sets $\mathscr{I} \subseteq \mathscr{P}(\mathbb{N})$ is called an ideal in X if and only if
(i) $\emptyset \in \mathscr{I}$.
(ii) $C, D \in \mathscr{I}$ implies that $C \cup D \in \mathscr{I}$ and
(iii) For $C \in \mathscr{I}$ and $D \subseteq C$, we have $D \in \mathscr{I}$."
Definition 2.7 [21] "A non-empty family of sets $\mathscr{F} \subseteq \mathscr{P}(\mathbb{N})$ is called a filter on X if and only if
(i) $\emptyset \notin \mathscr{F}$.
(ii) $C, D \in \mathscr{F}$ implies that $C \cap D \in \mathscr{F}$ and
(iii) For $C \in \mathscr{F}$ and $D \supseteq C$, we have $D \in \mathscr{F}$.

An ideal $\mathscr{I}$ is called non-trivial if $\mathscr{I} \neq \emptyset$ and $X \notin \mathscr{I}$. Obviously, $\mathscr{I} \subseteq \mathscr{P}(X)$ is a non-trivial ideal if and only if the class $\mathscr{F} = \mathscr{F}(\mathscr{I}) = \{X - C : C \in \mathscr{I}\}$ is a filter on X. The filter $\mathscr{F} = \mathscr{F}(\mathscr{I})$ is called the filter associated with the ideal $\mathscr{I}$."
Definition 2.8 [21] "A non-trivial ideal $\mathscr{I} \subseteq \mathscr{P}(\mathbb{N})$ is called an admissible ideal in X if and only if it contains all singletons i.e., if it contains $\{\{x\} : x \in X\}$."
Definition 2.9 [21] "A sequence $w = (w_k)$ is said to be ideal convergent (or $\mathscr{I}$ convergent) to w_0 if for every $\epsilon > 0$, the set $A(\epsilon) = \{k \in \mathbb{N} : |w_k - w_0| \geq \epsilon\}$ belongs to $\mathscr{I}$."

3. Main Results:

We begin in this section with the following definition:
Definition 3.1: Let $\mathscr{I} \subseteq \mathscr{P}(\mathbb{N})$ be an admissible ideal. A sequence $w = (w_k)$ in *NNS V* is called $\mathscr{I}$-statistically convergent (or $\mathscr{I}(\mathcal{S})$-convergent) to w_0 w.r.t the neutrosophic norm-(G, B, Y), if for each $\epsilon > 0, \varrho > 0$ and $\vartheta > 0$

$$\left\{s \in \mathbb{N} : \frac{1}{s}\left|\left\{k \leq s : G(w_k - w_0, \varrho) \leq 1 - \epsilon \text{ or } B(w_k - w_0, \varrho) \geq \epsilon, Y(w_k - w_0, \varrho) \geq \epsilon\right\}\right| \geq \vartheta\right\} \in \mathscr{I}.$$

In present case, we denote $\mathscr{I}(\mathcal{S}(G, B, Y)) - \lim\limits_{k\to\infty} w_k = w_0$.
Definition 3.2: Let $\mathscr{I} \subseteq \mathscr{P}(\mathbb{N})$ be an admissible ideal and $\theta = (k_s)$ be a lacunary sequence. A sequence $w = (w_k)$ in *NNS V* is called $\mathscr{I}$-lacunary statistically convergent (or $\mathscr{I}(\mathcal{S}_\theta)$-convergent) to w_0 w.r.t the neutrosophic norm-(G, B, Y), if for each $\epsilon > 0, \varrho > 0$ and $\vartheta > 0$

$$\left\{s \in \mathbb{N} : \frac{1}{h_s}\left|\left\{k \in I_s : G(w_k - w_0, \varrho) \leq 1 - \epsilon \text{ or } B(w_k - w_0, \varrho) \geq \epsilon, Y(w_k - w_0, \varrho) \geq \epsilon\right\}\right| \geq \vartheta\right\} \in \mathscr{I}.$$

In present case, we denote $\mathscr{I}(\mathcal{S}_\theta(G, B, Y)) - \lim\limits_{k\to\infty} w_k = w_0$.

Remark 3.1: If $\mathscr{I}_f$ represents the family of all finite subsets of the set of natural numbers. Then $\mathscr{I}(\mathcal{S}_\theta)$-convergence reduces to lacunary-statistical convergence as $\mathscr{I}_f$ is an admissible ideal.
Definition 3.3: Let $\mathscr{I} \subseteq \mathscr{P}(\mathbb{N})$ be an admissible ideal and $\theta = (k_s)$ be a lacunary sequence. A sequence $w = (w_k)$ in *NNS V* is called strongly $\mathscr{I}$-lacunary convergent (or $\mathscr{I}(N_\theta)$-convergent) to w_0 w.r.t the neutrosophic norm-(G, B, Y), if for each $\epsilon > 0$ and $\varrho > 0$,

$$\left\{s \in \mathbb{N} : \frac{1}{h_s}\sum_{k \in I_s} G(w_k - w_0, \varrho) \leq 1 - \epsilon \text{ or } \frac{1}{h_s}\sum_{k \in I_s} B(w_k - w_0, \varrho) \geq \epsilon, \frac{1}{h_s}\sum_{k \in I_s} Y(w_k - w_0, \varrho) \geq \epsilon\right\} \in \mathscr{I}.$$

In present case, we denote $\mathscr{I}(N_\theta(G, B, Y)) - \lim\limits_{k\to\infty} w_k = w_0$.

Theorem 3.1: If $\mathcal{I} \subseteq \mathscr{P}(\mathbb{N})$ is an admissible ideal, $\theta = (k_s)$ is a lacunary sequence and $w = (w_k)$ is a sequence in NNS V then

(I) $w_k \to w_0\big(\mathcal{I}(N_\theta(G,B,Y))\big) \Rightarrow w_k \to w_0\big(\mathcal{I}(\mathcal{S}_\theta(G,B,Y))\big)$ and the inclusion $\mathcal{I}(N_\theta(G,B,Y)) \subseteq \mathcal{I}(\mathcal{S}_\theta(G,B,Y))$ is proper for every ideal $\mathcal{I}$.

(II) $w_k \to w_0\big(\mathcal{I}(\mathcal{S}_\theta(G,B,Y))\big) \Rightarrow w_k \to w_0\big(\mathcal{I}(N_\theta(G,B,Y))\big)$ if $w \in l_\infty$, the space of all bounded sequences of V.

(III) $\mathcal{I}(\mathcal{S}_\theta(G,B,Y)) \cap l_\infty = \mathcal{I}(N_\theta(G,B,Y)) \cap l_\infty$.

Proof. (I) Let $\epsilon > 0$ and $\varrho > 0$, let $w_k \to w_0\big(\mathcal{I}(N_\theta(G,B,Y))\big)$. Now,

$$\begin{aligned}
&\sum_{k\in I_s}\Big(G(w_k - w_0,\varrho) \text{ or } B(w_k - w_0,\varrho), Y(w_k - w_0,\varrho)\Big)\\
&\quad= \sum_{\substack{k\in I_s\\ G(w_k-w_0,\varrho)\le 1-\epsilon \text{ or}\\ B(w_k-w_0,\varrho)\ge\epsilon, Y(w_k-w_0,\varrho)\ge\epsilon}} \Big(G(w_k - w_0,\varrho) \text{ or } B(w_k - w_0,\varrho), Y(w_k - w_0,\varrho)\Big)\\
&\quad+ \sum_{\substack{k\in I_s\\ G(w_k-w_0,\varrho)>1-\epsilon \ \&\\ B(w_k-w_0,\varrho)<\epsilon, Y(w_k-w_0,\varrho)<\epsilon}} \Big(G(w_k - w_0,\varrho) \text{ and } B(w_k - w_0,\varrho), Y(w_k - w_0,\varrho)\Big)\\
&\quad\ge \sum_{\substack{k\in I_s\\ G(w_k-w_0,\varrho)\le 1-\epsilon \text{ or}\\ B(w_k-w_0,\varrho)\ge\epsilon, Y(w_k-w_0,\varrho)\ge\epsilon}} \Big(G(w_k - w_0,\varrho) \text{ or } B(w_k - w_0,\varrho), Y(w_k - w_0,\varrho)\Big)\\
&\quad\ge \epsilon.\Big|\Big\{k \in I_s : G(w_k - w_0,\varrho) \le 1-\epsilon \text{ or } B(w_k - w_0,\varrho) \ge \epsilon, Y(w_k - w_0,\varrho) \ge \epsilon\Big\}\Big|.
\end{aligned}$$

So for $\vartheta > 0$,

$$\frac{1}{h_s}\Big|\Big\{k \in I_s : G(w_k - w_0,\varrho) \le 1-\epsilon \text{ or } B(w_k - w_0,\varrho) \ge \epsilon, Y(w_k - w_0,\varrho) \ge \epsilon\Big\}\Big| \ge \vartheta$$

$$\implies \frac{1}{h_s}\sum_{k\in I_s} G(w_k - w_0,\varrho) \le (1-\epsilon)\vartheta \text{ or } \frac{1}{h_s}\sum_{k\in I_s} B(w_k - w_0,\varrho) \ge \epsilon\vartheta, \frac{1}{h_s}\sum_{k\in I_s} Y(w_k - w_0,\varrho) \ge \epsilon\vartheta,$$

this implies

$$\Big\{s \in \mathbb{N} : \frac{1}{h_s}\Big|\Big\{k \in I_s : G(w_k - w_0,\varrho) \le 1-\epsilon \text{ or } B(w_k - w_0,\varrho) \ge \epsilon, Y(w_k - w_0,\varrho) \ge \epsilon\Big\}\Big| \ge \vartheta\Big\}$$

$$\subseteq \Big\{s \in \mathbb{N} : \frac{1}{h_s}\Big\{\sum_{k\in I_s} G(w_k - w_0,\varrho) \le 1-\epsilon \text{ or } \sum_{k\in I_s} B(w_k - w_0,\varrho) \ge \epsilon, \sum_{k\in I_s} Y(w_k - w_0,\varrho) \ge \epsilon\Big\} \ge \epsilon\vartheta\Big\} \in \mathcal{I}$$

as $w_k \to w_0\big(\mathcal{I}(N_\theta(G,B,Y))\big)$. Hence it follows that $w_k \to w_0\big(\mathcal{I}(\mathcal{S}_\theta(G,B,Y))\big)$.

In order to prove that the inclusion $\mathcal{I}(N_\theta(G,B,Y)) \subseteq \mathcal{I}(\mathcal{S}_\theta(G,B,Y))$ is proper. We define $w_k = 1, 2, 3, \ldots, [\sqrt{h_s}]$ for first $[\sqrt{h_s}]$ integers in I_s and $w_k = 0$ otherwise. It is obvious that the sequence w_k is unbounded. Then for $\varrho > 0$ and $\epsilon > 0$

$$\frac{1}{h_s}\Big|\Big\{k \in I_s : G(w_k - 0,\varrho) \le 1-\epsilon \text{ or } B(w_k - 0,\varrho) \ge \epsilon, Y(w_k - 0,\varrho) \ge \epsilon\Big\}\Big| \le \frac{[\sqrt{h_s}]}{h_s}$$

and for any $\vartheta > 0$, we get

$$\begin{aligned}
&\Big\{s \in \mathbb{N} : \frac{1}{h_s}\Big|\Big\{k \in I_s : G(w_k - 0,\varrho) \le 1-\epsilon \text{ or } B(w_k - 0,\varrho) \ge \epsilon, Y(w_k - 0,\varrho) \ge \epsilon\Big\}\Big| \ge \vartheta\Big\}\\
&\qquad\qquad \subseteq \Big\{s \in \mathbb{N} : \frac{[\sqrt{h_s}]}{h_s} \ge \vartheta\Big\} \in \mathcal{I}.
\end{aligned}$$

It follows that $w_k \to 0\Big(\mathcal{I}(\mathcal{S}_\theta(G,B,Y))\Big)$
Also

$$\frac{1}{h_s}\sum_{k\in I_s}\Big(G(w_k-0,\varrho) \text{ or } B(w_k-0,\varrho), Y(w_k-0,\varrho)\Big) = \frac{1}{h_s}\Bigg(\frac{[\sqrt{h_s}]\Big([\sqrt{h_s}]+1\Big)}{2}\Bigg)$$

then

$$\Big\{s\in\mathbb{N} : \frac{1}{h_s}\sum_{k\in I_s}G(w_k-0,\varrho)\le 1-\frac{1}{4} \text{ or } \frac{1}{h_s}\sum_{k\in I_s}B(w_k-0,\varrho)\ge\frac{1}{4}, \frac{1}{h_s}\sum_{k\in I_s}Y(w_k-0,\varrho)\ge\frac{1}{4}\Big\}$$
$$= \Bigg\{s\in\mathbb{N} : \frac{[\sqrt{h_s}]\Big([\sqrt{h_s}]+1\Big)}{h_s}\ge\frac{1}{2}\Bigg\} = \{u, u+1, u+2, \dots\} \in \mathcal{F}(\mathcal{I})$$

for some $u\in\mathbb{N}$. Since $\mathcal{I}$ is an admissible ideal. So $w_k \nrightarrow 0\Big(\mathcal{I}(N_\theta(G,B,Y))\Big)$.
(II) Let $w=(w_k)\in l_\infty$ s.t $\mathcal{I}(\mathcal{S}_\theta(G,B,Y)) - \lim_{k\to\infty} w_k = w_0$. Since $(w_k)\in l_\infty$ so $\exists\ M>0$ s.t $G(w_k-w_0,\varrho)\ge 1-M$ or $B(w_k-w_0,\varrho)\le M, Y(w_k-w_0,\varrho)\le M\ \forall\ k$. Let $\epsilon>0$ be arbitrary selected, now as in case (i) we can write

$$\frac{1}{h_s}\sum_{k\in I_s}\Big(G(w_k-w_0,\varrho) \text{ or } B(w_k-w_0,\varrho), Y(w_k-w_0,\varrho)\Big)$$
$$= \frac{1}{h_s}\sum_{\substack{k\in I_s\\ G(w_k-w_0,\varrho)\le 1-\epsilon \text{ or}\\ B(w_k-w_0,\varrho)\ge\epsilon, Y(w_k-w_0,\varrho)\ge\epsilon}}\Big(G(w_k-w_0,\varrho) \text{ or } B(w_k-w_0,\varrho), Y(w_k-w_0,\varrho)\Big)$$
$$+\frac{1}{h_s}\sum_{\substack{k\in I_s\\ G(w_k-w_0,\varrho)>1-\epsilon\ \&\\ B(w_k-w_0,\varrho)<\epsilon, Y(w_k-w_0,\varrho)<\epsilon}}\Big(G(w_k-w_0,\varrho) \text{ and } B(w_k-w_0,\varrho), Y(w_k-w_0,\varrho)\Big)$$
$$\le \frac{M}{h_s}\Big|\Big\{k\in I_s : G(w_k-w_0,\varrho)\le 1-\epsilon \text{ or } B(w_k-w_0,\varrho)\ge\epsilon, Y(w_k-w_0,\varrho)\ge\epsilon\Big\}\Big| + \epsilon.$$

Since $w_k \to w_0\Big(\mathcal{I}(\mathcal{S}_\theta(G,B,Y))\Big)$, so we have

$$A_{G,B,Y}(\epsilon,\varrho) = \Big\{s\in\mathbb{N} : \frac{1}{h_s}\Big|\Big\{k\in I_s : G(w_k-w_0,\varrho)\le 1-\epsilon \text{ or}$$
$$B(w_k-w_0,\varrho)\ge\epsilon, Y(w_k-w_0,\varrho)\ge\epsilon\Big\}\Big| \ge \frac{\epsilon}{M}\Big\} \in \mathcal{I}.$$

If $s\in A^C_{G,B,Y}(\epsilon,\varrho)$, then we have

$$\frac{1}{h_s}\sum_{k\in I_s}G(w_k-w_0,\varrho) > 1-2\epsilon \text{ or } \frac{1}{h_s}\sum_{k\in I_s}B(w_k-w_0,\varrho) < 2\epsilon, \frac{1}{h_s}\sum_{k\in I_s}Y(w_k-w_0,\varrho) < 2\epsilon,$$

and therefore, define a set $T_{G,B,Y}(\epsilon,\varrho)$ by

$$T_{G,B,Y}(\epsilon,\varrho) = \Big\{s\in\mathbb{N} : \frac{1}{h_s}\sum_{k\in I_s}G(w_k-w_0,\varrho)\le 1-2\epsilon \text{ or}$$
$$\frac{1}{h_s}\sum_{k\in I_s}B(w_k-w_0,\varrho)\ge 2\epsilon, \frac{1}{h_s}\sum_{k\in I_s}Y(w_k-w_0,\varrho)\ge 2\epsilon\Big\},$$

then $T_{G,B,Y}(\epsilon,\varrho)\subseteq A_{G,B,Y}(\epsilon,\varrho)$. Since the latter set belongs to $\mathcal{I}$ and therefore $T_{G,B,Y}(\epsilon,\varrho)\in\mathcal{I}$. Hence, $w_k \to w_0\Big(\mathcal{I}(N_\theta(G,B,Y))\Big)$.

(III) Follows easily from part (I) and part (II). □

Theorem 3.2 Let $w = (w_k)$ be a sequence in $NNS\ V$. If $\theta = (k_s)$ is a lacunary sequence with $\liminf_s q_s > 1$, then $w_k \to w_0\Big(\mathcal{I}(\mathcal{S}(G,B,Y))\Big) \Rightarrow w_k \to w_0\Big(\mathcal{I}(\mathcal{S}_\theta(G,B,Y))\Big)$.

Proof. Suppose that $\liminf_s q_s > 1$, then $\exists\, \beta > 0$ s.t $q_s \geq 1 + \beta$ for adequately large s,

$$\frac{h_s}{k_s} \geq \frac{\beta}{\beta+1}.$$

Since, $w_k \to w_0\Big(\mathcal{I}(\mathcal{S}(G,B,Y))\Big)$, for each $\epsilon > 0, \varrho > 0$ and for adequately large s, we have

$$\begin{aligned}
&\frac{1}{k_s}\left|\left\{k \leq k_s : G(w_k - w_0, \varrho) \leq 1-\epsilon \text{ or } B(w_k - w_0, \varrho) \geq \epsilon, Y(w_k - w_0, \varrho) \geq \epsilon\right\}\right| \\
&\qquad \geq \frac{1}{k_s}\left|\left\{k \in I_s : G(w_k - w_0, \varrho) \leq 1-\epsilon \text{ or } B(w_k - w_0, \varrho) \geq \epsilon, Y(w_k - w_0, \varrho) \geq \epsilon\right\}\right| \\
&\qquad \geq \frac{\beta}{(\beta+1)}\frac{1}{h_s}\left|\left\{k \in I_s : G(w_k - w_0, \varrho) \leq 1-\epsilon \text{ or } B(w_k - w_0, \varrho) \geq \epsilon, Y(w_k - w_0, \varrho) \geq \epsilon\right\}\right|.
\end{aligned}$$

For given $\vartheta > 0$, we get

$$\begin{aligned}
&\left\{s \in \mathbb{N} : \frac{1}{h_s}\left|\left\{k \in I_s : G(w_k - w_0, \varrho) \leq 1-\epsilon \text{ or } B(w_k - w_0, \varrho) \geq \epsilon, Y(w_k - w_0, \varrho) \geq \epsilon\right\}\right| \geq \vartheta\right\} \\
&\quad \subseteq \left\{s \in \mathbb{N} : \frac{1}{k_s}\left|\left\{k \leq k_s : G(w_k - w_0, \varrho) \leq 1-\epsilon \text{ or } B(w_k - w_0, \varrho) \geq \epsilon, Y(w_k - w_0, \varrho) \geq \epsilon\right\}\right| \geq \frac{\vartheta\beta}{(\beta+1)}\right\} \in \mathcal{I}.
\end{aligned}$$

This shows that $w_k \to w_0\Big(\mathcal{I}(\mathcal{S}_\theta(G,B,Y))\Big)$.□

To prove the following theorem, we admit that θ fulfills the property that for any set $D \in \mathcal{F}(\mathcal{I})$, $\bigcup\{r : k_{s-1} < r \leq k_s, s \in D\} \in \mathcal{F}(\mathcal{I})$.

Theorem 3.3 Let $w = (w_k)$ be a sequence in $NNS\ V$. If $\theta = (k_s)$ is a lacunary sequence with $\limsup_s q_s < \infty$, then $w_k \to w_0\Big(\mathcal{I}(\mathcal{S}_\theta(G,B,Y))\Big) \implies w_k \to w_0\Big(\mathcal{I}(\mathcal{S}(G,B,Y))\Big)$.

Proof. Suppose $\limsup_s q_s < \infty$. Then $\exists\, M > 0$ s.t $q_s < M\ \forall\ s$. Suppose that $w_k \to w_0\Big(\mathcal{I}(\mathcal{S}_\theta(G,B,Y))\Big)$. For $\epsilon > 0, \varrho > 0, \vartheta > 0$ and $\mu > 0$ define

$$D = \left\{s \in \mathbb{N} : \frac{1}{h_s}\left|\left\{k \in I_s : G(w_k - w_0, \varrho) \leq 1-\epsilon \text{ or } B(w_k - w_0, \varrho) \geq \epsilon, Y(w_k - w_0, \varrho) \geq \epsilon\right\}\right| < \vartheta\right\}.$$

and

$$H = \left\{u \in \mathbb{N} : \frac{1}{u}\left|\left\{k \leq u : G(w_k - w_0, \varrho) \leq 1-\epsilon \text{ or } B(w_k - w_0, \varrho) \geq \epsilon, Y(w_k - w_0, \varrho) \geq \epsilon\right\}\right| < \mu\right\}.$$

According to our presumption, it follows that $D \in \mathcal{F}(\mathcal{I})$. Now let

$$K_i = \frac{1}{h_i}\left|\left\{k \in I_i : G(w_k - w_0, \varrho) \leq 1-\epsilon \text{ or } B(w_k - w_0, \varrho) \geq \epsilon, Y(w_k - w_0, \varrho) \geq \epsilon\right\}\right| < \vartheta$$

$\forall\ i \in D$. Let u be any positive integer satisfying $k_{s-1} < u \leq k_s$ for some $s \in D$. Then

$$\begin{aligned}
&\frac{1}{u}\left|\left\{k \leq u : G(w_k - w_0, \varrho) \leq 1-\epsilon \text{ or} B(w_k - w_0, \varrho) \geq \epsilon, Y(w_k - w_0, \varrho) \geq \epsilon\right\}\right| \\
&\leq \frac{1}{k_{s-1}}\left|\{k \leq k_s : G(w_k - w_0, \varrho) \leq 1-\epsilon \text{ or } B(w_k - w_0, \varrho) \geq \epsilon, Y(w_k - w_0, \varrho) \geq \epsilon\right\}\right| \\
&= \frac{1}{k_{s-1}}\left|\{k \in I_1 : G(w_k - w_0, \varrho) \leq 1-\epsilon \text{ or } B(w_k - w_0, \varrho) \geq \epsilon, Y(w_k - w_0, \varrho) \geq \epsilon\right\}\right| \\
&+ \frac{1}{k_{s-1}}\left|\{k \in I_2 : G(w_k - w_0, \varrho) \leq 1-\epsilon \text{ or } B(w_k - w_0, \varrho) \geq \epsilon, Y(w_k - w_0, \varrho) \geq \epsilon\right\}\right| \\
&+ \ldots + \frac{1}{k_{s-1}}\left|\{k \in I_s : G(w_k - w_0, \varrho) \leq 1-\epsilon \text{ or } B(w_k - w_0, \varrho) \geq \epsilon, Y(w_k - w_0, \varrho) \geq \epsilon\right\}\right| \\
&= \left(\frac{k_1}{k_{s-1}}\right)\frac{1}{h_1}\left|\{k \in I_1 : G(w_k - w_0, \varrho) \leq 1-\epsilon \text{ or } B(w_k - w_0, \varrho) \geq \epsilon, Y(w_k - w_0, \varrho) \geq \epsilon\right\}\right| \\
&+ \left(\frac{k_2-k_1}{k_{s-1}}\right)\frac{1}{h_2}\left|\{k \in I_2 : G(w_k - w_0, \varrho) \leq 1-\epsilon \text{ or } B(w_k - w_0, \varrho) \geq \epsilon, Y(w_k - w_0, \varrho) \geq \epsilon\right\}\right| \\
&+ \ldots + \left(\frac{k_s-k_{s-1}}{k_{s-1}}\right)\frac{1}{h_s}\left|\{k \in I_s : G(w_k - w_0, \varrho) \leq 1-\epsilon \text{ or } B(w_k - w_0, \varrho) \geq \epsilon, Y(w_k - w_0, \varrho) \geq \epsilon\right\}\right| \\
&= \left(\frac{k_1}{k_{s-1}}\right)K_1 + \left(\frac{k_2-k_1}{k_{s-1}}\right)K_2 + \ldots + \left(\frac{k_s-k_{s-1}}{k_{s-1}}\right)K_s \leq \{\sup_{i\in D} K_i\}\frac{k_s}{k_{s-1}} < M\vartheta.
\end{aligned}$$

Choosing $\mu = \dfrac{\vartheta}{M}$ and also considering that $\bigcup\{u : k_{s-1} < u \leq k_s, s \in D\} \subset H$, where $D \in \mathcal{F}(\mathcal{I})$, so $H \subset F(\mathcal{I})$. This shows that $w_k \to w_0\Big(\mathcal{I}(\mathcal{S}(G, B, Y))\Big)$.□

Theorem 3.4 Let $w = (w_k)$ be a sequence in $NNS\ V$. If $\theta = (k_s)$ be a lacunary sequence with $1 < \liminf_s q_s \leq \limsup_s q_s < \infty$, then $w_k \to w_0\Big(\mathcal{I}(\mathcal{S}(G, B, Y))\Big) \iff w_k \to w_0\Big(\mathcal{I}(\mathcal{S}_\theta(G, B, Y))\Big)$.

Proof. This is a straightforward conclusion from Theorem (3.2) and Theorem (3.3).□

Theorem 3.5 Let $w = (w_k)$ be a sequence in $NNS\ V$ such that $\frac{1}{4}\epsilon_n \diamond \frac{1}{4}\epsilon_n < \frac{1}{2}\epsilon_n$ and $(1-\frac{1}{4}\epsilon_n)\circ(1-\frac{1}{4}\epsilon_n) > 1-\frac{1}{2}\epsilon_n$. If V is a Banach space, then $\mathcal{I}(\mathcal{S}_\theta(G, B, Y)) \cap l_\infty$ is a closed subset of l_∞.

Proof. Assume that $(w^n) = (w^n_k)$ is a convergent sequence in $\mathcal{I}(\mathcal{S}_\theta(G, B, Y)) \cap l_\infty$, that converges to $w_0 \in l_\infty$. We need to show that $w_0 \in \mathcal{I}(\mathcal{S}_\theta(G, B, Y)) \cap l_\infty$. Suppose that $w^n_k \to \xi_n\Big(\mathcal{I}(\mathcal{S}_\theta(G, B, Y))\Big)\ \forall\ n \in \mathbb{N}$. Consider a sequence (ϵ_n) of reducing positive integers such that $\epsilon_n \to 0$. For each $n \in \mathbb{N}$, $\exists\ M_n > 0$ s.t if $n \geq M_n$ and $\varrho > 0$ then $\sup_i B(w - w^i, \varrho) \leq \frac{1}{4}\epsilon_n\ \forall\ i \geq n$. Choose $0 < \vartheta < \frac{1}{5}$. Let

$$\begin{aligned}
H_{G,B,Y}(\epsilon_n, \varrho) = \Big\{s \in \mathbb{N} : \frac{1}{h_s}\Big|\Big\{k \in I_s : G(w^n_k - \xi_n, \varrho) \leq 1 - \frac{1}{4}\epsilon_n \text{ or} \\
B(w^n_k - \xi_n, \varrho) \geq \frac{1}{4}\epsilon_n, Y(w^n_k - \xi_n, \varrho) \geq \frac{1}{4}\epsilon_n\Big\}\Big| < \vartheta\Big\} \in \mathcal{F}(\mathcal{I})
\end{aligned}$$

and

$$\begin{aligned}
Q_{G,B,Y}(\epsilon_n, \varrho) = \Big\{s \in \mathbb{N} : \frac{1}{h_s}\Big|\Big\{k \in I_s : G(w^{n+1}_k - \xi_{n+1}, \varrho) \leq 1 - \frac{1}{4}\epsilon_n \text{ or} \\
B(w^{n+1}_k - \xi_{n+1}, \varrho) \geq \frac{1}{4}\epsilon_n, Y(w^{n+1}_k - \xi_{n+1}, \varrho) \geq \frac{1}{4}\epsilon_n\Big\}\Big| < \vartheta\Big\} \in \mathcal{F}(\mathcal{I}).
\end{aligned}$$

Since $H_{G,B,Y}(\epsilon_n, \varrho) \cap Q_{G,B,Y}(\epsilon_n, \varrho) \in \mathcal{F}(\mathcal{I})$ and $\emptyset \notin \mathcal{F}(\mathcal{I})$, choose $s \in H_{G,B,Y}(\epsilon_n, \varrho) \cap Q_{G,B,Y}(\epsilon_n, \varrho)$. Then $\frac{1}{h_s}\Big|\Big\{k \in I_s :$
$G(w^n_k - \xi_n, \varrho) \leq 1 - \frac{1}{4}\epsilon_n$ or $B(w^n_k - \xi_n, \varrho) \geq \frac{1}{4}\epsilon_n, Y(w^n_k - \xi_n, \eta) \geq \frac{1}{4}\epsilon_n$
$\bigvee G(w^{n+1}_k - \xi_{n+1}, \varrho) \leq 1 - \frac{1}{4}\epsilon_n$ or

$B(w_k^{n+1} - \xi_{n+1}, \varrho) \geq \frac{1}{4}\epsilon_n, Y(w_k^{n+1} - \xi_{n+1}, \varrho) \geq \frac{1}{4}\epsilon_n\Big\}\Big| < 2\vartheta < 1$.
Since $h_s \to \infty$ and $H_{G,B,Y}(\epsilon_n, \varrho) \cap Q_{G,B,Y}(\epsilon_n, \varrho) \in \mathcal{F}(\mathcal{I})$ is finite, we can select above s so that $h_s > 5$. So, $\exists$ a $k \in I_s$ for which we have simultaneously, $G(w_k^n - \xi_n, \varrho) > 1 - \frac{1}{4}\epsilon_n$ or $B(w_k^n - \xi_n, \varrho) < \frac{1}{4}\epsilon_n, Y(w_k^n - \xi_n, \varrho) < \frac{1}{4}\epsilon_n$ and $G(w_k^{n+1} - \xi_{n+1}, \varrho) > 1 - \frac{1}{4}\epsilon_n$ or $B(w_k^{n+1} - \xi_{n+1}, \varrho) < \frac{1}{4}\epsilon_n, Y(w_k^{n+1} - \xi_{n+1}, \varrho) < \frac{1}{4}\epsilon_n$. For given $\epsilon_n > 0$, choose $\frac{1}{2}\epsilon_n$ s.t $\frac{1}{2}\epsilon_n \diamond \frac{1}{2}\epsilon_n < \epsilon_n$ and $(1 - \frac{1}{2}\epsilon_n) \circ (1 - \frac{1}{2}\epsilon_n) > 1 - \epsilon_n$. Then it follows that

$$B\Big(\xi_n - w_k^n, \frac{\varrho}{2}\Big) \diamond B\Big(\xi_{n+1} - w_k^{n+1}, \frac{\varrho}{2}\Big) \leq \frac{1}{4}\epsilon_n \diamond \frac{1}{4}\epsilon_n < \frac{1}{2}\epsilon_n$$

and

$$B\Big(w_k^n - w_k^{n+1}, \varrho\Big) \leq \sup_n B\Big(w_0 - w_k^n, \frac{\varrho}{2}\Big) \diamond \sup_n B\Big(w_0 - w_k^{n+1}, \frac{\varrho}{2}\Big) \leq \frac{1}{4}\epsilon_n \diamond \frac{1}{4}\epsilon_n < \frac{1}{2}\epsilon_n.$$

Hence,

$$B\Big(\xi_n - \xi_{n+1}, \eta\Big) \leq \Big[B\Big(\xi_n - w_k^n, \frac{\varrho}{3}\Big) \diamond B\Big(w_k^{n+1} - \xi_{n+1}, \frac{\varrho}{3}\Big) \diamond B\Big(w_k^n - w_k^{n+1}, \frac{\varrho}{3}\Big)\Big] \leq \frac{1}{2}\epsilon_n \diamond \frac{1}{2}\epsilon_n < \epsilon_n.$$

Similarly, $Y\Big(\xi_n - \xi_{n+1}, \varrho\Big) < \epsilon_n$ and $G\Big(\xi_n - \xi_{n+1}, \varrho\Big) > 1 - \epsilon_n$. This emphasizes that $(\xi_n)_{n\in\mathbb{N}}$ is a Cauchy sequence in V and let $\xi_n \to \xi \in V$ as $n \to \infty$. We now show that $w_0 \to \xi\Big(\mathcal{I}(\mathcal{S}_\theta(G, B, Y))\Big)$. For any $\varrho > 0$ and $\epsilon > 0$, choose $n \in \mathbb{N}$ s.t $\epsilon_n < \frac{1}{4}\epsilon$, $\sup_n B(w_0 - w_k^n, \varrho) < \frac{1}{4}\epsilon$, $G(\xi_n - \xi, \varrho) > 1 - \frac{1}{4}\epsilon$ or $B(\xi_n - \xi, \varrho) < \frac{1}{4}\epsilon, Y(\xi_n - \xi, \varrho) < \frac{1}{4}\epsilon$.
Now,

$$\begin{aligned}\frac{1}{h_s}\Big|\Big\{k \in I_s : B(w_k - \xi, \varrho) \geq \epsilon\Big\}\Big| &\leq \frac{1}{h_s}\Big|\Big\{k \in I_s : B(w_k - w_k^n, \frac{\eta}{3}) \diamond \Big[B(w_k^n - \xi_n, \frac{\eta}{3}) \diamond B(\xi_n - \xi, \frac{\eta}{3})\Big] \geq \epsilon\Big\}\Big| \\ &\leq \frac{1}{h_s}\Big|\Big\{k \in I_s : B(w_k^n - \xi_n, \frac{\eta}{3}) \geq \frac{\epsilon}{2}\Big\}\Big|.\end{aligned}$$

Similarly

$$\frac{1}{h_s}|\{k \in I_s : Y(w_k - \xi, \varrho) \geq \epsilon\}| \leq \frac{1}{h_s}\Big|\Big\{k \in I_s : Y(w_k^n - \xi_n, \frac{\eta}{3}) \geq \frac{\epsilon}{2}\Big\}\Big|.$$

and

$$\frac{1}{h_s}|\{k \in I_s : G(w_k - \xi, \varrho) \leq 1 - \epsilon\}| > \frac{1}{h_s}\Big|\Big\{k \in I_s : G(w_k^n - \xi_n, \frac{\eta}{3}) \leq 1 - \frac{\epsilon}{2}\Big\}\Big|.$$

Thus, It follows

$$\begin{aligned}&\Big\{s \in \mathbb{N} : \frac{1}{h_s}|\{k \in I_s : G(w_k - \xi, \varrho) \leq 1 - \epsilon \text{ or } B(w_k - \xi, \varrho) \geq \epsilon, Y(w_k - \xi, \varrho) \geq \epsilon\}| \geq \vartheta\Big\} \\ &\quad\subset \Big\{s \in \mathbb{N} : \frac{1}{h_s}|\{k \in I_s : G(w_k^n - \xi_n, \frac{\varrho}{3}) \leq 1 - \frac{\epsilon}{2} \text{ or } B(w_k^n - \xi_n, \frac{\varrho}{3}) \geq \frac{\epsilon}{2}, Y(w_k^n - \xi_n, \frac{\varrho}{3}) \geq \frac{\epsilon}{2}\}| \geq \vartheta\Big\}\end{aligned}$$

for given $\vartheta > 0$. Hence, we have $w_0 \to \xi\Big(\mathcal{I}(\mathcal{S}_\theta(G, B, Y))\Big)$.□

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