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Erratum: S-Zariski Topology on S-Spectrum of Modules

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Corollary 3.14 in the published version is incorrect. The condition "every S-prime submodule of M is prime" is necessary. Consider the ring $R = M = \mathbb{Z} \times \mathbb{Z}$ and the multiplicatively closed subset $S = \mathbb{Z} \times \mathbb{Z}^*$, where $\mathbb{Z}^* = \mathbb{Z} - \{0\}$. Then $Spec_S(M) = \{n\mathbb{Z} \times \{0\} : n \in \mathbb{Z}\}$. Note that $(2\mathbb{Z} \times 0 : (0,1)) = \mathbb{Z} \times 0 = (3\mathbb{Z} \times 0 : (0,1))$ and $2\mathbb{Z} \times 0, 3\mathbb{Z} \times 0 \in Spec_S(M)$. By [1, Proposition 4], $\overline{\{2\mathbb{Z} \times 0\}} = \overline{\{3\mathbb{Z} \times 0\}}$. As $2\mathbb{Z} \times 0 \neq 3\mathbb{Z} \times 0$, $Spec_S(M)$ is not a T_0-space.

Thus the following is the corrected version of the mentioned result.

Corollary. *Suppose that M is a multiplication module. Then $Spec_S(M)$ is a T_0-space for both S-Zariski topology τ_S and quasi S-Zariski topology τ_S^* iff every S-prime submodule of M is prime.*

Proof. First note that $\tau_S = \tau_S^*$ since M is multiplication. Assume that every S-prime submodule is prime. Suppose $V_S(P) = V_S(Q)$ for $P, Q \in Spec_S(M)$. Then $S^{-1}(P : M) = S^{-1}(Q : M)$ by [2, Lemma 3.5] . As P, Q are S-prime submodules, they are also prime by the assumption. Then $(P : M)$ and $(Q : M)$ are prime ideals. So $S^{-1}(P : M) = S^{-1}(Q : M)$ implies that $(P : M) = (Q : M)$. Hence we obtain $P = (P : M)M = (Q : M)M = Q$ since M is a multiplication module. Thus $Spec_S(M)$ is T_0 by [2, Theorem 3.13]. The rest follows from the fact that $\tau_S \leq \tau_S^*$. Now assume $Spec_S(M)$ is T_0-space. Let $P \in Spec_S(M)$. Then there exists $s_P \in S$ such that $(P :_M s_P)$ is a prime submodule. Also by [2, Theorem 3.11], $V_S(P) = V_S((P :_M s_P))$. Since $Spec_S(M)$ is a T_0-space, we have $P = (P :_M s_P)$ is a prime submodule. □

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