Filomat 37:25 (2023), 8521–8535
https://doi.org/10.2298/FIL2325521T

Published by Faculty of Sciences and Mathematics,
University of Niš, Serbia
Available at: http://www.pmf.ni.ac.rs/filomat

Global scheme of the basic interactions and their geometrical interpretations

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Abstract.

Our space-time consists of three 3-dimensional spaces: space S, space rotations SR and time T. First are considered the basic possible 4 cases for exchange among them: 1. $r \to s$, 2. $s \to r$, 3. $r \to t$, and 4. $s \to t$, where $s \in S$, $r \in SR$, and $t \in T$. Analogous to the affine group of translations and rotations $\mathcal{A}$, it is considered a space group G_s of 6×6 matrices, which is isomorphic to the group $Spin(4)$. The space metric observed by the particles is found. Further are considered 4 generalized exchanges 1^*, 2^*, 3^* and 4^*, induced by the cases 1, 2, 3, and 4. The case 1^* leads to the electro-weak interaction, and it is a consequence of non-commutativity between one translation and one rotation in the space group G_s. The case 2^* leads to the strong interaction, and it is a consequence of non-commutativity between two translations in the space group G_s. It leads also to the galactic acceleration which is observed at the periphery of each galaxy, and now we do not need dark matter in order to explain the motion of the distant stars in the galaxies. The case 3^* leads to electromagnetic interaction, and it is a consequence of non-commutativity between one translation and one rotation in the affine group $\mathcal{A}$. The case 4^* leads to gravitational interaction and it is a consequence of non-commutativity between one translation and one "radial translation" in the affine group $\mathcal{A}$. The corresponding accelerations are deduced and for a fixed space positions they are of type $\mathbf{a} = \mathrm{rot}(\vec{\varphi})$ (gauge invariant), but the quantum and wave effects are neglected. It is also predicted a new gravity-weak interaction, which belongs to the case 2^*.

1. Introduction

The space and time draw attention from the old civilizations up to the XX century. They tried to present own view of the space-time. Basic postulate of the Theory of Relativity is the argument that there does not exist strong distinction between the space and time, i.e. they interfere, which is obvious from the Lorentz transformations. In the recent refs. ([1–4]) this idea was generalized. For each small body besides its 3 spatial coordinates, can be jointed also 3 degrees of freedom about its rotation in the space and also 3 degrees of freedom for the velocity of the considered body. The basic assumption is that these 3+3 degrees of freedom are of the same level and importance as the basic 3 spatial coordinates. Indeed, there are three 3-dimensional sets: space S which is homeomorphic to S^3, spatial rotations SR which is also homeomorphic

2020 *Mathematics Subject Classification*. Primary 22E43, 70S20, 83E99; Secondary 83D05, 85A05.
Keywords. Lie groups; Lorentz group; Gravitation; Electromagnetism; Strong interaction; Weak interaction; Gauge invariance.
Received: 08 December 2022; Accepted: 13 December 2022
Communicated by Mića Stanković and Zoran Rakić
Research is based on work from COST Action CaLISTA CA21109, supported by COST (European Cooperation in Science and Technology), and also supported by Ss. Cyril and Methodius University in Skopje [Grant NIP.UKIM.20-21.6]
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to S^3 and time T which is homeomorphic to $\mathbb{R}^3$. The space SR is homeomorphic to S^3 if it is considered as the group of quaternions with module 1, which is locally isomorphic to $SO(3,\mathbb{R})$. The existence on the space S, does not mean that the spatial rotations exist automatically. Each two of these sets may interfere analogously to the space and time in the Special Relativity. It is deduced in refs. [1, 2] that the general form of 3-dimensional time of one point is given by $\mathbf{n}t + (\mathbf{v}\times\mathbf{r})/c^2$, where t is the 1-parameter time, $\mathbf{n}$ is the unit vector of its velocity, $\mathbf{v}$ is its velocity and $\mathbf{r}$ is the radius vector of the considered point.

The group of Lorentz transformations $O^{\uparrow}_{+}(1,3)$ is known to be isomorphic to $SO(3,\mathbb{C})$, and if we consider this complex group as a group of real 6×6 matrices, this group is the required Lie group which connects the spaces S and T. The group which connects the spaces SR and T is the same group of transformations. This Lie group of transformations has Lie algebra which is determined by the matrices of type

$$\begin{bmatrix} X & Y \\ -Y & X \end{bmatrix}, \tag{1}$$

where X and Y are antisymmetric 3×3 matrices. This Lie group will be denoted by $G_{\rm t}$, because it connects the space T (temporal space) with the other two spaces. In ref. [3] the Lorentz transformations are converted as transformations in $S\times T$, given by 6×6 matrices.

The Lie group which connects the spaces S and SR has Lie algebra which consists of matrices of type

$$\begin{bmatrix} X & Y \\ Y & X \end{bmatrix}, \tag{2}$$

where X and Y are antisymmetric 3×3 matrices. This Lie group is generated by the following 6 matrices of type

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \cos\alpha & 0 & 0 & 0 & \sin\alpha \\ 0 & 0 & \cos\alpha & 0 & -\sin\alpha & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \sin\alpha & 0 & \cos\alpha & 0 \\ 0 & -\sin\alpha & 0 & 0 & 0 & \cos\alpha \end{bmatrix}, \begin{bmatrix} \cos\alpha & 0 & 0 & 0 & 0 & -\sin\alpha \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \cos\alpha & \sin\alpha & 0 & 0 \\ 0 & 0 & -\sin\alpha & \cos\alpha & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ \sin\alpha & 0 & 0 & 0 & 0 & \cos\alpha \end{bmatrix},$$
$$\begin{bmatrix} \cos\alpha & 0 & 0 & 0 & \sin\alpha & 0 \\ 0 & \cos\alpha & 0 & -\sin\alpha & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & \sin\alpha & 0 & \cos\alpha & 0 & 0 \\ -\sin\alpha & 0 & 0 & 0 & \cos\alpha & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \tag{3}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha & 0 & 0 & 0 \\ 0 & -\sin\alpha & \cos\alpha & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cos\alpha & \sin\alpha \\ 0 & 0 & 0 & 0 & -\sin\alpha & \cos\alpha \end{bmatrix}, \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ \sin\alpha & 0 & \cos\alpha & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\alpha & 0 & -\sin\alpha \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \sin\alpha & 0 & \cos\alpha \end{bmatrix},$$
$$\begin{bmatrix} \cos\alpha & \sin\alpha & 0 & 0 & 0 & 0 \\ -\sin\alpha & \cos\alpha & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\alpha & \sin\alpha & 0 \\ 0 & 0 & 0 & -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \tag{4}$$

where α is an arbitrary parameter. The matrices from (3) present "translations" along x, y, and z axes, while the matrices from (4) present rotations around the x, y, and z axes. This group will be denoted by $G_{\rm s}$ as a space group, which connects the spaces S and SR. The group G_s is isomorphic to the group Spin(4) ([4]). We denote by $\mathcal{A}$ the affine group of translations and rotations in the Euclidean space, and as a set of 6×6 matrices it can be proved that its Lie algebra has the form

$$\begin{bmatrix} X & Y \\ 0 & X \end{bmatrix}, \tag{5}$$

where X and Y are antisymmetric 3×3 matrices.

The elements of the space S are measured in length (meters), while the elements of spatial rotations SR are measured in angles (radians), so there exists a local constant as a coefficient of proportionality between these two spaces. This coefficient is called *radius of range* R. In this paper we determine the radii of range in case of the nucleons, galaxies and the universe, and they depend mainly on the mass. But not each body has its own radius of range. The radius of range is of the same importance as the velocity of light, which is a coefficient of proportionality between the space and time.

The multi-dimensional time was investigated also by another authors, for example in refs. [5–12].

2. Eight cases of exchanging among S, SR and T

In this section we present all possible 8 cases of exchanging among the spaces S, SR and T. The elements of these spaces will be denoted by s, r and t respectively. Firstly we consider the basic 4 cases, presented initially in [13]:

$$1.\ r \to s, \quad 2.\ s \to r, \quad 3.\ r \to t, \quad 4.\ s \to t.$$

The composite exchanges $s \to r \to t$ and $r \to s \to t$ are also admitted, while the exchange of type $x \to y \to x$ is not admitted and also $t \to s$ and $t \to r$ are not admitted, because it is impossible to constraint the time without constraint of s or r. Since the exchange $s \to r \to s$ is not possible, the case 3. must be of type $s \to r \to t$, and since the exchange $r \to s \to r$ is not possible, the case 4. must be of type $r \to s \to t$. We support these exchanges by simple examples.

The first case $r \to s$ means that if the rotation is not permitted, then the particles will be displaced, which is the induced spin motion. This induced spin velocity, or shortly spin velocity, was considered in many details in ref. [14]. Indeed, if a solid body moves including some rotations, each particle (or atom) of the body intends to rotate according to its spatial trajectory, but that rotation is constrained completely or partially because we consider a solid body. So, the constrained space rotation induces the spin velocity. This is the reason for circular motion of the spinning bodies. Moreover, the spin velocity of the Earth is considered in ref. [14], and as a consequence the change of the Earth's angular velocity with period of 6 months was deduced with accuracy 5% compared with the measurements. In ref. [15] the spin velocity is determined via the curvature and torsion of the trajectory of the considered particle.

The spin velocity is non-inertial and it will be denoted by capital letter $\mathbf{V}$ in order to distinguish from the classical inertial velocity $\mathbf{v}$. It is also important that if a coil moves with spin velocity in magnetic field, there will not appear an electromotive force. The reason is that the spin velocity means simply displacement in the space and there does not appear change of speed of time by the coefficient $\sqrt{1 - V^2/c^2}$. But if the spin velocity is constrained completely or partially, then the constrained part converts into inertial velocity such that $m_0 V = -m_0 v / \sqrt{1 - v^2/c^2}$, and hence $\mathbf{v} = -\mathbf{V} / \sqrt{1 + V^2/c^2}$. This is indeed the fourth case $s \to t$. Here inertial velocity between two bodies means that their temporal axes are not parallel in the observable 3+1-dimensional space time.

Let us return to the considered moving body and let $\mathbf{V}_i$ be the spin velocity of the i-th particle of the considered body. The global spin velocity is an average velocity $\mathbf{U}$. Then the spin velocity $\mathbf{V}_i - \mathbf{U}$ of the i-th particle of the body is constrained, and it converts into inertial velocity $\mathbf{U} - \mathbf{V}_i$ (case 4). So, we have composite case $r \to s \to t$ at almost each particle of the body.

The second case $s \to r$ means that if the space displacement is not permitted completely or partially, then it induces a spatial rotation. This can easily be interpreted if we consider a rigid body, which moves with a velocity $\mathbf{v}$. Assume that one point O of the rigid body is constrained to move. Then the body will start to rotate around the point O. Each point A of the body intends to rotate with angular velocity $\mathbf{w}_A = (\overrightarrow{OA} \times \mathbf{v})/|\overrightarrow{OA}|^2$. This means that the case $s \to r$ occurs. However, the rigid body will rotate globally with an averaged angular velocity $\mathbf{w}_{\text{aver}}$, which depends on the distribution of the mass. So at the i-th particle (point A for example) of the body the angular velocity $\mathbf{w}_A - \mathbf{w}_{\text{aver}}$ will be constrained. The angular velocity $\mathbf{w}_A - \mathbf{w}_{\text{aver}}$ can be non-zero if the point O is not fixed. This constrained angular velocity will be converted into inertial velocity $\mathbf{u} = -(\mathbf{w}_A - \mathbf{w}_{\text{aver}}) \times \overrightarrow{OA}$ and this is the third case $r \to t$. We conclude that almost at each particle there appears the composite case $s \to r \to t$.

Although we described the cases 1-4, we give additionally an evident example from the gravitation.

Assume that a non-rotating body initially rests with respect to the Earth on almost infinity distance. Assume that this body freely falls toward the Earth under the Earth's gravitation. When the body comes at the surface of the Earth, it is not permitted to be displaced further. So, this constraint will cause time displacement, such that the time will be slower. More precisely, if the velocity at the surface is equal to v, then the constraint for the space displacement will induce slower time for coefficient $\lambda = \sqrt{1-\frac{v^2}{c^2}}$. This is the case 4 ($s \to t$). Since $v \approx \sqrt{2GM/R}$, the time on the surface of the Earth is slower for coefficient $\lambda \approx \sqrt{1-\frac{2GM}{Rc^2}} \approx 1-\frac{GM}{Rc^2}$, which is also well known from the General Relativity up to approximation of c^{-2}. Note that before the body fall on the Earth, its speed of time is identical with speed of time far from the Earth, when it begun with free fall motion.

Although the results presented in the sections 1 and 2 are evident, we summarize them as **Axiom 1.** *The space-time consists of 3 sets: space* (S)*, which is homeomorphic to* S^3*, space rotations* (SR)*, which is also homeomorphic to* S^3*, and time* (T)*, which is homeomorphic to* $\mathbb{R}^3$*, and these sets can mutually exchange according to the cases 1, 2, 3, and 4.*

The last four exchanges, cases 1*, 2*, 3* and 4* are similar to the cases 1, 2, 3, 4 respectively. While the cases 1-4 describe the changes of positions (coordinates) of one particle if there are some constraints in the initial space, in cases 1*-4* we describe the fields which appear in case of constraints and describe the interactions between the particles. There are no classical constraints there, but there appear some incompatibilities, arising from the non-commutative groups. Before we consider them separately, to the end of this section we give some general comments about them.

In the cases 1* and 2* we should not use the Lorentz transformations. The time exists in these cases, but caused by another external reasons and it should not be actively used. The case 2* leads to strong interaction and galactic acceleration (which is observed at the periphery of each galaxy), the case 1* leads to weak interaction, the case 3* leads to the electromagnetic interaction and the case 4* leads to the gravitational interaction. It leads to global classification of the basic interactions in the nature. Let O_1 and O_2 be the centers of the bodies and X is close to O_2 which belongs to the second body, such that $\overrightarrow{O_2X} = (a,b,c)$ (Fig. 2), and let τ be translation for vector $\overrightarrow{O_2X} = (a,b,c)$. Then these 4 cases are the following (see also Table 1):

1^* The electro-weak interaction is a consequence of non-commutativity between τ and one rotation in the space group G_s. The rotation is partially constrained in G_s;

2^* The strong interaction is a consequence of non-commutativity between τ and one translation in the space group G_s. The translation is partially constrained in G_s;

3^* The electromagnetic interaction is a consequence of non-commutativity between τ and one rotation in the affine group $\mathcal{A}$. The rotation is partially constrained in $\mathcal{A}$;

4^* The gravitational interaction is a consequence of non-commutativity between τ and one "radial translation" in the affine group $\mathcal{A}$. The translation is partially constrained in $\mathcal{A}$.

Group of transformations	rotation	translation
G_s	electro-weak interaction	strong int. & gravity-weak int.
$\mathcal{A}$	electromagnetic interaction	gravitational interaction

Table 1: Global scheme of the basic interactions.

In case of the electromagnetic and gravitational interaction, the main role has the velocity of light c which "connects" the space and time. So these two interactions can be called *temporal interactions*. On the other hand, in case of strong and weak interaction the main role have the radii of range of the particles, which "connect" the space and the spatial rotations. So these two interactions can be called *spatial interactions*.

The strong, weak and electromagnetic interactions are studied in the Standard Model. The present geometrical approach does not consider the quantum/wave effects, but all accelerations are of type $\mathbf{a} = \text{rot}(\vec{\varphi})$ (gauge invariant). From this point of view the strong and weak interactions as spatial interactions are not compatible with the Standard Model, because the Dirac equation is based on the Special Relativity. So, this is the reason that the Quantum Electrodynamics gives the best results. We draw our attention to the reasons, which cause the interactions between the particles. In the recent ref. [13] the electromagnetic interaction and partially the gravitational interaction (cases 3^* and 4^*) were studied in many details, including the magnetic fields of the spinning bodies. So, in section 5 we give brief view of the electromagnetic field. The main attention in this paper will be on the cases 1^* and 2^*, and also some new viewpoint of the gravitational interaction.

In all 4 interactions, the fields are described by the relative acceleration $\mathbf{a}$ between the bodies as a result of the interaction and the relative angular velocity/precession $\mathbf{w}$ between the bodies is also a result of the interaction. In case of electromagnetism these two fields are analogous to the electric field $\mathbf{E}$ and magnetic field $\mathbf{H} = c\mathbf{B}$. These two fields are components of the well known antisymmetric tensor field F of rank 2. Analogously, in case of gravitation, the fields $\mathbf{a}$ and $c\mathbf{w}$ are components of an antisymmetric tensor field ϕ of rank 2 [13]. In case of spatial interactions (strong and weak), instead of the pair $(\mathbf{a}, c\mathbf{w})$, we need to consider a pair $(\mathbf{a}, v_0\mathbf{w})$, where the velocity v_0 is of local character, which may depend on the energy of the particles and so on. We do not have universal constant $v_0 = c$ in cases 1^* and 2^*. There $\mathbf{a}$ and $v_0\mathbf{w}$ may not be components of an antisymmetric 4×4 or equivalently 6×6 tensor of rank 2. Such tensor does not exist. But, we prove in the next section that $\mathbf{a} = v_0\mathbf{w}$, or $\mathbf{a} = -v_0\mathbf{w}$, and so the vectors $\mathbf{a}$ and $\mathbf{w}$ are collinear.

3. Strong interaction

We start with the strong interaction as interaction in the basic space $S \times SR$. In this case if we consider composition of two translations, the final result contains also a spatial rotation for an angle $\vec{\varphi}$. It follows since the product of two matrices of type (3) has a component of type (4), i.e. contains rotation for angle $\vec{\varphi}$. This angle is not a function of the temporal parameter t, but it is a function of the spatial coordinates, and so instead of differentiation by t we should apply the operator rotor. Half of this quantity is admitted, and half is not admitted. The procedure is the following. We determine the angle $\vec{\varphi}$ which corresponds to composition of two translations (or translation and rotation) in the plane determined by the radius vector (x, y, z) and arbitrary small vector (a, b, c). Then we apply the operator rotor and we multiply by $v_0^2/2$ in order to obtain the required acceleration, which has radial direction, parallel to the radius vector (x, y, z). This procedure is general and will be applied also in case of the other interactions. In case of electromagnetic and gravitational interaction, where the time is actual, then $v_0 = c$ as a universal constant. After determination of the acceleration we determine the angular velocity by using the corresponding Lie group G_s or G_t, as we mentioned at the end of section 2.

Example 1. Let us consider the vector field $\mathbf{v}(x, y, z) = (-yw, xw, 0)$. Since $\text{rot}\mathbf{v} = (0, 0, 2w)$, obviously when it is admitted we have angular velocity $\mathbf{w} = (0, 0, 2w)$, and when it is not admitted, it converts into centripetal acceleration $(a_x, a_y, a_z) = \mathbf{v} \times (-2\mathbf{w})$. These two effects do not occur simultaneously, but they change periodically in a very short interval Δt as a process of discretization, because these two effects are mutually dependent: in the first time interval Δt the body rotates around its axis with angular velocity $2w$, in the next interval Δt it falls toward the center with centripetal acceleration $2\mathbf{v} \times (-\mathbf{w})$, and it periodically repeats. So, we observe that the average angular velocity is $(0, 0, w)$, and the average centripetal acceleration is $\mathbf{v} \times (-\mathbf{w})$.

The previous comments can be summarized by the following **Axiom 2:** *The time is a discrete quantity with two extremely small steps $(\Delta t)_{\text{rot}}$ and $(\Delta t)_{\text{trans}}$ with equal duration. During the time $(\Delta t)_{\text{trans}}$ the rotations are not admitted and they are converted into displacements, which are admitted and which are observed as motions and* *accelerations. During the time $(\Delta t)_{\rm rot}$ the translations are not admitted and they are converted into rotations, which are admitted.*

Theorem 1. *In case of weak and strong interaction, i.e. in the Lie group G_s, it holds $\mathbf{a} = v_0\mathbf{w}$ or $\mathbf{a} = -v_0\mathbf{w}$.*

Proof. First, for comparison we refer to the known isomorphism $\begin{bmatrix} X & Y \\ -Y & X \end{bmatrix} \mapsto X+iY$ of Lie algebras, which defines local isomorphism between the Lie groups G_t and $SO(3,\mathbb{C})$. In case of the space group G_s, we have the following mapping $F_1\Big(\begin{bmatrix} X & Y \\ Y & X \end{bmatrix}\Big) = X + Y$. It is a homomorphism because

$$F_1\Big(\begin{bmatrix} X & Y \\ Y & X \end{bmatrix}\begin{bmatrix} X' & Y' \\ Y' & X' \end{bmatrix} - \begin{bmatrix} X' & Y' \\ Y' & X' \end{bmatrix}\begin{bmatrix} X & Y \\ Y & X \end{bmatrix}\Big)$$

$$= F_1\Big(\begin{bmatrix} X & Y \\ Y & X \end{bmatrix}\begin{bmatrix} X' & Y' \\ Y' & X' \end{bmatrix}\Big) - F_1\Big(\begin{bmatrix} X' & Y' \\ Y' & X' \end{bmatrix}\begin{bmatrix} X & Y \\ Y & X \end{bmatrix}\Big)$$

$$= F_1\Big(\begin{bmatrix} XX' + YY' & XY' + YX' \\ XY' + YX' & XX' + YY' \end{bmatrix}\Big) - F_1\Big(\begin{bmatrix} X'X + Y'Y & X'Y + Y'X \\ X'Y + Y'X & X'X + Y'Y \end{bmatrix}\Big) =$$

$$= XX' + YY' + XY' + YX' - X'X - Y'Y - X'Y - Y'X =$$

$$= (X + Y)(X' + Y') - (X' + Y')(X + Y) =$$

$$= F_1\Big(\begin{bmatrix} X & Y \\ Y & X \end{bmatrix}\Big)F_1\Big(\begin{bmatrix} X' & Y' \\ Y' & X' \end{bmatrix}\Big) - F_1\Big(\begin{bmatrix} X' & Y' \\ Y' & X' \end{bmatrix}\Big)F_1\Big(\begin{bmatrix} X & Y \\ Y & X \end{bmatrix}\Big).$$

This homomorphism of Lie algebras gives a homomorphism from the Lie group G_s into $SO(3,\mathbb{R})$, from 6 dimensions into 3 dimensions. Since the matrix $X + Y$ is given by

$$X + Y = \begin{bmatrix} 0 & v_0w_z + a_z & -(v_0w_y + a_y) \\ -(v_0w_z + a_z) & 0 & v_0w_x + a_x \\ v_0w_y + a_y & -(v_0w_x + a_x) & 0 \end{bmatrix}, \tag{6}$$

we conclude that $\mathbf{a}+v_0\mathbf{w}$ transforms as a vector in the Euclidean space. Analogously, the following mapping $F_2\Big(\begin{bmatrix} X & Y \\ Y & X \end{bmatrix}\Big) = X - Y$ is also homomorphism of groups. In this case we obtain that $\mathbf{a} - v_0\mathbf{w}$ transforms as a vector in the Euclidean space. Moreover, there do not exist another such homomorphisms. Hence we have only 2 such vectors $\mathbf{a} \pm v_0\mathbf{w}$, which correspond to two opposite intrinsic angular momentums of the considered particle. It is not possible both of them to be non-zero vector fields, because then both $\mathbf{a}$ and $v_0\mathbf{w}$ would be vector fields, and also $\mathbf{a} + \lambda v_0\mathbf{w}$ would be vector field for arbitrary $\lambda \in \mathbb{R}$. It is in a contradiction with the argument that $F\Big(\begin{bmatrix} X & Y \\ Y & X \end{bmatrix}\Big) = X + \lambda Y$ is not a homomorphism when $\lambda \neq \pm 1$. So, the other field is 0 vector field. Hence $\mathbf{a} - v_0\mathbf{w} \equiv 0$, or $\mathbf{a} + v_0\mathbf{w} \equiv 0$, and thus $\mathbf{a} = v_0\mathbf{w}$ or $\mathbf{a} = -v_0\mathbf{w}$. □

Before we consider this force in more details, we find the metric close to the particles. The angular quantities of the group G_s which correspond to the translations, are indeed quotients l/R. In the space part of G_s the biggest circles from the body with radius of range R are with radius R. Our calculations refer when only the center of the particle or close to it is an observer and let it be at point A (Fig. 1), and let B be a point on arc distance d from A. If B' is the projection of B on the line AO, and B'' is projection of B on the tangent line at A, then the distance d from A is observed as

$$|AB''| = |BB'| = R\sin\angle AOB = R\sin\frac{d}{R} = d\Big(\frac{R}{d}\sin\frac{d}{R}\Big).$$

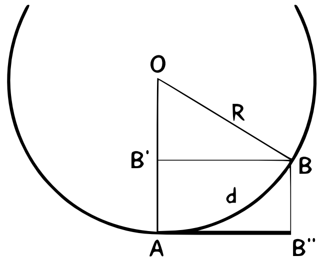

Figure 1: The observation from a body with radius of range R.

Arbitrary vector $\mathbf{s}$ which is orthogonal to the arc AB on distance d is observed by the same coefficient of contraction, as vector $\mathbf{s}|BB'|/d = \mathbf{s}\frac{R}{d}\sin\frac{d}{R}$. It can be interpreted on Fig. 1, by using similar spherical triangles with coefficient $\frac{R}{d}\sin\frac{d}{R}$. If the small vector $\mathbf{s}$ is tangent to the arc AB at B, then it is observed as $\mathbf{s}(R\sin\frac{d}{R})' = \mathbf{s}\cos\frac{d}{R}$. So, we proved the following theorem.

Theorem 2. *The space distance ds observed from the center of the particle in spherical coordinates r, θ and ϕ is given by*

$$(ds)^2 = \Big(\cos\frac{r}{R}\Big)^2(dr)^2 + \Big(\frac{R}{r}\sin\frac{r}{R}\Big)^2 r^2[(d\phi)^2 + \sin^2\phi(d\theta)^2] =$$

$$= R^2\Big[\Big(d(\sin\frac{r}{R})\Big)^2 + \Big(\sin\frac{r}{R}\Big)^2((d\phi)^2 + \sin^2\phi(d\theta)^2)\Big], \tag{7}$$

and hence the Riemann-Christoffel tensor of curvature is 0.

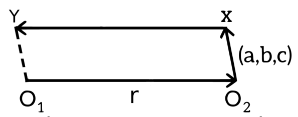

Figure 2: The strong interaction is a consequence of non-commutativity of translations for the vectors $\mathbf{r}$ and (a, b, c) in $S \times SR$.

Now let us consider two nucleons with centers at O_1 and O_2 with radii of range R_1 and R_2 respectively, and $\mathbf{r} = \overrightarrow{O_1O_2}$ (Fig.2). The non-commutativity of the translations obtains by the angle of two translations: translation for vector $\overrightarrow{O_1X}$ observed by O_1, and then translation by vector $-\mathbf{r}$ observed by the point X, or almost the same by O_2. In G_s the angle as a result of two translations is analogous to rotation as a consequence of two rotations, i.e. the vector product of two vectors and we use the coefficient 1/2 analogous to the Thomas precession. The endpoint of translation is a point Y which is close to O_1, where almost there is no rotation between these two points. We use the notations $k_1 = \frac{R_1}{r}\sin\frac{r}{R_1}$, $k_2 = \frac{R_2}{r}\sin\frac{r}{R_2}$, and $k_1^* = \cos\frac{r}{R_1}$. Without loss of generality we assume that the vector $\mathbf{r}$ is parallel to the z-axis, i.e. $x = y = 0$, and as a consequence of Theorem 2, the vector $\overrightarrow{O_1X}$ is observed from O_1 as $k_1\mathbf{r} + (k_1a, k_1b, k_1^*c)$, while the vector $-\mathbf{r}$ from X is observed as $-k_2\mathbf{r}$. The normalization should be done with respect to the distance r. Using the form of matrices given in section 1, the angle φ is given by

$$\vec{\varphi} = \frac{-1}{2}\Big[\frac{k_1\mathbf{r} + (k_1a, k_1b, k_1^*c)}{r} \times \frac{-k_2\mathbf{r}}{r}\Big] = -k_1k_2\frac{\mathbf{r}\times(a,b,c)}{2r^2}.$$

Further we obtain

$$\mathrm{rot}\vec{\varphi} = -k_1k_2\frac{\mathbf{r}}{r^2},$$

$$\frac{1}{2}\mathrm{rot}\vec{\varphi} = -\frac{1}{2}\sin\frac{r}{R_1}\sin\frac{r}{R_2}\frac{R_1R_2}{r^2}\frac{\mathbf{r}}{r^2}. \tag{8}$$

Assume that the rotation is not admitted. The relative acceleration between the two bodies is given by

$$\mathbf{a}_{\mathrm{rel}} = -\frac{v_0^2}{2}\sin\frac{r}{R_1}\sin\frac{r}{R_2}\frac{R_1R_2}{r^2}\frac{\mathbf{r}}{r^2}. \tag{9}$$

Let us denote by $\mathbf{a}_1$ and $\mathbf{a}_2$ the accelerations of the first and the second body, then

$$\mathbf{a}_{\mathrm{rel}} = \mathbf{a}_2 - \mathbf{a}_1, \quad \mathbf{a}_1 = -\frac{m_2}{m_1+m_2}\mathbf{a}_{\mathrm{rel}}, \quad \mathbf{a}_2 = \frac{m_1}{m_1+m_2}\mathbf{a}_{\mathrm{rel}}.$$

The forces toward the first and toward the second body are opposite

$$\mathbf{f}_1 = m_1\mathbf{a}_1 = -\frac{m_1m_2}{m_1+m_2}\mathbf{a}_{\mathrm{rel}}, \quad \mathbf{f}_2 = m_2\mathbf{a}_2 = \frac{m_1m_2}{m_1+m_2}\mathbf{a}_{\mathrm{rel}}.$$

Assume that the space displacement is not admitted. Then it appears a relative rotation of the two bodies which is given by

$$\mathbf{w}_{\mathrm{rel}} = -\frac{v_0}{2}\sin\frac{r}{R_1}\sin\frac{r}{R_2}\frac{R_1R_2}{r^2}\frac{\mathbf{r}}{r^2}. \tag{10}$$

Let us denote by $\mathbf{w}_1$ and $\mathbf{w}_2$ the angular velocities of the first and the second body, then

$$\mathbf{w}_{\mathrm{rel}} = \mathbf{w}_2 - \mathbf{w}_1, \quad \mathbf{w}_1 = -\frac{I_2}{I_1+I_2}\mathbf{w}_{\mathrm{rel}}, \quad \mathbf{w}_2 = \frac{I_1}{I_1+I_2}\mathbf{w}_{\mathrm{rel}},$$

where I_1 and I_2 are the moments of inertia of the two bodies. Then the induced angular momentums of the first and the second body are opposite

$$\mathbf{L}_1 = I_1\mathbf{w}_1 = -\frac{I_1I_2}{I_1+I_2}\mathbf{w}_{\mathrm{rel}}, \quad \mathbf{L}_2 = I_2\mathbf{w}_2 = \frac{I_1I_2}{I_1+I_2}\mathbf{w}_{\mathrm{rel}}.$$

Further in case of equal particles, we have $R_1 = R_2 = R$, $m_1 = m_2 = m$, and $I_1 = I_2 = I$. Hence,

$$\mathbf{L}_2 = -\frac{Iv_0}{4}\Big(\sin\frac{r}{R}\Big)^2\frac{R^2}{r^3}\frac{\mathbf{r}}{r}, \tag{11}$$

$$\mathbf{f}_2 = -\frac{mv_0^2R^2}{4r^3}\Big(\sin\frac{r}{R}\Big)^2\frac{\mathbf{r}}{r}, \tag{12}$$

and so the strong nuclear force is always attractive in this case. In case of nucleons, the local parameter v_0 is very close to the boundary value c and we will use below approximately that $v_0 = c$. It is a consequence since the nucleons move with velocities close to c. Indeed, if there are no high velocities of the nucleons, then there are no contractions of the nucleons and there is no enough space for all of them in the nuclei.

At the end we use the fact that at distance of one femptometer the strong nuclear force is about 137 times stronger than the electromagnetic, i.e. Coulomb's force. Hence we are able to estimate the radius of range of the nucleons R. So, if we put approximately from (12) that

$$m_{\mathrm{p}}\frac{c^2R^2}{4r^3}\Big(\sin\frac{r}{R}\Big)^2 \approx 137\frac{e^2}{4\pi\epsilon_0r^2}$$

where $r = 1$ fm, m_{p} is the mass of the proton and e is its charge, we obtain $R \approx 1.40$ fm. This is an expected result because R is comparable with the radius of the proton. We assume further that this universal constant for the radius of range of proton is $R = a_0\alpha^2/2 = 1.4089$ fm, where a_0 is the Bohr radius and α is the fine-structure constant. It is known that the strong interaction is observable on distance 1-3 fm where it binds the protons and neutrons together in order to form the nucleus of the atom, and also on smaller scale less than 0.8 fm that holds the quarks together to form the proton. The strong interaction has the following important property: it does not exist between two particles which have different temporal positions, because the strong interaction is a spatial interaction. It is sufficient to exist a slight time displacement in order to dissappear the strong interaction and it is very important for stability of the nucleus. The strong interaction occurs almost immediately, much faster than the light.

4. Galactic acceleration

There are no limitations of the masses, moments of inertia and the radii of range in section 3. It means that we may assume that $m_2 << m_1$, $I_2 << I_1$ and $R_2 = \infty$, i.e. m_2 and I_2 of a test body can be neglected. Let us consider a galaxy with radius of range R. The application of the interaction caused by 2^* is more convenient in case of galaxies and the universe compared with the nucleons and gluons, because the quantum perturbations disappear now. Then each star can be considered as a test body. Having in mind that the matrix $X + Y$ is given by (6), analogously to the previous section the equation of motion is given by

$$\frac{\mathbf{w}}{v_0} + \frac{\mathbf{a}}{v_0^2} = \frac{1}{2}\mathrm{rot}\Big(\frac{-1}{2}\sin\frac{r}{R}\frac{R}{r^2}\Big[\frac{\mathbf{r}}{r}\times(a,b,c)\Big]\Big),$$

$$\frac{\mathbf{w}}{v_0} + \frac{\mathbf{a}}{v_0^2} = -\sin\frac{r}{R}\frac{R}{2r^2}\frac{\mathbf{r}}{r}. \tag{13}$$

This is the equation of motion according to the Lie group G_s. When the angular velocity is non-admitted, i.e. $\mathbf{w} = 0$, the required space displacement is observed by the acceleration

$$\mathbf{a} = -\sin\frac{r}{R}\frac{v_0^2 R}{2r^2}\frac{\mathbf{r}}{r}. \tag{14}$$

When the space displacement is not-admitted, i.e. $\mathbf{a} = 0$, then we obtain the angular velocity

$$\mathbf{w} = -\sin\frac{r}{R}\frac{v_0 R}{2r^2}\frac{\mathbf{r}}{r}. \tag{15}$$

The acceleration (14) was not included in the equations of motion previously, and so the dark matter was introduced in order to avoid contradiction with the Newtonian gravitation. We will see further that there is no need for dark matter in order to explain the motion of the distant stars in the galaxies. The angular velocity (15) is also not known to the present science. In case of the solar system, using that $R = 17$ kpc (see eq. (19) below) this angular velocity has a period of 315 million years, which is more than one galactic year and so it is difficult to be detected. This is the reason that the ecliptic plane is inclined toward the galactic plane.

We make estimation of the local constants v_0 and R. Let us denote by v_{N} the expected velocity of a star by using of the Newtonian gravity theory, and v is the observed velocity of a star. Then we have the following equilibrium condition

$$\frac{v_0^2 R}{2r^2}\sin\frac{r}{R} + \frac{v_{\mathrm{N}}^2}{r} = \frac{v^2}{r}, \tag{16}$$

because the total acceleration toward the center of the galaxy (left side) is equal to the centripetal acceleration (right side). Hence,

$$v_0 = \sqrt{2(v^2 - v_N^2)\Big(\frac{R}{r}\sin\frac{r}{R}\Big)^{-1}}. \tag{17}$$

Let us consider two stars on distances r_1 and r_2, with velocities v_1 and v_2, and velocities $v_{1\mathrm{N}}$ and $v_{2\mathrm{N}}$ predicted by the Newton's theory, then from (17) we obtain

$$\frac{\sin\frac{r_1}{R}}{\sin\frac{r_2}{R}} = \frac{r_1(v_1^2 - v_{1\mathrm{N}}^2)}{r_2(v_2^2 - v_{2\mathrm{N}}^2)}, \tag{18}$$

and hence R can be found. For example, in case of Milky Way we can use $r_1 = 8.122$ kpc, $v_1 \approx 233.6$ km/s, $v_{1\mathrm{N}} \approx 45$ km/s for the Sun, and use some recent measurements (see ref. [16], fig. 2) for another star, for example $r_2 = 12$ kpc, $v_2 \approx 227$ km/s and $v_{2\mathrm{N}} \approx 36$ km/s. The solution of eq. (18) is $R = 17$ kpc. Further according to (17) we obtain $v_0 = 330.4$ km/s. For each galaxy, the graph of the function $v(r)$ determined by (16) has the same form as on fig.2 from ref. [16]. So, there is no contradiction with the Newton's theory and we do not need now dark matter to explain the motion of the distant stars.

The galactic acceleration is of special interest for the galaxies, because it is attractive for its stars on distance less than πR. Moreover, if two close galaxies have different masses, then the force as a function of the distance between them can be attractive and repulsive. The repulsive acceleration helps to avoid collision on short distance between their centers, i.e. black holes.

Let M be the mass of a chosen galaxy. The radius of range R of the Milky Way satisfies the equality

$$\frac{MG}{R^2} = cH, \tag{19}$$

where H is the Hubble constant. Using that for the Milky Way $M \approx 3 \times 10^{42}$ kg (NASA 2019) we obtain again that its radius of range is $\approx 0.539 \times 10^{21}$ m ≈ 17 kpc. Indeed, this is basic eq. and probably it is true for each galaxy, where M is its mass and R is its radius of range.

The basic formula (19) is convenient to apply also in case of the universe. Naturally, the radius of range is equal to $R = c/H$. We neglect the history of the universe, but consider the cold universe at the present time. The universe as S^3 manifold should be considered as a boundary of 4-dimensional ball with radius R^*, for example $x_1^2 + x_2^2 + x_3^2 + x_4^2 = (R^*)^2$. Then the volume of the universe is equal to $2\pi^2(R^*)^3$. The radius R^* should be chosen as follows. Observed from any point of the universe, for example $(0, 0, 0, R^*)$ it seems as 3-dimensional sphere without the point $(0, 0, 0, -R^*)$. Moreover, the mass inside the semi 3-dimensional sphere $0 < r < \pi R$, i.e. $0 < x_4 < R^*$, has attractive force toward the chosen point, while the mass inside the semi 3-dimensional sphere $\pi R < r < 2\pi R$, i.e. $-R^* < x_4 < 0$ is a repulsive force from the chosen point according to (14). Further it repeats periodically. Hence we have a simple eq. $2\pi R = \pi R^*$, $R^* = 2R = 2c/H$. So, for the volume of the universe we obtain

$$V = 2\pi^2(R^*)^3 = 16\pi^2 c^3/H^3 = 3.52 \times 10^{80} \text{ m}^3. \tag{20}$$

The standard estimation of the volume of the universe is 3.57×10^{80} m^3, where there is very good coincidence. Now let us return to the mass M of the universe. Having in mind that $R = c/H$, from (19) we obtain that $M = \frac{c^3}{HG}$. But, the mass of our universe is twice larger, which can not be explained in this paper, but in a incoming paper. So

$$M = \frac{2c^3}{HG}, \tag{21}$$

and the universe has mass $M = 1.76 \times 10^{53}$ kg. The standard estimation of the mass of the universe is 1.5×10^{53} kg, which is 14.77% less than our estimation according to the formula (21). Hence the density of the universe is

$$\rho = \frac{H^2}{8\pi^2 G}. \tag{22}$$

Since H decreases with the time, probably the universe will stop to expand, when the right side of (22) will take the known critical value of the density.

5. Electromagnetic interaction

In section 2 it was explained why the time in gravitational field is slower for coefficient $1 - GM/(rc^2)$. There it was not important that the particle moved under gravitation. For example, assume that we have two charged bodies with charges e_1 and e_2 (Fig.3). Assume that the second body has mass m_2 and that it radiates waves with frequency ν_0. If the second body is placed close to the first body on distance r between their centers and assume that the distance r remains constant. Then according to the case 4, analogously

Figure 3: The frequency ν changes when the distance r changes.

to the gravitational case now we have slower speed of time and simple calculations show that the new frequency is given by

$$\nu = \nu_0\Big(1 + \frac{e_1 e_2}{4\pi\epsilon_0 r m_2 c^2}\Big). \tag{23}$$

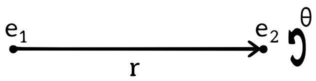

Figure 4: The second charged particle is rotated by the first charged particle (and vice versa).

The first charged body rotates the second charged body. We assume an **Axiom 3** [13] that the angle of rotation is equal to (Fig.4)

$$\vec{\theta} = \frac{e_1 e_2}{4\pi\epsilon_0 r^2 m_2 c^2}\mathbf{r}, \tag{24}$$

where $\mathbf{r} = (x, y, z)$ is the radius vector from the center of the first charged body toward the second body. This axiom will be used for deriving the Coulomb's force. So, this geometrical assumption gives us a more fundamental understanding of the charge as a physical phenomena.

Let us consider two charged particles with centers at O_1 and O_2 and let (a, b, c) be a small vector for translation. We choose the coordinate system such that the angle of rotation is $(0, 0, \theta)$, i.e $x = y = 0$. Then the non-commutativity between the rotation for angle $\vec{\theta}$ and translation for vector (a, b, c) in the group $\mathcal{A}$ leads to translation of the point O_2 for vector $\overrightarrow{O_2O_2'} = (a(\cos\theta - 1) - b\sin\theta, a\sin\theta + b(\cos\theta - 1), 0)$. It just follows from

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

$$\cdot \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 0 & 0 & a' \\ 0 & 1 & 0 & b' \\ 0 & 0 & 1 & c' \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{25}$$

where $(a', b', c') = (a(\cos\theta - 1) - b\sin\theta, a\sin\theta + b(\cos\theta - 1), 0)$. This translation gives an angle of rotation

$$\overrightarrow{O_2O_1O_2'} = (a(\cos\theta - 1) - b\sin\theta, a\sin\theta + b(\cos\theta - 1), 0)/r.$$

The unadmitted translation leads to the Coulomb's acceleration/force

$$\mathbf{a} = \frac{c^2}{2}\mathrm{rot}\,\Big(\frac{1}{r}(a(\cos\theta - 1) - b\sin\theta, a\sin\theta + b(\cos\theta - 1), 0)\Big) = (\sin\theta)c^2\frac{\mathbf{r}}{r^2} \approx \theta c^2 \frac{\mathbf{r}}{r^2},$$

$$\mathbf{a} = \frac{e_1 e_2}{4\pi\epsilon_0 r^3 m_2}\mathbf{r} + O(c^{-4}), \quad \mathbf{f} = \frac{e_1 e_2}{4\pi\epsilon_0 r^3}\mathbf{r} + O(c^{-4}). \tag{26}$$

Hence the electric field caused by the first charged body is

$$\mathbf{E} = \frac{e_1}{4\pi\epsilon_0 r^3}\mathbf{r} + O(c^{-4}). \tag{27}$$

Analogously to the geodetic precession now it appears the following precession $\Omega = \frac{g}{2}(\mathbf{v}\times\mathbf{a})/c^2$, where g is the g-factor, and it also appears angular velocity $d\vec{\theta}/dt$.

6. Gravitational interaction

Analogously to the Axiom 3 for the charged bodies, in case of mass we give the following **Axiom 4**: *A point-mass body with mass M radially translates each point by 3-vector with magnitude GM/c^2.* It means that if a mass M is at the coordinate origin $O(0,0,0)$, then a point $A(x,y,z)$ will have new coordinates $(x,y,z)(1+\frac{GM}{rc^2})$, where $r = \sqrt{x^2+y^2+z^2}$. It is easy to see that this *gravitational translation* preserves the lengths of radial vectors, while a small vector $\mathbf{s}$ which is orthogonal to the radial direction transforms into $\mathbf{s}(1+\frac{GM}{rc^2})$.

This gravitational translation should be combined with translation for a small vector (a,b,c), and the non-commutativity leads to the gravitational acceleration. Let a point A has a radius vector $\mathbf{r} = (x,y,z)$. If we apply first translation for vector (a,b,c) and then gravitational translation, we obtain

$$A(x,y,z) \to (x+a, y+b, z+c) \to B\Big((x+a,y+b,z+c)\Big(1+\frac{GM}{r'c^2}\Big)\Big),$$

where $r' = r$ if we neglect the small lengths a,b,c. If we apply gravitational translation and then translation for vector (a,b,c), we obtain

$$A(x,y,z) \to (x,y,z)(1+\frac{GM}{rc^2}) \to B'\Big((x,y,z)\Big(1+\frac{GM}{rc^2}\Big) + (a,b,c)\Big).$$

The non-commutativity of both translations gives an oriented angle

$$\overrightarrow{\angle BOB'} = \frac{\overrightarrow{OB}\times\overrightarrow{OB'}}{|OB|\cdot|OB'|},$$

$$\overrightarrow{\angle BOB'} = -\frac{GM}{rc^2}\frac{(yc-zb, za-xc, xb-ya)}{r(r+GM/c^2)}.$$

Half of this angle is not admitted and it induces acceleration given by

$$\mathbf{g} = \frac{c^2}{2}\mathrm{rot}\overrightarrow{\angle BOB'} = -c^2\frac{GM}{rc^2}\frac{(x,y,z)}{r(r+GM/c^2)} = -\mathbf{r}\frac{GM}{r^2(r+GM/c^2)} \approx -\mathbf{r}\frac{GM}{r^3}. \tag{28}$$

The other half which is admitted induces the known precessions in gravitational field for moving test body.

In an incoming paper it will be shown that it leads to the same known predictions in gravitation, which are experimentally verified. The previous introduction of the gravitation is initiated by the following remark.

Remark. The 3-metric $g_{ij} = (1+\frac{2GM}{rc^2})\delta_{ij}$ usually interpretes that all small lengths on distance r are observed to be enlarged for coefficient $1+GM/(rc^2)$ from an observer far from massive bodies. If this is true, we come to a contradiction. Imagine a large number n of small balls whose centers $O_1, O_2, \cdots, O_n$ lie on the circle $x^2+y^2=r^2, z=0$, where at the coordinate origin $O(0,0,0)$ there is a mass M. Assume that for a local observer A on the mentioned circle each two neighbouring balls mutually touch. Then this is also true according to an observer far from the massive bodies. Indeed, both observers see that

$$\angle O_iOO_{i+1} = \angle O_{i+1}OO_i = \pi/2 - \pi/n, \tag{29}$$

for each $i = 1, 2, \cdots, n$, where $O_{n+1} = O_1$. Then according to (29), the observer far from the massive bodies sees that the radial distance r to be $r(1 + GM/(rc^2)) = r + GM/c^2$. Let us consider an observer A' on radial distance r' from the coordinate origin, where $r' \approx r$ and O, A and A' are collinear. Both observers A and A' say that the distance between them is $|r' - r|$. Also the observer far from the massive bodies says that this distance is equal $|(r' + \frac{GM}{c^2}) - (r + \frac{GM}{c^2})| = |r' - r|$, which is in a contradiction to our assumption.

In the General Relativity is using the isotropic coordinate ρ such that $r = \rho\Big(1 + \frac{GM}{2\rho c^2}\Big)^2$. If we neglect the terms of order c^{-4} we get $r = \rho + \frac{GM}{c^2}$, which leads to our definition of gravitation. Since this radial transformation does not preserve the Euclidean metric, we obtained the Newtonian acceleration, while the variation of speed of time is a further consequence, but not conversely.

7. Electro-weak and Gravity-weak interactions

Let us consider two charged particles with centers at O_1 and O_2, radii of range R_1 and R_2 and the coefficients k_1 and k_1^* have the same meaning as in section 3, and let (a, b, c) be a small vector for translation. Only the charged particles cause rotation, which was described in section 5. Thus we use the same steps as in the section 5, in order to obtain electro-weak interaction. The rotations remain unchanged in both cases as in section 5, and there is only change in the vector (a, b, c). Without loss of generality assume that the vector $\mathbf{r}$ is parallel to the z-axis, i.e. $x = y = 0$. Then according to Theorem 2, the vector (a, b, c) from the first particle is observed as $(k_1 a, k_1 b, k_1^* c)$. Although the basic group is G_s, the calculations are analogous as in $\mathcal{A}$. Analogously to section 5, in this case leads to translation for vector

$$\overrightarrow{O_2O_2}' = (k_1 a(\cos\theta - 1) - k_1 b \sin\theta, k_1 a \sin\theta + k_1 b(\cos\theta - 1), 0)$$

and it corresponds to angle $\angle\overrightarrow{O_2O_1O_2}' = (k_1 a(\cos\theta - 1) - k_1 b \sin\theta, k_1 a \sin\theta k_1 b(\cos\theta - 1), 0)/r$.

The unadmitted translation leads to the acceleration/force of the second body toward the first body

$$\mathbf{a} = \frac{v_0^2}{2}\mathrm{rot}\Big(\frac{1}{r}(k_1 a(\cos\theta - 1) - k_1 b \sin\theta, k_1 a \sin\theta + k_1 b(\cos\theta - 1), 0)\Big) = \frac{v_0^2}{c^2}\Big(\frac{R_1}{r}\sin\frac{r}{R_1}\Big)\frac{e_1 e_2}{4\pi\epsilon_0 r^3 m_2}\mathbf{r} + O(c^{-6}), \quad (30)$$

$$\mathbf{f} = \frac{v_0^2}{c^2}\Big(\frac{R_1}{r}\sin\frac{r}{R_1}\Big)\frac{e_1 e_2}{4\pi\epsilon_0 r^3}\mathbf{r} + O(c^{-6}). \quad (31)$$

Symmetrically, the force of the first charged body toward the second charged body is given by

$$\mathbf{f} = -\frac{v_0^2}{c^2}\Big(\frac{R_2}{r}\sin\frac{r}{R_2}\Big)\frac{e_1 e_2}{4\pi\epsilon_0 r^3}\mathbf{r} + O(c^{-6}). \quad (32)$$

If $R_1 \neq R_2$, then these two forces are not opposite, and the symmetry is broken now. In a special case, when the mutual distance r between the two charged bodies is very close to 0 and v_0 is close to c, i.e. in case of high energies, then the weak interaction leads to the electromagnetic interaction.

Analogously to the strong interaction, the following angular velocity appears now

$$\mathbf{w} = \frac{v_0}{c^2}\Big(\frac{R_1}{r}\sin\frac{r}{R_1}\Big)\frac{e_1 e_2}{4\pi\epsilon_0 r^3 m_2}\mathbf{r} + O(c^{-6}), \quad (33)$$

instead of $\Omega = \frac{g}{2}(\mathbf{v} \times \mathbf{a})/c^2$. The angular velocity $d\vec{\theta}/dt$ also appears, analogously to section 5.

Similarly to the electro-weak, we have also gravity-weak interaction. This is close to gravitation via the group $\mathcal{A}$, but the observer will be a particle with radius of range R_1. So we follow the section 6. We need to change the following coordinates there: x, y, z and also a and b should be multiplied by $k_1 = \frac{R_1}{r}\sin\frac{r}{R_1}$, while c should be multiplied by $k_1^* = \cos\frac{r}{R_1}$. It should be replaced into the coordinates of B and B', while both vectors $\overrightarrow{OB}$ and $\overrightarrow{OB'}$ should be divided by $(r + \frac{GM}{c^2})$ as in section 6. The calculation shows that the required acceleration is

$$\mathbf{g} = \frac{v_0^2}{2}\mathrm{rot}\angle\overrightarrow{BOB'} = -\frac{v_0^2}{c^2}\Big(\frac{R_1}{r}\sin\frac{r}{R_1}\Big)^2\frac{GM}{r^2(r+GM/c^2)}\mathbf{r} \approx -\frac{v_0^2}{c^2}\Big(\frac{R_1}{r}\sin\frac{r}{R_1}\Big)^2\frac{GM}{r^3}\mathbf{r}. \tag{34}$$

Analogously to the strong interaction, the following angular velocity appears

$$\mathbf{w} = -\frac{v_0}{c^2}\Big(\frac{R_1}{r}\sin\frac{r}{R_1}\Big)^2\frac{GM}{r^2(r+GM/c^2)}\mathbf{r} \approx -\frac{v_0}{c^2}\Big(\frac{R_1}{r}\sin\frac{r}{R_1}\Big)^2\frac{GM}{r^3}\mathbf{r}. \tag{35}$$

In case of gravity-weak interaction the symmetry is also broken as in case of electro-weak interaction.

The gravity-weak interaction is much smaller than the gravitational interaction and so it is not experimentally detected and it is unknown.

8. Some comments about future investigation

It is obvious that the strong interaction occurs in the group G_s, because otherwise it would not exist, since the translations in the affine group $\mathcal{A}$ commute. Close to the centers of the elementary particles the affine group $\mathcal{A}$, which requires flat Euclidean space, is probably not admitted and that is the reason for appearance of the electro-weak and gravity-weak interactions inside the group G_s. But on larger distance from the center of the particle, where the flat space is admitted and hence the affine group $\mathcal{A}$, there the gravitational and electromagnetic interactions can be realized. Then the gravity-weak and electro-weak interactions transform into gravitational and electromagnetic interaction respectively, while the strong interaction can not be transformed in anything. So it has influence to the distant stars in galaxies, where the black hole has its own radius of range. The border, where the affine group close to the particle is admitted, in future should be studied, and the quantum and wave effects should be considered. It is also needed to find an explicite formula or description for the local parameter v_0.

Analogous to the Dirac equation, in case of the group G_s should be considered a simpler equation, which will lead to the wave theory for the strong and weak interactions, including the particles which transfer these two interactions. For example, it is convenient to start from $\mathbf{a} = \pm v_0\mathbf{w}$ according to Theorem 1. Hence it follows that $a^2 = v_0^2 w^2$ and by multiplication with m^2d^2 where m is the mass of the particle and d is a distance, we obtain $E^2 = L^2w^2$ where $L = mdv_0$ is the angular momentum. Finally we may write the equation $(E - Lw)(E + Lw) = 0$. The sign $\pm$ in eq. $E = \pm Lw$ depends of the sign of the spin of the particle. Moreover, the equation $E = \hbar w = h\nu$ is just the energy of the photon.

This paper gives also some other possibilities for research, because this model is invariant if the gradient $(\partial F/\partial a, \partial F/\partial b, \partial F/\partial c)$ is added to the oriented angle $\vec{\varphi}$.

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