Filomat 37:9 (2023), 2647–2658
https://doi.org/10.2298/FIL2309647B

Published by Faculty of Sciences and Mathematics,
University of Niš, Serbia
Available at: http://www.pmf.ni.ac.rs/filomat

Soft interval spaces and soft interval sequence spaces

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Abstract. In this article, we present the two new kinds of examples of soft quasilinear spaces namely "$\widetilde{I\mathbb{R}^n}$ soft interval space" and "$\widetilde{Is}$, $\widetilde{Il_2}$, $\widetilde{Il_\infty}$ and $\widetilde{Ic_0}$ soft interval sequence spaces". We give some properties of these spaces. We study completeness of $\widetilde{I\mathbb{R}^n}$ soft normed quasilinear spaces. Further, we obtain some results on these soft interval spaces and soft interval sequence spaces related to concepts of soft quasilinear dependence-independence and solid-floored.

1. Introduction

In 1999, Molodtsov [1] introduced the soft set theory. Then he showed various applications of this theory on economics, engineering, medical science, etc. Then, Maji et al. [2], presented several operations on soft sets. After, Das and Samanta introduced the notions of soft element in [3], soft real number in [4]. After these studies, they worked on soft linear spaces, soft normed linear spaces , soft linear operators, soft inner product spaces and their some properties in [5], [6], [7] and [8]. Later, Yazar and et al. introduced the soft normed space in a new point of view, soft inner product space and soft Hilbert space on soft linear spaces respectively in [9], [10].

On the other side, in 1986, Aseev presented the notions of quasilinear spaces and normed quasilinear spaces in [11]. Then in [12], Levent and Yılmaz researched bounded quasilinear interval-valued functions and in [13] , they investigated new function space consists of set valued functions. After, Yılmaz and Bozkurt, in [14], [15], [16] and [17] introduced the concept of quasilinear inner product and presented some properties of quasilinear inner product spaces.

After these studies on soft linear spaces and quasilinear spaces, Bozkurt, in [18], introduced the notions of soft quasilinear spaces and soft normed quasilinear spaces. Afterwards, Bozkurt and Gönci, in [19], they given definitions of soft inner product quasilinear spaces and soft Hilbert quasilinear spaces. They worked on some properties of soft inner product quasilinear spaces.

In this paper, we presented the two new kinds of examples of soft quasilinear spaces namely "$\widetilde{I\mathbb{R}^n}$ soft interval space" and "$\widetilde{Is}$, $\widetilde{Il_2}$, $\widetilde{Il_\infty}$ and $\widetilde{Ic_0}$ soft interval sequence spaces". We given some properties of these spaces. We studied completeness of $\widetilde{I\mathbb{R}^n}$ soft normed quasilinear spaces. We showed that $\widetilde{I\mathbb{R}^n}$ is a soft inner product quasilinear space with the inner product we defined on $\widetilde{I\mathbb{R}^n}$. Further, we obtain some results on these soft interval spaces and soft interval sequence spaces related to concepts of soft quasilinear dependence-independence and solid-floored.

2020 *Mathematics Subject Classification*. 03E72, 08A72, 06F99, 08A02
Keywords. Soft set, soft quasilinear space, soft normed quasilinear space, soft interval space, soft interval sequence space
Received: 19 April 2022; Accepted: 16 August 2022
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2. Preliminaries

In this section, we will give some notions related to soft set theory and some basic notions such as soft quasilinear spaces, soft normed quasilinear spaces and soft inner product quasilinear spaces.

Let Q be an universe and P be a set of parameters, $P(Q)$ indicates the power set of Q and B be a non-empty subset of P.

Definition 2.1. *[1] A pair (G, P) is called a soft set over Q, where G is a mapping defined by $G : P \to P(Q)$.*

Definition 2.2. *[6] A soft set (G, P) over Q is said to be an absolute soft set represented by $\widetilde{Q}$, if for every $\lambda \in P$, $G(\lambda) = Q$. A soft set (G, P) over Q is said to be a null soft set represented by Φ, if for every $\lambda \in P$, $G(\lambda) = \emptyset$.*

Definition 2.3. *[4] Let Q be an non-empty set and P be a nonempty parameter set. Then a function $q : P \to Q$ is said to be soft element of Q. A soft element q of Q is said belongs to a soft set G of Q, which is denoted by $q \in Q$, if $q(\lambda) \in G(\lambda)$, $\lambda \in P$. So, for a soft set G of Q with respect to the index set P, we get $G(\lambda) = \left\{q(\lambda), \lambda \widetilde{\in} P\right\}$. A soft set (G, P) for which $G(\lambda)$ is a singleton set, $\forall \lambda \in P$ can be determined with a soft element by simply determined the singleton set with the element that it contains $\forall \lambda \in P$.*

The set of all soft sets (G, P) over Q will be described by $S\left(\widetilde{Q}\right)$ for which $G(\lambda) \neq \emptyset$, for all $\lambda \in P$ and the collection of all soft elements of (G, P) over Q will be denoted by $SE\left(\widetilde{Q}\right)$.

Now let's give a definition that is meaningful in soft quasilinear spaces that are not in soft linear spaces.

Definition 2.4. *[18] Let Q be a quasilinear space and P be a parameter set. Let G be a soft set over (Q, P). G is said to be a soft quasilinear space of Q if $Q(\lambda)$ is a quasilinear subspace of Q for every $\lambda \in P$.*

[18] We use the notation $\widetilde{q}, \widetilde{w}, \widetilde{z}$ to indicate soft quasi vectors of a soft quasilinear space and $\widetilde{a}, \widetilde{b}, \widetilde{c}$ to specify soft real numbers. If a soft quasi element $\widetilde{q}$ has an inverse i.e. $\widetilde{q} - \widetilde{q} = \overline{\theta}$ such that $\widetilde{q}(\lambda) - \widetilde{q}(\lambda) = \overline{\theta}(\lambda)$ for every $\lambda \in P$ then it is called regular. If a soft quasi element $\widetilde{q}$ has no inverse, then it is called singular. Also, $\widetilde{Q}_r$ express for the set of all soft regular elements in $\widetilde{Q}$ and $\widetilde{Q}_s$ imply the sets of all soft singular elements in $\widetilde{Q}$.

Definition 2.5. *[18] Let $\widetilde{Q}$ be the absolute soft quasilinear space i.e. $\widetilde{Q}(\lambda) = Q$ for every $\lambda \in P$. Then a mapping $\|.\| : SE\left(\widetilde{Q}\right) \longrightarrow \mathbb{R}(P)$ is said to be soft norm on the soft quasilinear space $\widetilde{Q}$, if $\|.\|$ satisfies the following conditions:*

i) $\|\widetilde{q}\| \widetilde{\geq} 0$ if $\widetilde{q} \neq \overline{\theta}$ for every $\widetilde{q} \in \widetilde{Q}$,

ii) $\|\widetilde{q} + \widetilde{w}\| \widetilde{\leq} \|\widetilde{q}\| + \|\widetilde{w}\|$ for every $\widetilde{q}, \widetilde{w} \in \widetilde{Q}$,

iii) $\|\widetilde{\alpha} \cdot \widetilde{q}\| = |\widetilde{\alpha}| \cdot \|\widetilde{q}\|$ for every $\widetilde{q} \in \widetilde{Q}$ and for every soft scalar $\widetilde{\alpha}$,

iv) if $\widetilde{q} \widetilde{\preceq} \widetilde{w}$, then $\|\widetilde{q}\| \widetilde{\leq} \|\widetilde{w}\|$ for every $\widetilde{q}, \widetilde{w} \in \widetilde{Q}$,

v) if for any $\varepsilon \widetilde{>} 0$ there exists an element $\widetilde{q}_\varepsilon \in \widetilde{Q}$ such that, $\widetilde{q} \widetilde{\preceq} \widetilde{w} + \widetilde{q}_\varepsilon$ and $\|\widetilde{q}_\varepsilon\| \widetilde{\leq} \varepsilon$ then $\widetilde{q} \widetilde{\preceq} \widetilde{w}$ for any soft elements $\widetilde{q}, \widetilde{w} \in \widetilde{Q}$.

A soft quasilinear space $\widetilde{Q}$ with a soft norm$\|.\|$ on $\widetilde{Q}$ is called soft normed quasilinear space and is indicated by $\left(\widetilde{Q}, \|.\|\right)$ or $\left(\widetilde{Q}, \|.\|, P\right)$.

Theorem 2.6. *[19] If a soft norm $\|.\|$ on soft normed quasilinear space $\widetilde{Q}$ satisfied the condition " $\xi \in Q$, and $\lambda \in P$, $\left\{\|\widetilde{q}\|(\lambda) = \xi\right\}$ is a singleton set.". If for every $\lambda \in P$, $\|.\|_\lambda : Q \to \mathbb{R}^+$ be a mapping such that for every $\xi \in Q$, $\|\xi\|_\lambda = \|\widetilde{q}\|(\lambda)$, where $\widetilde{q} \in \widetilde{Q}$ such that $\widetilde{q}(\lambda) = \xi$. Then for every $\lambda \in P$, $\|.\|_\lambda$ is a norm on quasilinear space Q.*

Let $\widetilde{Q}$ be a soft normed quasilinear space. Then, soft Hausdorff or soft norm metric on $\widetilde{Q}$ is defined by

$$h_Q(\widetilde{q}, \widetilde{w}) = \inf\left\{\widetilde{r} \widetilde{\geq} 0 : \widetilde{q} \widetilde{\preceq} \widetilde{w} + \widetilde{q}_1^r, \widetilde{w} \widetilde{\preceq} \widetilde{q} + \widetilde{q}_2^r, \|\widetilde{q}_i^r\| \widetilde{\leq} \widetilde{r}\right\}.$$

Definition 2.7. *[18] Let $\widetilde{Q}$ be a soft quasilinear space. $(\widetilde{q_k})_{k=1}^{n} \subset \widetilde{Q}$ and $(\widetilde{\alpha_k})_{k=1}^{n}$ are soft scalars. If*

$$\widetilde{\theta}\widetilde{\preceq}\widetilde{\alpha_1}\cdot\widetilde{q_1}+\widetilde{\alpha_2}\cdot\widetilde{q_2}+...+\widetilde{\alpha_n}\cdot\widetilde{q_n}$$

implies $\widetilde{\alpha_1}=\widetilde{\alpha_2}=...=\widetilde{\alpha_n}=\widetilde{0}$, then $(\widetilde{q_k})_{k=1}^{n}$ is said to be quasilinear independent, otherwise $(\widetilde{q_k})_{k=1}^{n}$ is said to be quasilinear dependent.

Definition 2.8. *[19] Let $\widetilde{Q}$ be a soft quasilinear space, $\widetilde{W}\subseteq\widetilde{Q}$ and $\widetilde{q}\in\widetilde{W}$. The set*

$$F_{\widetilde{q}}^{\widetilde{W}}=\{\widetilde{m}\in\widetilde{W}_r:\widetilde{m}\widetilde{\preceq}\widetilde{q}\},$$

is called floor in $\widetilde{W}$ of $\widetilde{q}$. If $\widetilde{W}=\widetilde{Q}$ then we will say only floor of $\widetilde{q}$ and written shortly $F_{\widetilde{q}}$ instead of $F_{\widetilde{q}}^{\widetilde{Q}}$.

Definition 2.9. *[19] Let $\widetilde{Q}$ be a soft quasilinear space, $\widetilde{Q}$ is called a solid floored soft quasilinear space whenever*

$$\widetilde{q}=\sup\{\widetilde{m}\in\widetilde{W}_r:\widetilde{m}\widetilde{\preceq}\widetilde{q}\}$$

for every $\widetilde{q}\in\widetilde{Q}$. Otherwise, $\widetilde{Q}$ is called a non-solid floored soft quasilinear space.

Theorem 2.10. *[19] Absolute soft quasilinear space $\left(\widetilde{\Omega_C(\mathbb{R})}\right)$ is a solid floored.*

Definition 2.11. *[19] Let $\widetilde{Q}$ be a soft quasilinear space. Consolidation of floor of $\widetilde{Q}$ is the smallest solid floored soft quasilinear space $\widehat{(\widetilde{Q})}$ containing $\widetilde{Q}$, namely, if there exists different solid floored soft quasilinear space $\widetilde{W}$ including $\widetilde{Q}$, then $\widehat{(\widetilde{Q})}\widetilde{\subseteq}\widetilde{W}$.*

Definition 2.12. *[19] Let $\widetilde{Q}$ be the absolute soft quasilinear space i.e. $\widetilde{Q}(\lambda)=Q$, $\forall\lambda\in P$. Then a mapping*

$$\langle.\rangle:SE\left(\widetilde{Q}\right)\times SE\left(\widetilde{Q}\right)\to\Omega(\mathbb{R})(P)$$

is said to be a soft quasi inner product on the soft quasilinear space $\widetilde{Q}$, if $\langle.\rangle$ satisfies the following conditions:

i) $\langle\widetilde{q},\widetilde{w}\rangle\in(\Omega(\mathbb{R}))_r\equiv\widetilde{\mathbb{R}}$ if $\widetilde{q},\widetilde{w}\in\widetilde{Q}_r$,

ii) $\langle\widetilde{q}+\widetilde{w},\widetilde{z}\rangle\widetilde{\subseteq}\langle\widetilde{q},\widetilde{z}\rangle+\langle\widetilde{w},\widetilde{z}\rangle$ for all $\widetilde{q},\widetilde{w},\widetilde{z}\in\widetilde{Q}$,

iii) $\langle\widetilde{\alpha}\cdot\widetilde{q},\widetilde{w}\rangle=\widetilde{\alpha}\cdot\langle\widetilde{q},\widetilde{w}\rangle$ for all $\widetilde{q},\widetilde{w}\in\widetilde{Q}$ and for every soft scalar $\widetilde{\alpha}$,

iv) $\langle\widetilde{q},\widetilde{w}\rangle=\langle\widetilde{w},\widetilde{q}\rangle$ for all $\widetilde{q},\widetilde{w}\in\widetilde{Q}$,

v) $\langle\widetilde{q},\widetilde{w}\rangle\widetilde{\supseteq}\widetilde{0}$ if $\widetilde{q}\in\widetilde{Q}_r$ and $\langle\widetilde{q},\widetilde{q}\rangle=\left\{\widetilde{0}\right\}\Leftrightarrow\widetilde{q}=\{\theta\}$,

vi) $\left\|\langle\widetilde{q},\widetilde{w}\rangle\right\|_{\Omega(\mathbb{R})}=\sup\left\{\left\|\langle x,y\rangle\right\|:x\in F_{\widetilde{q}}^{\widehat{(\widetilde{Q})}},y\in F_{\widetilde{w}}^{\widehat{(\widetilde{Q})}}\right\}$,

vii) $\langle\widetilde{q},\widetilde{w}\rangle\widetilde{\subseteq}\langle\widetilde{z},\widetilde{v}\rangle$ if $\widetilde{q}\widetilde{\preceq}\widetilde{z}$ and $\widetilde{w}\widetilde{\preceq}\widetilde{v}$ for all $\widetilde{q},\widetilde{w},\widetilde{z},\widetilde{v}\in\widetilde{Q}$,

viii) $\forall\widetilde{\epsilon}\widetilde{\geq}0$, $\exists\widetilde{q_{\widetilde{\epsilon}}}\in\widetilde{Q}$ such that $\widetilde{q}\widetilde{\preceq}\widetilde{w}+\widetilde{q_{\widetilde{\epsilon}}}$ and $\langle\widetilde{q_{\widetilde{\epsilon}}},\widetilde{q_{\widetilde{\epsilon}}}\rangle\widetilde{\subseteq}S_{\widetilde{\epsilon}}(\theta)$ then $\widetilde{q}\widetilde{\preceq}\widetilde{w}$.

A soft quasilinear space $\widetilde{Q}$ with a soft quasi inner product $\langle.\rangle$ on $\widetilde{Q}$ is called a soft quasilinear inner product space and denoted by $\left(\widetilde{Q},\langle.\rangle,P\right)$.

Remark 2.13. *If $\widetilde{Q}$ is a soft linear space, then above conditions are determined by conditions of the real soft inner product spaces. Moreover, a regular subspace $\widetilde{Q}_r$ of a soft quasilinear inner product space $\widetilde{Q}$ is a soft (linear) inner product space with the same inner product.*

Definition 2.14. *[19] A soft quasi vector $\widetilde{q}$ of soft quasilinear inner product space $\widetilde{Q}$ is said to be orthogonal to soft quasi element $\widetilde{w} \in \widetilde{Q}$ if*

$$\left\| \langle \widetilde{q}, \widetilde{w} \rangle \right\|_{\Omega(\mathbb{R})} = \overline{0}.$$

It is also denoted by $\widetilde{q} \perp \widetilde{w}$. Let $\widetilde{M}$ be a non-null soft quasi subset of soft quasilinear inner product space $\widetilde{Q}$ such that $\widetilde{M}(\lambda) \neq \emptyset$ for every $\lambda \in P$. If a soft quasi vector $\widetilde{q}$ of soft quasilinear inner product space $\widetilde{Q}$ orthogonal to every soft quasi vectors of $\widetilde{M}$, then we say that $\widetilde{q}$ is orthogonal to $\widetilde{M}$ and we write $\widetilde{q} \perp \widetilde{M}$. A non-null orthonormal soft quasi subset $\widetilde{M}$ of soft quasilinear inner product space $\widetilde{Q}$ such that $\widetilde{M}(\lambda) \neq \emptyset$ for every $\lambda \in P$ is a orthogonal soft quasi subset in $\widetilde{Q}$ whose soft quasi vectors have norm $\overline{1}$; that is, for all $\widetilde{q}, \widetilde{w} \in \widetilde{M}$

$$\left\| \langle \widetilde{q}, \widetilde{w} \rangle \right\|_{\Omega(\mathbb{R})} = \begin{cases} \overline{0}, & \widetilde{q} = \widetilde{w} \\ \overline{1}, & \widetilde{q} \neq \widetilde{w} \end{cases}.$$

3. Main Results

This section introduces the two new examples of soft quasillinear spaces as well as some properties of them. The first one is soft interval spaces and other one is soft interval sequence spaces.

Let $I\mathbb{R}$ is the whole compact-convex subset of real numbers and $\widetilde{I\mathbb{R}}$ be a set of all soft quasi vectors defined from parameter set M to $I\mathbb{R}$ i.e.

$$\widetilde{I\mathbb{R}} = \{\widetilde{q} : \widetilde{q}(\lambda) = q \in I\mathbb{R},\ \lambda \in M\}.$$

So, a set of two dimensional soft quasi vectors is represented by

$$\widetilde{I\mathbb{R}^2} = \{\widetilde{q} = (\widetilde{q_1}, \widetilde{q_2}) : \widetilde{q}(\lambda) = (\widetilde{q_1}(\lambda), \widetilde{q_2}(\lambda)) = (q_1, q_2) \in I\mathbb{R}^2,\ \lambda \in M\}.$$

Thus, we can define a set of n- dimensional soft quasi vectors by

$$\widetilde{I\mathbb{R}^n} = \left\{ \begin{array}{c} (\widetilde{q}) = (\widetilde{q_1}, \widetilde{q_2}, ..., \widetilde{q_n}) : \\ \widetilde{q}(\lambda) = (\widetilde{q_1}(\lambda), \widetilde{q_2}(\lambda), ..., \widetilde{q_n}(\lambda)) = (q_1, q_2, ..., q_n) \in I\mathbb{R}^n,\ \lambda \in M \end{array} \right\}.$$

We know that $I\mathbb{R}^n$ is an example of quasilinear spaces from [16].

Theorem 3.1. *Let $\widetilde{q}, \widetilde{w} \in \widetilde{I\mathbb{R}^n}$. The set $\widetilde{I\mathbb{R}^n}$ is a soft quasilinear space according to the following operations and partial order relation;*

i) $(\widetilde{q} + \widetilde{w})(\lambda) = (\widetilde{q})(\lambda) + (\widetilde{w})(\lambda)$,
ii) $(\widetilde{\alpha} \cdot \widetilde{q})(\lambda) = \widetilde{\alpha}(\lambda) \cdot (\widetilde{q})(\lambda)$ for every soft scalar $\widetilde{\alpha}$,
iii) $\widetilde{q} \widetilde{\preceq} \widetilde{w} \Leftrightarrow \widetilde{q_i} \widetilde{\preceq} \widetilde{w_i} \Leftrightarrow (\widetilde{q_i})(\lambda) \leq_{I\mathbb{R}} (\widetilde{w_i})(\lambda)$ for every $\lambda \in M$.
From Theorem 3.1, we get that $\widetilde{I\mathbb{R}^n}$ is a different example of soft quasilinear spaces.

Remark 3.2. *The soft quasilinear space $\widetilde{I\mathbb{R}^n}$ and $\widetilde{\Omega_C(\mathbb{R}^n)}$ are different from each other for $n \neq 1$. For example;*

$$\begin{array}{ccl} \widetilde{q} & : & M \to \Omega_C(\mathbb{R}^2) \\ \lambda & \to & \widetilde{q}(\lambda) = \{(a,b) : a = 0, -2 \leq b \leq 2\} \end{array}$$

is a soft quasi element of $\widetilde{\Omega_C(\mathbb{R}^2)}$, but it is not soft quasi vector of $\widetilde{I\mathbb{R}^2}$. Also,

$$\begin{array}{ccl} \widetilde{w} & : & M \to I\mathbb{R}^2 \\ \lambda & \to & \widetilde{w}(\lambda) = \{(a,b) : a = [1,2], b = [-2,3]\} \end{array}$$

is a soft quasi element of $\widetilde{I\mathbb{R}^2}$ but it is not soft quasi vector of $\widetilde{\Omega_C(\mathbb{R}^2)}$.

Example 3.3. *Let $\widetilde{q},\widetilde{w}\in\widetilde{I\mathbb{R}^3}$ and λ is a parameter such that*
$\widetilde{q}(\lambda)=(\widetilde{q_1}(\lambda),\widetilde{q_2}(\lambda),\widetilde{q_3}(\lambda))=([-1,2],[0,1],\{-1\})$
$\widetilde{w}(\lambda)=(\widetilde{w_1}(\lambda),\widetilde{w_2}(\lambda),\widetilde{w_3}(\lambda))=(\{1\},\{0\},\{-1\})$.
$\widetilde{w}\in\left(\widetilde{I\mathbb{R}^2}\right)_r$ *because of* $(\widetilde{w}-\widetilde{w})(\lambda)=(\widetilde{w_1}(\lambda)-\widetilde{w_1}(\lambda),\widetilde{w_2}(\lambda)-\widetilde{w_2}(\lambda),\widetilde{w_3}(\lambda)-\widetilde{w_3}(\lambda))=(\{0\},\{0\},\{0\})=\widetilde{\emptyset}$. *But,* $\widetilde{q}\in\left(\widetilde{I\mathbb{R}^2}\right)_s$ *since* $(\widetilde{q}-\widetilde{q})(\lambda)=(\widetilde{q_1}(\lambda)-\widetilde{q_1}(\lambda),\widetilde{q_2}(\lambda)-\widetilde{q_2}(\lambda),\widetilde{q_3}(\lambda)-\widetilde{q_3}(\lambda))=([-3,3,[-1,1],\{0\})\neq\widetilde{\emptyset}$. *Further, for every parameter λ we get $\widetilde{w}\widetilde{\preceq}\widetilde{q}$ because of* $(\widetilde{w})(\lambda)\subseteq(\widetilde{q})(\lambda)$.

Theorem 3.4. $\widetilde{I\mathbb{R}^n}$ *is a soft normed quasilinear space with*

$$\left\|\widetilde{q}\right\|(\lambda)=\sup_{1\leq i\leq n}\left\|\widetilde{q_i}(\lambda)\right\|_{\widetilde{I\mathbb{R}}} \tag{1}$$

for every $\widetilde{q}\in\widetilde{I\mathbb{R}^n}$ and parameter λ.

Proof. Clearly, $\left\|\widetilde{q}\right\|\widetilde{\geq}\widetilde{0}$ since $\left\|\widetilde{q_i}\right\|_{\widetilde{I\mathbb{R}}}\widetilde{\geq}\widetilde{0}$. If $\left\|\widetilde{q}\right\|=\widetilde{0}$, then we get $\sup_{1\leq i\leq n}\left\|\widetilde{q_i}\right\|_{\widetilde{I\mathbb{R}}}=\widetilde{0}$. This gives $\widetilde{q_i}(\lambda)=q_i=0$ for every λ parameters and $1\leq i\leq n$. So, we obtain $\widetilde{q}=\widetilde{\theta}$. On the other hand, if $\widetilde{q}=\widetilde{\theta}$, the we get $\widetilde{q_i}=\widetilde{\theta}$ for every $1\leq i\leq n$. Thus, we obtain $\left\|\widetilde{q}\right\|=\widetilde{0}$ because of $\widetilde{I\mathbb{R}}$ is a soft normed quasilinear space with norm $\|.\|_{\widetilde{I\mathbb{R}}}$. For every $\widetilde{q},\widetilde{w}\in\widetilde{I\mathbb{R}^n}$, we find $\left\|\widetilde{q}+\widetilde{w}\right\|=\sup_{1\leq i\leq n}\left\|\widetilde{q_i}+\widetilde{w_i}\right\|_{\widetilde{I\mathbb{R}}}\widetilde{\leq}\sup_{1\leq i\leq n}\left\|\widetilde{q_i}\right\|_{\widetilde{I\mathbb{R}}}+\sup_{1\leq i\leq n}\left\|\widetilde{w_i}\right\|_{\widetilde{I\mathbb{R}}}=\left\|\widetilde{q}\right\|+\left\|\widetilde{w}\right\|$. Clearly, $\left\|\widetilde{\alpha}\cdot\widetilde{q}\right\|=\widetilde{\alpha}\cdot\left\|\widetilde{q}\right\|$ for every $\widetilde{q},\widetilde{w}\in\widetilde{I\mathbb{R}^n}$. If $\widetilde{q}\widetilde{\preceq}\widetilde{w}$ for $\widetilde{q},\widetilde{w}\in\widetilde{I\mathbb{R}^n}$, then we have $\widetilde{q_i}\widetilde{\preceq}\widetilde{w_i}$ for every $1\leq i\leq n$ and $\widetilde{q}=(\widetilde{q_1},\widetilde{q_2},...,\widetilde{q_n})\in\widetilde{I\mathbb{R}^n}$, $\widetilde{w}=(\widetilde{w_1},\widetilde{w_2},...,\widetilde{w_n})\in\widetilde{I\mathbb{R}^n}$. Since $\widetilde{I\mathbb{R}}$ is a soft normed quasilinear space with $\left\|\widetilde{q_i}\right\|_{\widetilde{I\mathbb{R}}}=\sup_{a\in\widetilde{q_1}(\lambda)}|a|$ for a parameter λ, we obtain $\left\|\widetilde{q_i}\right\|_{\widetilde{I\mathbb{R}}}\widetilde{\leq}\left\|\widetilde{w_i}\right\|_{\widetilde{I\mathbb{R}}}$. This gives $\left\|\widetilde{q}\right\|\widetilde{\leq}\left\|\widetilde{w}\right\|$. Let for any $\widetilde{\epsilon}\widetilde{\geq}\widetilde{0}$ there exists a soft quasi vector $\widetilde{q_\epsilon}\in\widetilde{I\mathbb{R}^n}$ such that $\widetilde{q}\widetilde{\preceq}\widetilde{w}+\widetilde{q_\epsilon}$ and $\left\|\widetilde{q_\epsilon}\right\|\widetilde{\leq}\widetilde{\epsilon}$. From Theorem 3.1 and definition of norm, we get $\widetilde{q_i}\widetilde{\preceq}\widetilde{w_i}+\widetilde{q_{\epsilon_i}}$ and $\left\|\widetilde{q_{\epsilon_i}}\right\|_{\widetilde{I\mathbb{R}}}\widetilde{\leq}\widetilde{\epsilon}$ for every $1\leq i\leq n$. Thus, we find $\widetilde{q_i}\widetilde{\preceq}\widetilde{w_i}$ for every $1\leq i\leq n$ since $\widetilde{I\mathbb{R}}$ is a soft normed quasilinear space with norm $\|.\|_{\widetilde{I\mathbb{R}}}$. This gives $\widetilde{q}\widetilde{\preceq}\widetilde{w}$. □

Example 3.5. *Let $\mathbb{R}$ be a parameter set and $\widetilde{q},\widetilde{w}\in\widetilde{I\mathbb{R}^2}$ such that*

$$\begin{aligned}\widetilde{q}\quad &: \quad \mathbb{R}\to I\mathbb{R}^2\\ n\quad &\to \quad \widetilde{q}(n)=([-n,n],[0,n])\end{aligned}$$

and

$$\begin{aligned}\widetilde{w}\quad &: \quad \mathbb{R}\to I\mathbb{R}^2\\ n\quad &\to \quad \widetilde{w}(n)=(\{n\},\{-n\}).\end{aligned}$$

From here, we find $\left\|\widetilde{q}\right\|(n)=\sup_{1\leq i\leq 2}\left\|\widetilde{q_i}(n)\right\|_{\widetilde{I\mathbb{R}}}=\sup\{n,n\}=n$ *and* $\left\|\widetilde{w}\right\|(n)=\sup_{1\leq i\leq 2}\left\|\widetilde{w_i}(n)\right\|_{\widetilde{I\mathbb{R}}}=\sup\{n,n\}=n$.

The soft Hausdorff metric on $\widetilde{I\mathbb{R}^n}$ is defined by

$$h_{\widetilde{I\mathbb{R}^n}}(\widetilde{q},\widetilde{w})=\inf\left\{\widetilde{\epsilon}\widetilde{\geq}\widetilde{0}:\widetilde{q}\widetilde{\preceq}\widetilde{w}+\widetilde{q_1^{\epsilon}},\ \widetilde{w}\widetilde{\preceq}\widetilde{q}+\widetilde{q_1^{\epsilon}},\ \left\|\widetilde{q_j^{\epsilon}}\right\|\widetilde{\leq}\widetilde{\epsilon},\ j=1,2\right\}$$

for every $\widetilde{q},\widetilde{w}\in\widetilde{I\mathbb{R}^n}$.

Theorem 3.6. *The soft normed quasilinear space $\widetilde{I\mathbb{R}^n}$ is a soft Banach space with (1) norm.*

Proof. Let $\widetilde{q^n}$ be a Cauchy sequence in $\widetilde{I\mathbb{R}^n}$ such that for every $\widetilde{\epsilon}\widetilde{\geq}\widetilde{0}$ there exists $n_0 \in \mathbb{N}$ such that for every $n, m > n_0$, we find

$$\widetilde{q^n}\widetilde{\leq}\widetilde{q^m} + \widetilde{q^{\epsilon}_{1n}},\ \widetilde{q^m}\widetilde{\leq}\widetilde{q^n} + \widetilde{q^{\epsilon}_{2n}},\ \left\|\widetilde{q^{\epsilon}_{jn}}\right\|\widetilde{\leq}\widetilde{\epsilon}.$$

For every parameter λ and for every $1 \leq i \leq n$, we obtain

$$\widetilde{q^n_i}(\lambda)\widetilde{\leq}\widetilde{q^m_i}(\lambda) + \left(\widetilde{q^{\epsilon}_{1n}}\right)_i(\lambda),\ \widetilde{q^m_i}(\lambda)\widetilde{\leq}\widetilde{q^n_i}(\lambda) + \left(\widetilde{q^{\epsilon}_{2n}}\right)_i(\lambda) \tag{2}$$

and $\left\|\widetilde{q^{\epsilon}_{jn}}\right\|(\lambda) = \sup\left\|\left(\widetilde{q^{\epsilon}_{jn}}\right)_i(\lambda)\right\|\widetilde{\leq}\widetilde{\epsilon}(\lambda)$ from (1). So, we get $h_{\widetilde{I\mathbb{R}}}(\widetilde{q^n_i},\widetilde{q^n_i})\widetilde{\leq}\widetilde{\epsilon}$ for every $i \in [1, n]$. This gives $\widetilde{q^n_i}$ is a Cauchy sequence in $\widetilde{I\mathbb{R}}$. Since $\widetilde{I\mathbb{R}}$ is a complete, there is a $\widetilde{q_i} \in \widetilde{I\mathbb{R}}$ such that $h_{\widetilde{I\mathbb{R}}}(\widetilde{q^n_i},\widetilde{q_i})\widetilde{\leq}\widetilde{\epsilon}$ and $\widetilde{q} = (\widetilde{q_1},\widetilde{q_2},...,\widetilde{q_n}) \in \widetilde{I\mathbb{R}^n}$ because of $\widetilde{q_i} \in \widetilde{I\mathbb{R}}$ for every $i \in [1, n]$. If we take $m \to \infty$ at (2), for every $\widetilde{\epsilon}\widetilde{\geq}\widetilde{0}$ there exists $n_0 \in \mathbb{N}$ such that for every $n > n_0$, we have

$$\widetilde{q^n_i}(\lambda)\widetilde{\leq}\widetilde{q_i}(\lambda) + \left(\widetilde{q^{\epsilon}_{1n}}\right)_i(\lambda),\ \widetilde{q_i}(\lambda)\widetilde{\leq}\widetilde{q^n_i}(\lambda) + \left(\widetilde{q^{\epsilon}_{2n}}\right)_i(\lambda),\ \left\|\left(\widetilde{q^{\epsilon}_{jn}}\right)_i(\lambda)\right\|\widetilde{\leq}\widetilde{\epsilon}(\lambda).$$

From (1), we find $\left\|\widetilde{q^{\epsilon}_{jn}}\right\|\widetilde{\leq}\widetilde{\epsilon}$. Further, we get

$$\widetilde{q^n}\widetilde{\leq}\widetilde{q} + \widetilde{q^{\epsilon}_{1n}},\ \widetilde{q}\widetilde{\leq}\widetilde{q^n} + \widetilde{q^{\epsilon}_{2n}}.$$

This gives, $\widetilde{q^n}$ converges to $\widetilde{q}$ in $\widetilde{I\mathbb{R}^n}$. So, we can say $\widetilde{I\mathbb{R}^n}$ is a soft Banach space with (1) norm.

Definition 3.7. *(Soft $\Omega-$Space) Let $\widetilde{Q}$ be a soft normed quasilinear space. If*

$$\left\|\widetilde{q}\right\| \widetilde{\leq} \left\|\widetilde{B}_{\widetilde{Q}}\right\| \Rightarrow \widetilde{q}\widetilde{\leq}\widetilde{B}_{\widetilde{Q}}$$

satisfy for an element $\widetilde{B}_{\widetilde{Q}} \neq \widetilde{\theta}$, then we say that $\widetilde{Q}$ is a soft $\Omega-$space.

Theorem 3.8. *$\widetilde{I\mathbb{R}^n}$ is a soft Ω-space with norm (1).*

Proof. Let $\widetilde{B}_{\widetilde{q}} = \{\widetilde{w}\}$ and $\widetilde{w}(\lambda) = ([-2, 2], [-2, 2], ..., [-2, 2]) \in I\mathbb{R}^n$. Then, we obtain $\left\|\widetilde{q}\right\| = \left\|\widetilde{q}\right\|(\lambda) = \sup_{1\leq i\leq n}\left\|\widetilde{q_i}(\lambda)\right\|_{\widetilde{I\mathbb{R}}} \widetilde{\leq} \left\|\widetilde{B_{\widetilde{q}}}\right\|(\lambda) = \widetilde{2}$ from definition of soft $\Omega-$space. So, we have $\left\|\widetilde{q_i}\right\|_{\widetilde{I\mathbb{R}}} \widetilde{\leq} 2$ for every $i \in [1, n]$. This gives $\widetilde{q_i}(\lambda)\widetilde{\subseteq}[-2, 2] = \widetilde{B}_{\widetilde{q_i}}(\lambda)$. Thus, $\widetilde{I\mathbb{R}^n}$ is a soft Ω-space.

Theorem 3.9. *The soft normed quasilinear space $\widetilde{I\mathbb{R}^n}$ is a soft quasilinear inner product space with*

$$\langle\widetilde{q},\widetilde{w}\rangle = \sum_{i=1}^{n}\langle\widetilde{q_i},\widetilde{w_i}\rangle_{\widetilde{I\mathbb{R}}}. \tag{3}$$

Proof. Clearly, $\langle\widetilde{q_i},\widetilde{w_i}\rangle_{\widetilde{I\mathbb{R}}} \in \Omega(\mathbb{R})$ for every $i \in [1, n]$. For every $\widetilde{q},\widetilde{w},\widetilde{z} \in \widetilde{I\mathbb{R}^n}$, we get

$$\begin{aligned}
\langle\widetilde{q}+\widetilde{w},\widetilde{z}\rangle &= \sum_{i=1}^{n}\langle\widetilde{q_i}+\widetilde{w_i},\widetilde{z_i}\rangle_{\widetilde{I\mathbb{R}}} \\
&\widetilde{\leq} \sum_{i=1}^{n}\langle\widetilde{q_i},\widetilde{z_i}\rangle_{\widetilde{I\mathbb{R}}} + \sum_{i=1}^{n}\langle\widetilde{w_i},\widetilde{z_i}\rangle_{\widetilde{I\mathbb{R}}} \\
&= \langle\widetilde{q},\widetilde{z}\rangle + \langle\widetilde{w},\widetilde{z}\rangle
\end{aligned}$$

because of $\widetilde{I\mathbb{R}}$ *is a soft inner product space with soft inner product* $\langle , \rangle_{\widetilde{I\mathbb{R}}}$. *Also, we find* $\langle \widetilde{\alpha} \cdot \widetilde{q}, \widetilde{w} \rangle = \sum_{i=1}^{n} \langle \widetilde{\alpha} \cdot \widetilde{q_i}, \widetilde{w_i} \rangle_{\widetilde{I\mathbb{R}}} = \widetilde{\alpha} \cdot \sum_{i=1}^{n} \langle \widetilde{q_i}, \widetilde{w_i} \rangle_{\widetilde{I\mathbb{R}}} = \widetilde{\alpha} \cdot \langle \widetilde{q}, \widetilde{w} \rangle$. *Clearly, if* $\langle \widetilde{q}, \widetilde{q} \rangle = \sum_{i=1}^{n} \langle \widetilde{q_i}, \widetilde{q_i} \rangle_{\widetilde{I\mathbb{R}}} = \widetilde{0}$, *then* $\widetilde{q} = \widetilde{\theta}$. *The converse is similarly true. If we take* $\widetilde{q} \in \left(\widetilde{I\mathbb{R}^n}\right)_r$, *then we get* $\langle \widetilde{q}, \widetilde{q} \rangle = \sum_{i=1}^{n} \langle \widetilde{q_i}, \widetilde{q_i} \rangle_{\widetilde{I\mathbb{R}}} \widetilde{\geq} \widetilde{0}$. *For every* $\widetilde{q}, \widetilde{w} \in \widetilde{I\mathbb{R}^n}$, *we get*

$$\begin{aligned} \left\| \langle \widetilde{q}, \widetilde{w} \rangle \right\|_{\widetilde{\Omega(\mathbb{R})}} (\lambda) &= \sup \{ |x| : x \in \langle \widetilde{q}, \widetilde{w} \rangle (\lambda) \} \\ &= \sup \left\{ |x| : x \in \sum_{i=1}^{n} \langle \widetilde{q_i}, \widetilde{w_i} \rangle_{\widetilde{I\mathbb{R}}} (\lambda) \right\} \\ &= \sup \left\{ |x| : x \in \sum_{i=1}^{n} \{ ab : a \in \widetilde{q_i}(\lambda), b \in \widetilde{w_i}(\lambda) \} \right\} \\ &= \sup \left\{ \left| \sum_{i=1}^{n} ab \right| : a \in F_{\widetilde{q_i}(\lambda)}, b \in F_{\widetilde{w_i}(\lambda)} \right\} \\ &= \sup \left\{ \| \langle a, b \rangle \|_{\widetilde{\Omega(\mathbb{R})}} : a \in F^{\left(\widetilde{I\mathbb{R}^n}\right)}_{\widetilde{q}(\lambda)}, b \in F^{\left(\widetilde{I\mathbb{R}^n}\right)}_{\widetilde{w}(\lambda)} \right\}. \end{aligned}$$

Let $\widetilde{q} \widetilde{\preceq} \widetilde{w}$ *and* $\widetilde{z} \widetilde{\preceq} \widetilde{k}$ *for every* $\widetilde{q}, \widetilde{w}, \widetilde{z}, \widetilde{k} \in \widetilde{I\mathbb{R}^n}$. *Then, we find* $\widetilde{q_i} \widetilde{\preceq} \widetilde{w_i}$ *and* $\widetilde{z_i} \widetilde{\preceq} \widetilde{k_i}$ *for every* $\widetilde{q_i}, \widetilde{w_i}, \widetilde{z_i}, \widetilde{k_i} \in \widetilde{I\mathbb{R}}$. *Since* $\widetilde{I\mathbb{R}}$ *is a soft quasilinear inner product space with inner product* $\langle , \rangle_{\widetilde{I\mathbb{R}}}$, *we obtain*

$$\langle \widetilde{q_i}, \widetilde{z_i} \rangle_{\widetilde{I\mathbb{R}}} \widetilde{\leq} \left\langle \widetilde{w_i}, \widetilde{k_i} \right\rangle_{\widetilde{I\mathbb{R}}}.$$

From (3), we get $\langle \widetilde{q}, \widetilde{z} \rangle \widetilde{\leq} \left\langle \widetilde{w}, \widetilde{k} \right\rangle$. *For every* $\widetilde{\epsilon} \geq 0$ *there exists a* $\widetilde{x_\epsilon} \in \widetilde{I\mathbb{R}^n}$ *such that* $\widetilde{q} \widetilde{\preceq} \widetilde{w} + \widetilde{x_\epsilon}$ *and* $\langle \widetilde{x_\epsilon}, \widetilde{x_\epsilon} \rangle \widetilde{\leq} \widetilde{\epsilon}$. *From Theorem 3.1 and (3), we obtain* $\widetilde{q_i} \widetilde{\preceq} \widetilde{w_i} + \widetilde{x_{i\epsilon}}$ *and* $\langle \widetilde{x_{i\epsilon}}, \widetilde{x_{i\epsilon}} \rangle_{\widetilde{I\mathbb{R}}} \widetilde{\leq} \widetilde{\epsilon}$. *Because of* $\widetilde{I\mathbb{R}}$ *is a soft quasilinear inner product space with inner product* $\langle , \rangle_{\widetilde{I\mathbb{R}}}$, *we obtain* $\widetilde{q_i} \widetilde{\preceq} \widetilde{w_i}$ *for every* $i \in [1, n]$. *This gives* $\widetilde{q} \widetilde{\preceq} \widetilde{w}$.

Now, let's we define an another norm on soft quasilinear space $\widetilde{I\mathbb{R}^n}$.

Theorem 3.10. $\widetilde{I\mathbb{R}^n}$ *is a soft normed quasilinear space with*

$$\left\| \widetilde{q} \right\| (\lambda) = \left(\sum_{i=1}^{n} \left\| \widetilde{q_i} (\lambda) \right\|_{\widetilde{I\mathbb{R}}}^2 \right)^{1/2} \tag{4}$$

for every $\widetilde{q} \in \widetilde{I\mathbb{R}^n}$ *and parameter* λ. *Further,* $\widetilde{I\mathbb{R}^n}$ *is a soft Banach quasilinear space with norm (4).*

Proof. *The norm (4) is satisfies all soft norm axioms. So,* $\widetilde{I\mathbb{R}^n}$ *is a soft normed quasilinear space with norm (4). We only show that* $\widetilde{I\mathbb{R}^n}$ *is a soft Banach quasilinear space with norm (4). Let* $\widetilde{q^n}$ *be a Cauchy sequence in* $\widetilde{I\mathbb{R}^n}$ *such that for every* $\widetilde{\epsilon} \widetilde{\geq} \widetilde{0}$ *there exists* $n_0 \in \mathbb{N}$ *such that for every* $n, m > n_0$, *we find*

$$\widetilde{q^n} \widetilde{\preceq} \widetilde{q^m} + \widetilde{q^{\widetilde{\epsilon}}_{1n}}, \ \widetilde{q^m} \widetilde{\preceq} \widetilde{q^n} + \widetilde{q^{\widetilde{\epsilon}}_{2n}}, \ \left\| \widetilde{q^{\widetilde{\epsilon}}_{jn}} \right\| \widetilde{\leq} \widetilde{\epsilon}.$$

For every parameter λ *and for every* $i \in [1, n]$, *we obtain*

$$\widetilde{q^n_i} (\lambda) \widetilde{\preceq} \widetilde{q^m_i} (\lambda) + \left(\widetilde{q^{\widetilde{\epsilon}}_{1n}} \right)_i (\lambda), \ \widetilde{q^m_i} (\lambda) \widetilde{\preceq} \widetilde{q^n_i} (\lambda) + \left(\widetilde{q^{\widetilde{\epsilon}}_{2n}} \right)_i (\lambda) \tag{5}$$

and $\left\| \widetilde{q^{\widetilde{\epsilon}}_{jn}} \right\| (\lambda) = \left(\sum_{i=1}^{n} \left\| \left(\widetilde{q^{\widetilde{\epsilon}}_{jn}} \right)_i (\lambda) \right\|_{\widetilde{I\mathbb{R}}}^2 \right)^{1/2} \widetilde{\leq} \widetilde{\epsilon} (\lambda)$ *from (4). So, we get* $\left\| \left(\widetilde{q^{\widetilde{\epsilon}}_{jn}} \right)_i (\lambda) \right\|_{\widetilde{I\mathbb{R}}} \widetilde{\leq} \widetilde{\epsilon} (\lambda)$ *and* $h_{\widetilde{I\mathbb{R}}}(\widetilde{q^n_i}, \widetilde{q^n_i}) \widetilde{\leq} \widetilde{\epsilon}$ *for every* $i \in [1, n]$. *This gives* $\widetilde{q^n_i}$ *is a Cauchy sequence in* $\widetilde{I\mathbb{R}}$. *Since* $\widetilde{I\mathbb{R}}$ *is a complete, there is a* $\widetilde{q_i} \in \widetilde{I\mathbb{R}}$ *such that* $h_{\widetilde{I\mathbb{R}}}(\widetilde{q^n_i}, \widetilde{q_i}) \widetilde{\leq} \widetilde{\epsilon}$

and $\widetilde{q} = (\widetilde{q_1}, \widetilde{q_2}, ..., \widetilde{q_n}) \in \widetilde{I\mathbb{R}^n}$ *because of* $\widetilde{q_i} \in \widetilde{I\mathbb{R}}$ *for every* $i \in [1, n]$. *If we take* $m \to \infty$ *at (5), for every* $\widetilde{\epsilon} \widetilde{\geq} \widetilde{0}$ *there exists* $n_0 \in \mathbb{N}$ *such that for every* $n > n_0$, *we have*

$$\widetilde{q_i^n}(\lambda) \widetilde{\leq} \widetilde{q_i}(\lambda) + \left(\widetilde{w_{1n}^{\epsilon}}\right)_i(\lambda), \ \widetilde{q_i}(\lambda) \widetilde{\leq} \widetilde{q_i^n}(\lambda) + \left(\widetilde{w_{2n}^{\epsilon}}\right)_i(\lambda), \left\| \left(\widetilde{w_{jn}^{\epsilon}}\right)_i(\lambda) \right\| \widetilde{\leq} \frac{\widetilde{\epsilon}}{\sqrt{n}}(\lambda).$$

From (4), we find $\left\| \widetilde{w_{jn}^{\epsilon}} \right\| \widetilde{\leq} \widetilde{\epsilon}$. *Further, we get*

$$\widetilde{q^n} \widetilde{\leq} \widetilde{q} + \widetilde{w_{1n}^{\epsilon}}, \ \widetilde{q} \widetilde{\leq} \widetilde{q^n} + \widetilde{w_{2n}^{\epsilon}}.$$

This gives, $\widetilde{q^n}$ *converges to* $\widetilde{q}$ *in* $\widetilde{I\mathbb{R}^n}$. *So, we can say* $\widetilde{I\mathbb{R}^n}$ *is a soft Banach space with (4) norm.*

Example 3.11. *Let us take soft quasilinear inner product space* $\widetilde{I\mathbb{R}^3}$ *and* $A = \{\widetilde{q}, \widetilde{w}, \widetilde{z}\}$ *is a soft quasi set of soft quasi vectors of* $\widetilde{I\mathbb{R}^3}$ *such that* $\widetilde{q}(\lambda) = (\{1\}, \{0\}, \{0\})$, $\widetilde{w}(\lambda) = (\{0\}, \{1\}, \{0\})$ *and* $\widetilde{z}(\lambda) = (\{0\}, \{0\}, \{1\})$ *for a parameter* λ. *A is a orthonormal subset of* $\widetilde{I\mathbb{R}^3}$.

Example 3.12. *Let* $B = \left\{\widetilde{m}, \widetilde{n}, \widetilde{k}\right\}$ *is an another soft quasi set of soft quasi vectors of* $\widetilde{I\mathbb{R}^3}$ *such that* $\widetilde{m}(\lambda) = ([-1, 2], [5, 6], \{0\})$, $\widetilde{n}(\lambda) = (\{0\}, \{5\}, \{0\})$ *and* $\widetilde{k}(\lambda) = ([-1, 0], \{6\}, \{0\})$ *for a parameter* λ. *Floor of soft quasi vector* $\widetilde{m}$ *at B is* $F_{\widetilde{m}}^{B} = \left\{\widetilde{l} \in B_r : \widetilde{l}(\lambda) \widetilde{\leq} \widetilde{m}(\lambda)\right\} = \widetilde{n}$. *Also, floor of soft quasi vector* $\widetilde{m}$ *at* $\widetilde{I\mathbb{R}^3}$ *is* $F_{\widetilde{m}}^{\widetilde{I\mathbb{R}^3}} = \left\{\widetilde{l} \in \left(\widetilde{I\mathbb{R}^3}\right)_r : \widetilde{l}(\lambda) \widetilde{\leq} \widetilde{m}(\lambda)\right\}$. $F_{\widetilde{m}}^{\widetilde{I\mathbb{R}^3}} = \left\{\widetilde{l}(\lambda) = \left(\widetilde{l_1}(\lambda), \widetilde{l_2}(\lambda), \widetilde{l_3}(\lambda)\right) : -1 \leq \widetilde{l_1}(\lambda) \leq 2, 5 \leq \widetilde{l_2}(\lambda) \leq 6, \widetilde{l_3}(\lambda) = 0\right\}$ *for a parameter* λ.

Theorem 3.13. *Soft quasilinear space* $\widetilde{I\mathbb{R}^n}$ *is a solid-floored quasilinear space.*

Proof. Let $\widetilde{q} = \widetilde{q_n}$ is a soft quasi vector of $\widetilde{I\mathbb{R}^n}$. From Definition 2.9, we obtain $\widetilde{q} = \sup\{\widetilde{m} \in \widetilde{I\mathbb{R}^n}_r : \widetilde{m} \widetilde{\leq} \widetilde{q}\}$. Since $\widetilde{I\mathbb{R}^n}$ is a quasilinear space, if $\widetilde{m} \widetilde{\leq} \widetilde{q}$, then we find $\widetilde{m_i} \widetilde{\leq} \widetilde{q_i}$ and $\widetilde{m_i}, \widetilde{q_i} \in \widetilde{I\mathbb{R}}$ for every $1 \leq i \leq n$. For a λ parameter, we have $\widetilde{m_i}(\lambda) \widetilde{\leq} \widetilde{q_i}(\lambda)$ for $\widetilde{m_i}(\lambda), \widetilde{q_i}(\lambda) \in I\mathbb{R}$. Thus, we obtain $\widetilde{q_i}(\lambda) = \sup F_{\widetilde{q_i}(\lambda)} = \left\{\widetilde{m_i}(\lambda) \in I\mathbb{R} : \widetilde{m_i}(\lambda) \widetilde{\leq} \widetilde{q_i}(\lambda)\right\}$ since $I\mathbb{R}$ is a solid-floored quasilinear space [16]. This gives $\widetilde{q_i} = \sup F_{\widetilde{q_i}}$ for every $1 \leq i \leq n$. So, we get $\widetilde{q} = \sup F_{\widetilde{q}}$.

Example 3.14. *Let* $A = \{\widetilde{q}, \widetilde{w}, \widetilde{z}\}$ *is a soft quasi set of soft quasi vectors of* $\widetilde{I\mathbb{R}^2}$ *such that* $\widetilde{q}(\lambda) = ([1, 2], \{1\})$, $\widetilde{w}(\lambda) = (\{1\}, [1, 3])$ *and* $\widetilde{z}(\lambda) = ([-1, 5], \{0\})$ *for a parameter* λ. *The soft quasi set A is quasilinear dependent because of*

$$\begin{aligned} & (\{0\}, \{0\})(\lambda) \widetilde{\subseteq} \left(\widetilde{\alpha} \cdot \widetilde{q} + \widetilde{\beta} \cdot \widetilde{w} + \widetilde{\gamma} \cdot \widetilde{z}\right)(\lambda) \\ = \ & \widetilde{\alpha}(\lambda) \cdot \widetilde{q}(\lambda) + \widetilde{\beta}(\lambda) \cdot \widetilde{w}(\lambda) + \widetilde{\gamma}(\lambda) \cdot \widetilde{z}(\lambda) \\ = \ & \widetilde{\alpha}(\lambda) \cdot ([1, 2], \{1\}) + \widetilde{\beta}(\lambda) \cdot (\{1\}, [1, 3]) + \widetilde{\gamma}(\lambda) \cdot ([-1, 5], \{0\}) \end{aligned}$$

for $\widetilde{\alpha}(\lambda) = \widetilde{\gamma}(\lambda) = 1$ *and* $\widetilde{\beta}(\lambda) = -1$. *If we take soft quasi set* $B = \{\widetilde{m}, \widetilde{n}\}$ *of* $\widetilde{I\mathbb{R}^2}$ *such that* $\widetilde{m}(\lambda) = ([2, 3], \{0\})$, $\widetilde{n}(\lambda) = (\{0\}, [1, 2])$ *for a parameter* λ. *The soft quasi set B is quasilinear independent because of*

$$\begin{aligned} & (\{0\}, \{0\})(\lambda) \widetilde{\subseteq} \left(\widetilde{\alpha} \cdot \widetilde{m} + \widetilde{\beta} \cdot \widetilde{n}\right)(\lambda) \\ = \ & \widetilde{\alpha}(\lambda) \cdot \widetilde{m}(\lambda) + \widetilde{\beta}(\lambda) \cdot \widetilde{n}(\lambda) \\ = \ & \widetilde{\alpha}(\lambda) \cdot ([2, 3], \{0\}) + \widetilde{\beta}(\lambda) \cdot (\{0\}, [1, 2]) \end{aligned}$$

for $\widetilde{\alpha}(\lambda) = \widetilde{\beta}(\lambda) = 0$. *Also,* $\dim \widetilde{I\mathbb{R}^2} = 2$.

Let us consider soft normed quasilinear space $\widetilde{I\mathbb{R}^n}$. A soft quasi vector $\widetilde{q}$ of $\widetilde{I\mathbb{R}^n}$ is a soft quasi regular element of $\widetilde{I\mathbb{R}^n}$ if and only if $\widetilde{q}-\widetilde{q}=\widetilde{0}$. Otherwise, we say that $\widetilde{q}$ is a soft quasi singular element of $\widetilde{I\mathbb{R}^2}$. For example; in Example 3.11, the soft quasi vectors $\widetilde{q},\widetilde{w},\widetilde{z},\widetilde{n}$ of $\widetilde{I\mathbb{R}^3}$ are regular with respect to parameter λ, but $\widetilde{m},\widetilde{k}$ are singular with respect to parameter λ.

Now, we will define a new soft interval space. The set

$$\widetilde{Is}=\left\{\widetilde{q}=(\widetilde{q_n})=(\widetilde{q_1},\widetilde{q_2},...)\in\widetilde{I\mathbb{R}^\infty}:(\widetilde{q_n}(\lambda))=(\widetilde{q_1}(\lambda),\widetilde{q_2}(\lambda),...,\widetilde{q_n}(\lambda),...)\in I\mathbb{R}^\infty,\ \lambda\in M\right\}$$

is called all soft interval sequences space. The interval sequence space $\widetilde{Is}$ is a soft quasilinear space with

$$(\widetilde{q_n}+\widetilde{w_n})(\lambda)=(\widetilde{q_n})(\lambda)+(\widetilde{w_n})(\lambda), \tag{6}$$

$$(\widetilde{\alpha}\cdot\widetilde{q_n})(\lambda)=\widetilde{\alpha}(\lambda)\cdot(\widetilde{q_n})(\lambda) \tag{7}$$

$$\widetilde{q_n}\widetilde{\preceq}\widetilde{w_n}\Leftrightarrow(\widetilde{q_n})(\lambda)\preceq_{I\mathbb{R}}(\widetilde{w_n})(\lambda) \tag{8}$$

for $(\widetilde{q_n})=(\widetilde{q_1},\widetilde{q_2},...),(\widetilde{w_n})=(\widetilde{w_1},\widetilde{w_2},...)\in\widetilde{Is}$, soft real scalar $\widetilde{\alpha}$ and parameter λ.

Example 3.15. *Let $\widetilde{q},\widetilde{w}\in\widetilde{Is}$. Then*

$$\widetilde{d}(\widetilde{q},\widetilde{w})=\sum_{i=1}^{\infty}\frac{1}{2^i}\left[\frac{h_{\widetilde{I\mathbb{R}}}(\widetilde{q_i},\widetilde{w_i})}{1+h_{\widetilde{I\mathbb{R}}}(\widetilde{q_i},\widetilde{w_i})}\right]$$

is a soft metric on soft quasilinear space $\widetilde{Is}$. Here, $h_{\widetilde{I\mathbb{R}}}$ represents the soft Hausdorff metric on soft quasilinear space $\widetilde{I\mathbb{R}}$.

Definition 3.16. *Let $\widetilde{q}=(\widetilde{q_n})=(\widetilde{q_1},\widetilde{q_2},...)\in\widetilde{I\mathbb{R}^\infty}$, $(\widetilde{q_n}(\lambda))=(\widetilde{q_1}(\lambda),\widetilde{q_2}(\lambda),...,\widetilde{q_n}(\lambda),...)\in I\mathbb{R}^\infty$ for $\lambda\in M$. If $\sup\left\{\|\widetilde{q_i}\|_{\widetilde{I\mathbb{R}}}\right\}<\infty$, for a soft quasi sequence $(\widetilde{q_n}(\lambda))=(\widetilde{q_1}(\lambda),\widetilde{q_2}(\lambda),...,\widetilde{q_n}(\lambda),...)\in I\mathbb{R}^\infty$ then the set of these soft quasi sequences is called as a soft quasi $\widetilde{Il_\infty}$ set. $\widetilde{Il_\infty}$ is a soft quasilinear space with (6), (7) and (8). The sum operation and soft scalar multiplication are well-defined because of*

$$\begin{aligned}&\sup\left\{\|\widetilde{q_i}+\widetilde{w_i}\|_{\widetilde{I\mathbb{R}}}\right\}\widetilde{\leq}\sup\left\{\|\widetilde{q_i}\|_{\widetilde{I\mathbb{R}}}+\|\widetilde{w_i}\|_{\widetilde{I\mathbb{R}}}\right\}\\&\widetilde{\leq}\sup\left\{\|\widetilde{q_i}\|_{\widetilde{I\mathbb{R}}}\right\}+\sup\left\{\|\widetilde{w_i}\|_{\widetilde{I\mathbb{R}}}\right\}\\<\ &\infty\end{aligned}$$

and

$$\begin{aligned}&\sup\left\{\|\widetilde{\alpha}\cdot\widetilde{q_i}\|_{\widetilde{I\mathbb{R}}}\right\}=\sup\left\{|\widetilde{\alpha}|\,\|\widetilde{q_i}\|_{\widetilde{I\mathbb{R}}}\right\}\\=\ &|\widetilde{\alpha}|\cdot\sup\left\{\|\widetilde{q_i}\|_{\widetilde{I\mathbb{R}}}\right\}\\<\ &\infty.\end{aligned}$$

Theorem 3.17. *Let $\widetilde{q}=\widetilde{q_n}\in\widetilde{Il_\infty}$ and the function $\|.\|:\widetilde{Il_\infty}\times\widetilde{Il_\infty}\to\mathbb{R}(M)$ is defined as*

$$\|\widetilde{q}\|=\sup\left\{\|\widetilde{q_n}\|_{\widetilde{I\mathbb{R}}}:n\in\mathbb{N}\right\} \tag{9}$$

where $M=\mathbb{R}$. The function $\|.\|$ is a soft normed on soft quasilinear space $\widetilde{Il_\infty}$ and $\left(\widetilde{Il_\infty},\|.\|\right)$ is a soft normed quasilinear space.

Proof. We will only prove the last condition because of the other conditions of soft normed quasilinear space are very easy. Let $\widetilde{q}\widetilde{\preceq}\widetilde{w}+\widetilde{q}^{\epsilon}$ and $\|\widetilde{q}^{\epsilon}\|\widetilde{\leq}\widetilde{\epsilon}$ for every $\widetilde{q},\widetilde{w}\in\widetilde{Il\infty}$. Then, we obtain $\left\|\widetilde{q}_n^{\epsilon}\right\|_{\widetilde{I\mathbb{R}}}\widetilde{\leq}\widetilde{\epsilon}$ for all $n\in\mathbb{N}$. Moreover, we have $\widetilde{q_n}\widetilde{\preceq}\widetilde{w_n}+\widetilde{q}_n^{\epsilon}$ for all $n\in\mathbb{N}$ and $\widetilde{q_n},\widetilde{w_n}\in\widetilde{I\mathbb{R}}$ from (8). Since $\widetilde{I\mathbb{R}}$ is a soft normed quasilinear space, we obtain $\widetilde{q_n}\widetilde{\preceq}\widetilde{w_n}$ for every $n\in\mathbb{N}$. This gives $\widetilde{q}\widetilde{\preceq}\widetilde{w}$. □

Theorem 3.18. *Let* $\widetilde{q} = \{\widetilde{q_n}\}, \widetilde{w} = \{\widetilde{w_n}\} \in \widetilde{Il_\infty}$ *and the function* $\widetilde{d} : \widetilde{Il_\infty} \times \widetilde{Il_\infty} \to \mathbb{R}(M)$ *is defined as*

$$d(\widetilde{q}, \widetilde{w}) = \sup\left\{h_{\widetilde{I\mathbb{R}}}(\widetilde{q_n}, \widetilde{w_n}) : n \in \mathbb{N}\right\} \tag{10}$$

where $M = \mathbb{R}$. *The function d is a soft metric on soft quasilinear space* $\widetilde{Il_\infty}$.

Proof. Firstly, the metric (10) is well-defined. $d(\widetilde{q}, \widetilde{w}) \widetilde{\geq} \overline{0}$ since $h_{\widetilde{I\mathbb{R}}}(\widetilde{q_n}, \widetilde{w_n}) \widetilde{\geq} \overline{0}$ for every $\widetilde{q}, \widetilde{w} \in \widetilde{Il_\infty}$ and $n \in \mathbb{N}$. If $d(\widetilde{q}, \widetilde{w}) = \overline{0}$, then we get $h_{\widetilde{I\mathbb{R}}}(\widetilde{q_n}, \widetilde{w_n}) = \overline{0}$ for every $n \in \mathbb{N}$. Since h is a soft Hausdorff metric on $\overline{I\mathbb{R}}$, we obtain $\widetilde{q_n} = \widetilde{w_n}$ for every $n \in \mathbb{N}$. This gives $\widetilde{q} = \widetilde{w}$. Clearly $d(\widetilde{q}, \widetilde{w}) = d(\widetilde{w}, \widetilde{q})$ for every $\widetilde{q}, \widetilde{w} \in \widetilde{Il_\infty}$. Since h is a soft Hausdorff metric on $\overline{I\mathbb{R}}$, we write

$$h_{\widetilde{I\mathbb{R}}}(\widetilde{q_n}, \widetilde{z_n}) \widetilde{\leq} h_{\widetilde{I\mathbb{R}}}(\widetilde{q_n}, \widetilde{w_n}) + h_{\widetilde{I\mathbb{R}}}(\widetilde{w_n}, \widetilde{z_n})$$

for every $\widetilde{q_n}, \widetilde{w_n}, \widetilde{z_n} \in \widetilde{I\mathbb{R}}$. Then, we have

$$\sup\{h_{\widetilde{I\mathbb{R}}}(\widetilde{q_n}, \widetilde{z_n}) : n \in \mathbb{N} \widetilde{\leq}\} \widetilde{\leq} \sup\{h_{\widetilde{I\mathbb{R}}}(\widetilde{q_n}, \widetilde{w_n}) : n \in \mathbb{N}\} + \sup\{h_{\widetilde{I\mathbb{R}}}(\widetilde{w_n}, \widetilde{z_n}) : n \in \mathbb{N}\}.$$

This gives $d(\widetilde{q}, \widetilde{z}) \widetilde{\leq} d(\widetilde{q}, \widetilde{w}) + d(\widetilde{w}, \widetilde{z})$ for every $\widetilde{q}, \widetilde{w}, \widetilde{z} \in \widetilde{Il_\infty}$.

Definition 3.19. *Let* $\widetilde{q} = (\widetilde{q_n}) = (\widetilde{q_1}, \widetilde{q_2}, ...) \in \widetilde{I\mathbb{R}^\infty}$, $(\widetilde{q_n}(\lambda)) = (\widetilde{q_1}(\lambda), \widetilde{q_2}(\lambda), ..., \widetilde{q_n}(\lambda), ...) \in I\mathbb{R}^\infty$ *for* $\lambda \in M$. *If* $\widetilde{q_n} \to \widetilde{0}$, *for a soft quasi sequence* $(\widetilde{q_n}(\lambda)) = (\widetilde{q_1}(\lambda), \widetilde{q_2}(\lambda), ..., \widetilde{q_n}(\lambda), ...) \in I\mathbb{R}^\infty$ *then the set of these soft quasi sequences is called as a soft quasi* $\widetilde{Ic_0}$ *set. Namely, the soft set* $\widetilde{Ic_0}$ *consist of all soft convergent sequence* $\widetilde{q_n}$ *whose limit is* $\widetilde{0}$. $\widetilde{Ic_0}$ *is a soft quasilinear space with (6), (7) and (8).*

Theorem 3.20. *The soft quasilinear space* $\widetilde{Ic_0}$ *is a soft normed quasilinear space with the norm (9).*

Proof. The proof is similar to Theorem 3.17.

Definition 3.21. *Let* $\widetilde{q} = (\widetilde{q_n}) = (\widetilde{q_1}, \widetilde{q_2}, ...) \in \widetilde{I\mathbb{R}^\infty}$, $(\widetilde{q_n}(\lambda)) = (\widetilde{q_1}(\lambda), \widetilde{q_2}(\lambda), ..., \widetilde{q_n}(\lambda), ...) \in I\mathbb{R}^\infty$ *for* $\lambda \in M$. *If* $\sum_{i=1}^{\infty} \left\|\widetilde{q_n}\right\|_{\widetilde{I\mathbb{R}}} < \infty$, *for a soft quasi sequence* $(\widetilde{q_n}(\lambda)) = (\widetilde{q_1}(\lambda), \widetilde{q_2}(\lambda), ..., \widetilde{q_n}(\lambda), ...) \in I\mathbb{R}^\infty$, *then the space of these soft quasi sequences is called as soft quasi* $\widetilde{Il_2}$ *space.* $\widetilde{Il_2}$ *is not a soft linear space.*

Theorem 3.22. $\widetilde{Il_2}$ *is a soft normed quasilinear space with norm*

$$\left\|\widetilde{q}\right\| = \left(\sum_{i=1}^{\infty} \left\|\widetilde{q_i}\right\|^2_{\widetilde{I\mathbb{R}}}\right)^{\frac{1}{2}}. \tag{11}$$

Proof. $\widetilde{Il_2}$ is a soft quasilinear space with (6), (7) and (8). Since

$$\left\|\widetilde{q} + \widetilde{w}\right\| \widetilde{\leq} \left(\sum_{i=1}^{\infty} \left\|\widetilde{q_i}\right\|^2_{\widetilde{I\mathbb{R}}}\right)^{\frac{1}{2}} + \left(\sum_{i=1}^{\infty} \left\|\widetilde{w_i}\right\|^2_{\widetilde{I\mathbb{R}}}\right)^{\frac{1}{2}} < \infty$$

and

$$\left\|\widetilde{\alpha} \cdot \widetilde{q}\right\| = \widetilde{\alpha} \cdot \left(\sum_{i=1}^{\infty} \left\|\widetilde{q_i}\right\|^2_{\widetilde{I\mathbb{R}}}\right)^{\frac{1}{2}} < \infty,$$

sum operation and scalar multiplication is well defined. Further, the norm in (11) satisfies all norm axioms in $\widetilde{Il_2}$. Additionally, the soft quasilinear space $\widetilde{Il_2}$ is not complete with norm (11) because of the quasilinear space Il_2 is not complete.

Corollary 3.23. *$\widetilde{Il_2}$ is not a soft Banach quasilinear space since $\widetilde{q}^n \to \widetilde{q} \Rightarrow \widetilde{q_i}^n \to \widetilde{q}_i$ but $\widetilde{q_i}^n \to \widetilde{q}_i \Longrightarrow \widetilde{q}^n \nrightarrow \widetilde{q}$.*

Proof. We know that $\widetilde{Il_2}$ is a soft quasilinear space with norm (11). Let $\widetilde{q^n}$ be a convergent to $\widetilde{q} \in \widetilde{I\mathbb{R}^n}$ such that for every $\widetilde{\epsilon}\widetilde{\geq}\widetilde{0}$ there exists $n_0 \in \mathbb{N}$ such that for every $n > n_0$, we find

$$\widetilde{q^n}\widetilde{\leq}\widetilde{q} + \widetilde{q}^{\widetilde{\epsilon}}_{1n},\ \widetilde{q}\widetilde{\leq}\widetilde{q^n} + \widetilde{q}^{\widetilde{\epsilon}}_{2n},\ \left\|\widetilde{q}^{\widetilde{\epsilon}}_{jn}\right\|\widetilde{\leq}\widetilde{\epsilon}.$$

For every parameter λ and for every $1 \leq i < \infty$, we obtain

$$\widetilde{q_i^n}(\lambda)\,\widetilde{\leq}\widetilde{q_i}(\lambda) + \widetilde{\left(q^{\widetilde{\epsilon}}_{1n}\right)_i}(\lambda),\ \widetilde{q_i}(\lambda)\,\widetilde{\leq}\widetilde{q_i^n}(\lambda) + \widetilde{\left(q^{\widetilde{\epsilon}}_{2n}\right)_i}(\lambda).$$

From (11), we have $\left\|\widetilde{\left(q^{\widetilde{\epsilon}}_{jn}\right)_i}\right\|_{\widetilde{I\mathbb{R}}}\widetilde{\leq}\widetilde{\epsilon}$ for $j = 1,2$. This gives $\widetilde{q_i^n} \to \widetilde{q_i}$ in $\widetilde{I\mathbb{R}}$. Now, we assume that $\widetilde{q_i}^n \to \widetilde{q_i}$ for every $\widetilde{\epsilon}\widetilde{\geq}\widetilde{0}$ there exists $n_0 \in \mathbb{N}$ such that for every $n > n_0$. But, $\widetilde{q^n} \nrightarrow \widetilde{q}$ since $\left\|\widetilde{q}^{\widetilde{\epsilon}}_{jn}\right\| = \left(\sum_{i=1}^{\infty}\left\|\widetilde{\left(q^{\widetilde{\epsilon}}_{jn}\right)_i}\right\|^2_{\widetilde{I\mathbb{R}}}\right)^{\frac{1}{2}}\widetilde{\nleq}\widetilde{\epsilon}$ may not be satisfy.

Example 3.24. *$\widetilde{Il_2}$, $\widetilde{Il_\infty}$ and $\widetilde{Ic_0}$ are soft solid floored quasilinear spaces. If we take $\widetilde{q} \in \widetilde{Il_2}$, then we have $F_{\widetilde{q}} = \left\{\widetilde{w} \in \left(\widetilde{Il_2}\right)_r : \widetilde{w}\widetilde{\leq}\widetilde{q}\right\}$. This gives $F_{\widetilde{q}(\lambda)} = \left\{\widetilde{w}(\lambda) \in (Il_2)_r : \widetilde{w}(\lambda)\,\widetilde{\leq}\widetilde{q}(\lambda)\right\}$ for every parameter λ. Since $\widetilde{Il_2}$ soft quasilinear space, we obtain $\widetilde{w_i}(\lambda)\,\widetilde{\leq}\widetilde{q_i}(\lambda)$ for every $\widetilde{w_i}(\lambda) \in \mathbb{R}$, $\widetilde{q_i}(\lambda) \in I\mathbb{R}$ and $1 \leq i < \infty$. From [16], $I\mathbb{R}$ is a solid-floored quasilinear space, we obtain $\widetilde{q_i}(\lambda) = \sup\left\{\widetilde{w_i}(\lambda) \in \mathbb{R} : \widetilde{w_i}(\lambda)\,\widetilde{\leq}\widetilde{q_i}(\lambda)\right\}$ for every $1 \leq i < \infty$. So, we have $\widetilde{q}(\lambda) = \sup F_{\widetilde{q}(\lambda)}$ because of $\widetilde{Il_2}$ is a soft quasilinear space with relation "$\widetilde{\leq}$". We can show that $\widetilde{Il_\infty}$ and $\widetilde{Ic_0}$ are solid-floored similar to $\widetilde{Il_2}$. But the singular subsapce of $\widetilde{Il_2}$, $\widetilde{Il_\infty}$ and $\widetilde{Ic_0}$ are soft non-solid floored quasilinear spaces.*

For example, if we take $\widetilde{q} \in \widetilde{(Ic_0)}_s$ such that $\widetilde{q}(\lambda) = (\{0\}, [-2,2], \{0\}, \{0\}, ...)$, then $F^{\widetilde{(Ic_0)_s}}_{\widetilde{q}} = \left\{\widetilde{w} \in \left(\widetilde{(Ic_0)_s}\right)_r : \widetilde{w}\widetilde{\leq}\widetilde{q}\right\}$. For every parameter λ, we obtain $F^{\widetilde{(Ic_0)_s}}_{\widetilde{q}(\lambda)} = \left\{\widetilde{w}(\lambda) \in ((Ic_0)_s)_r : \widetilde{w}(\lambda)\,\widetilde{\leq}\widetilde{q}(\lambda)\right\} = (\{0\}, \{0\}, \{0\}, ...)$. So, $\sup F^{\widetilde{(Ic_0)_s}}_{\widetilde{q}} \neq \widetilde{q}$. This gives $\widetilde{Ic_0}$ is a non-solid floored quasilinear space.

Example 3.25. *Let us consider soft quasilinear space $\widetilde{Ic_0}$ and $W = \{\widetilde{q_1}, \widetilde{q_2}, \ldots\} \subset \widetilde{Ic_0}$ such that $\widetilde{q_1}(\lambda) = ([2,3], 0, 0, 0, ...)$, $\widetilde{q_2}(\lambda) = (0, [2,3], 0, 0, ...)$, ... for every parameter λ. The set W ql-independent in $\widetilde{Ic_0}$ because of*

$$\begin{aligned}
\widetilde{\theta}(\lambda) &= (\overline{0}(\lambda), \overline{0}(\lambda), \overline{0}(\lambda), ...)\widetilde{\leq}\widetilde{\alpha}_1(\lambda)\cdot\widetilde{q_1}(\lambda) + \widetilde{\alpha}_2(\lambda)\cdot\widetilde{q_2}(\lambda) + ... + \widetilde{\alpha}_n(\lambda)\cdot\widetilde{q_n}(\lambda) \\
&= \widetilde{\alpha}_1(\lambda)\cdot([2,3], 0, 0, 0, ...) + \widetilde{\alpha}_2(\lambda)\cdot(0, [2,3], 0, 0, ...) + ... \\
&\quad + \widetilde{\alpha}_n(\lambda)\cdot(0, 0, ...[2,3], 0, , ...) \\
&\Leftrightarrow \widetilde{\alpha}_1(\lambda) = \widetilde{\alpha}_2(\lambda) = ... = \widetilde{\alpha}_n(\lambda) = \overline{0}(\lambda).
\end{aligned}$$

4. Conclusions and Future Works

In our study, a special examples of soft quasilinear spaces, which are soft interval spaces and soft interval sequence spaces have been introduced. Moreover, some related properties and examples of soft interval spaces and soft interval sequence spaces have been given. Finally, related theorems including soft quasilinear theory and many conclusions are researched. Some algebraic properties of soft interval spaces and soft interval sequence spaces such as basis, dimensions and properness will be studied in further investigations depending on the descriptions of soft interval and soft interval sequence spaces given in this research.

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