



On interpolative \mathcal{R} -Meir-Keeler contractions of rational forms

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Abstract. In this article, the notion of rational interpolative Meir-Keeler type contraction is discussed. The existence and uniqueness of a fixed point for interpolative Meir-Keeler contraction of rational Das-Gupta are investigated. The obtained results improve and generalize the existing results on the topic in the recent literature.

1. Introduction

In 1922, Banach [14] gave famous result so called Banach contraction principle. After that fixed point theory become one of the important research areas of mathematics. Banach fixed point theorem has many generalization in the literature. In 1968 Kannan [29] removed the continuity property and give a new variant. Another interesting contraction is the uniform contraction given by Meir and Keeler [43]. In 2018, Karapınar [30] enriched the concept of interpolation and establishing a new contraction known as interpolative Kannan type contraction. There are many articles on this direction; see e.g. [1–4, 7–13, 15, 16, 18, 20–22, 24, 25, 27, 31–40, 44, 47].

For the sake of the completeness, we remind the notion of Meir-Keeler contraction, as follows.

Definition 1.1. [43] Let (X, d) be a complete metric space and $f : X \rightarrow X$ be a mapping. We say that f is a Meir-Keeler contraction on the space X , if for every $\varepsilon > 0$, there exists $\delta(\varepsilon) > 0$ such that

$$\varepsilon \leq d(u, v) < \varepsilon + \delta \Rightarrow d(fu, fv) < \varepsilon, \quad (1)$$

for every $u, v \in X$.

Theorem 1.2. [43] Let (X, d) be a complete metric space and $f : X \rightarrow X$ be a Meir-Keeler contraction-contraction. Then, f has a unique fixed point.

In (1976), M. S. Khan [41] proved the following fixed point theorem.

Theorem 1.3. Let (X, d) be a complete metric space and let $f : X \rightarrow X$ be a mapping satisfying the following condition:

$$d(f(x), f(y)) \leq \mu \frac{d(x, f(x))d(x, f(y)) + d(y, f(y))d(y, f(x))}{d(x, f(y)) + d(y, f(x))}, \quad \mu \in (0, 1) \quad (1.2)$$

Then f has a unique fixed point $u \in X$. Moreover, for all $x_0 \in X$, the sequence $\{f^n(x_0)\}$ converges to u .

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Remark 1.4. It was shown by B. Fisher [19] that Theorem 1.3 is incorrect and needed the extra condition, which is $d(x, f(y)) + d(y, f(x)) = 0$ implies that $d(f(x), f(y)) = 0$ for the theorem to hold. Thus, the true version of Theorem 1.1 is as follows.

Theorem 1.5. Let (X, d) be a complete metric space and let $f : X \rightarrow X$ be a mapping satisfying the following condition:

$$d(f(x), f(y)) < \mu \frac{d(x, f(x))d(x, f(y)) + d(y, f(y))d(y, f(x))}{d(x, f(y)) + d(y, f(x))}, \quad \mu \in (0, 1) \quad (1.3)$$

$$\text{if } d(x, f(y)) + d(y, f(x)) \neq 0 \quad (\star)$$

and

$$d(f(x), f(y)) = 0 \text{ if } d(x, f(y)) + d(y, f(x)) = 0. \quad (\star\star)$$

Then f has a unique fixed point $u \in X$. Moreover, for all $x_0 \in X$, the sequence $\{f^n(x_0)\}$ converges to u .

Some variations of Theorem 1.2 and its extensions are established by several authors (see [5, 6, 17, 19, 23, 28, 42, 45, 46, 48, 49, 51]). Very recently, Karapinar [31] introduce interpolative Kannan-Meir-Keeler type contraction. Definition 1.6 and Theorem 1.7 are given below.

Definition 1.6. [31] Let (X, d) be a complete metric space. A mapping $f : X \rightarrow X$ is said to be an interpolative Kannan-Meir-Keeler type contraction on X (on short, KMK-contraction), if there exists $\gamma \in (0, 1)$ such that for every $u, v \in X \setminus \text{Fix}(f)$ we have

(1) given $\varepsilon > 0$, there exists $\delta > 0$ such that

$$\varepsilon < [d(u, fu)]^\gamma [d(v, fv)]^{1-\gamma} < \varepsilon + \delta \implies d(fu, fv) \leq \varepsilon. \quad (2)$$

Also we have

(2)

$$d(fu, fv) < [d(u, fu)]^\gamma [d(v, fv)]^{1-\gamma}. \quad (3)$$

Theorem 1.7. [31] On a complete metric space (X, d) , any interpolative KMK-contraction $f : X \rightarrow X$ has a fixed point.

2. Main Results

We start this section by giving the definition of interpolative Meir-Keeler contraction of rational Das-Gupta.

Definition 2.1. Let (X, d) be a metric space and $f : X \rightarrow X$ be a mapping. We say that f is an interpolative Meir-Keeler contraction of rational Das-Gupta on X if there exists $\gamma \in (0, 1)$, such that for every $u, v \in X \setminus \text{Fix}(f)$ we have

(1) given $\varepsilon > 0$, there exists $\delta(\varepsilon) > 0$ so that

$$\varepsilon \leq \mathcal{R}(u, v) < \varepsilon + \delta \implies d(fu, fv) < \varepsilon, \quad (4)$$

(2)

where

$$\mathcal{R}(u, v) = (d(u, v))^\gamma \left(\frac{d(v, fv) \cdot (1 + d(u, fu))}{1 + d(u, v)} \right)^{1-\gamma} \tag{5}$$

Lemma 2.2. *Let (X, d) be a metric space and $f : X \rightarrow X$ be an interpolative Meir-Keeler contraction of rational Das-Gupta on X . Then, we have*

$$d(fu, fv) < \mathcal{R}(u, v), \tag{6}$$

Theorem 2.3. *On a complete metric space (X, d) , any interpolative Meir-Keeler contraction of rational Das-Gupta $f : X \rightarrow X$ has a fixed point.*

Proof. Let $u_0 \in X$, be an arbitrary point and the sequence $\{u_p\}$, be defined by the following rule:

$$u_p = fu_{p-1} = f^p u_0, \text{ for all } p \in \mathbb{N}. \tag{7}$$

(We can assume that $u_p \neq u_{p+1}$ for every $p \in \mathbb{N} \cup \{0\}$. If not, then there exists $p_0 \in \mathbb{N} \cup \{0\}$ such that $u_{p_0} = u_{p_0+1} = fu_{p_0}$. Therefore, we get that u_{p_0} is a fixed point of the mapping f , which close the proof.) Thus, by (6), letting $u = u_{p-1}$ and $v = u_p$, we have

$$\begin{aligned} d(u_p, u_{p+1}) &= d(fu_{p-1}, fu_p) < \mathcal{R}(u_{p-1}, u_p) \\ &= (d(u_{p-1}, u_p))^\gamma \left(\frac{d(u_p, fu_p) \cdot (1 + d(u_{p-1}, fu_{p-1}))}{1 + d(u_{p-1}, u_p)} \right)^{1-\gamma} \\ &= (d(u_{p-1}, u_p))^\gamma (d(u_p, u_{p+1}))^{1-\gamma} \end{aligned}$$

Therefore, we get that $d(u_p, u_{p+1}) < d(u_{p-1}, u_p)$, which shows that sequence $\{d(u_p, u_{p+1})\}$ is strictly decreasing. On the other hand, since $d(u_p, u_{p+1}) > 0$, for every $p \in \mathbb{N} \cup \{0\}$, we have that the sequence $\{d(u_p, u_{p+1})\}$ is convergent, and then, there exists a point $\ell \geq 0$ such that $\lim_{p \rightarrow \infty} d(u_p, u_{p+1}) = \ell$. We claim that $\ell = 0$. On the contrary, if $\ell > 0$, taking $\varepsilon = \ell$, from (4), we have that there exists $\delta(\ell) > 0$ such that

$$\ell < \mathcal{R}(u_{p-1}, u_p) < \ell + \delta(\ell) \text{ implies } d(fu_{p-1}, fu_p) \leq \ell. \tag{8}$$

On the other hand, since $d(u_p, u_{p+1}) \searrow \ell$, for $\delta(\ell) > 0$ we can find $N \in \mathbb{N}$, such that

$$\begin{aligned} \ell &< d(u_p, u_{p+1}) < \mathcal{R}(u_{p-1}, u_p) \\ &= (d(u_{p-1}, u_p))^\gamma \left(\frac{d(u_p, fu_p) \cdot (1 + d(u_{p-1}, fu_{p-1}))}{1 + d(u_{p-1}, u_p)} \right)^{1-\gamma} \\ &= (d(u_{p-1}, u_p))^\gamma (d(u_p, u_{p+1}))^{1-\gamma} \\ &\leq d(u_{p-1}, u_p) < \ell + \delta(\ell), \end{aligned}$$

for any $p \geq N$. Therefore, taking (8) into account, it follows that

$$d(u_p, u_{p+1}) < \ell, \text{ for any } p \geq N, \tag{9}$$

a contradiction. Hence, we find $\lim_{p \rightarrow \infty} d(u_p, u_{p+1}) = \ell = 0$.

We claim now that $\{u_p\}$ is a Cauchy sequence. Based on (6), we have

$$\begin{aligned} d(u_p, u_q) &< \mathcal{R}(u_{p-1}, u_{q-1}) \\ &= (d(u_{q-1}, u_{p-1}))^\gamma \left(\frac{d(u_{q-1}, fu_{q-1}) \cdot (1 + d(u_{p-1}, fu_{p-1}))}{1 + d(u_{q-1}, u_{p-1})} \right)^{1-\gamma} \end{aligned} \tag{10}$$

Letting $p, q \rightarrow \infty$, the right hand tends to zero. Hence, we find that $\lim_{p,q \rightarrow \infty} d(u_p, u_q) = 0$

Therefore, the sequence $\{u_p\}$ is Cauchy and by the completeness of the space X it follows that there exists $u^* \in X$ such that

$$\lim_{p \rightarrow \infty} u_p = u^*, \tag{11}$$

and also, $\lim_{p \rightarrow \infty} fu_p = u^*$.

By (6) we have

$$\begin{aligned} 0 \leq d(u^*, fu^*) &\leq d(u^*, u_{p+1}) + d(u_{p+1}, fu^*) = d(u_{p+1}, u_{p+1}) + d(fu_p, fu^*) \\ &< d(u^*, u_{p+1}) + \mathcal{R}(u^*, u_p) \\ &= d(u^*, u_{p+1}) + (d(u^*, u_p))^\gamma \left(\frac{d(u_p, fu_p) \cdot (1 + d(u^*, fu^*))}{1 + d(u^*, u_p)} \right)^{1-\gamma} \end{aligned}$$

Therefore, $d(u^*, fu^*) = 0$, that is, u^* is a fixed point of the mapping f . \square

Example 2.4. Let $X = [1, +\infty)$ and the distance $d : X \times X \rightarrow [0, +\infty)$, with $d(u, v) = \begin{cases} \max\{u, v\}, & \text{if } u \neq v \\ 0, & \text{if } u = v \end{cases}$.

Let also the mapping $f : X \rightarrow X$, be defined by

$$fu = \begin{cases} 1, & \text{for } u \in [1, 2] \\ \frac{1}{2}, & \text{for } u \in (2, +\infty). \end{cases}$$

Since $d(fu, fv) = 0$ for any $u, v \in [1, 2]$, respectively $u, v \in (2, +\infty)$, it follows that (6) holds for these cases. If $u \in (2, +\infty)$ and $v \in [1, 2]$, we have

$$\begin{aligned} d(fu, fv) &= \max\left\{1, \frac{1}{2}\right\} = 1, \quad d(u, v) = \max\{u, v\} = u, \\ d(u, fu) &= \max\left\{u, \frac{1}{2}\right\} = u, \quad d(v, fv) = \max\{v, 1\} = v, \\ \mathcal{R}(u, v) &= [d(u, v)]^\gamma \left[\frac{d(v, fv)(1 + d(u, fu))}{1 + d(u, v)} \right]^{1-\gamma} = u^\gamma \left(\frac{v(1 + u)}{1 + u} \right)^{1-\gamma} = u^\gamma v^{1-\gamma}. \end{aligned}$$

Thus, choosing for example $\gamma = \frac{1}{2}$,

$$d(u, v) < \sqrt{uv} = \mathcal{R}(u, v),$$

for any $u \in (2, +\infty)$ and $v \in [1, 2]$. One can see that (6) holds for any $u, v \in X$; that is, f is an interpolative Meir-Keeler contraction of rational Das-Gupta on X . Thus, by Theorem 2.3, the mapping f has a fixed point.

A mapping $\varphi : [0, \infty) \rightarrow [0, \infty)$ is called a *comparison function* if it is increasing and $\varphi^n(t) \rightarrow 0, n \rightarrow \infty$, for any $t \in [0, \infty)$. We denote by Φ , the class of the comparison functions $\varphi : [0, \infty) \rightarrow [0, \infty)$. For more details and illustrative examples, see e.g. [50]. Among them, we recall the following essential result.

Lemma 2.5. (Rus [50]) If $\varphi : [0, \infty) \rightarrow [0, \infty)$ is a comparison function, then:

- (1) each iterate φ^k of $\varphi, k \geq 1$, is also a comparison function;
- (2) φ is continuous at 0;
- (3) $\varphi(t) < t$, for any $t > 0$.

Let Ψ be the family of nondecreasing functions $\psi : [0, \infty) \rightarrow [0, \infty)$ so that $\sum_{n=1}^{\infty} \psi^n(t) < \infty$ for each $t > 0$,

where ψ^n is the n -th iterate of ψ . It is clear that if $\Phi \subset \Psi$ (see e.g. [26]) and hence, by Lemma 2.5 (3), for $\psi \in \Psi$ we have $\psi(t) < t$, for any $t > 0$.

Remark 2.6. Every function $\psi \in \Psi$ is called a (c)-comparison function. It is easy to prove that if ψ is a (c)-comparison function, then $\psi(t) < t$ for any $t > 0$ and $\psi(0) = 0$.

Definition 2.7. Let (X, d) be a metric space and $f : X \rightarrow X$ be a mapping. We say that f is an interpolative Meir-Keeler contraction of rational Barada type on X if there exist $\psi \in \Psi$ and $\gamma \in [0, 1)$, such that for every $u, v \in X$ we have

(1) given $\varepsilon > 0$, there exists $\delta(\varepsilon) > 0$ so that

$$\varepsilon \leq \psi(\mathcal{B}(u, v)) < \varepsilon + \delta \implies d(fu, fv) < \varepsilon, \tag{12}$$

(2)

where

$$\mathcal{B}(u, v) = (d(u, v))^\gamma \left(\frac{d(u, fu)d(u, fv) + d(v, fv)d(v, fu)}{\max\{d(u, fv), d(v, fu)\}} \right)^{1-\gamma}. \tag{13}$$

Lemma 2.8. Let (X, d) be a metric space and $f : X \rightarrow X$ be an interpolative Meir-Keeler contraction of rational Das-Gupta on X . Then, we have

$$d(fu, fv) < \psi(\mathcal{B}(u, v)), \tag{14}$$

Proof. We define the sequence $\{x_k\}$ in X by $x_{k+1} = f(x_k)$ for all $k \geq 0$. If $x_{k_0} = x_{k_0+1}$ for some k_0 , then x_{k_0} is a fixed point of f . Now, we assume that $x_k \neq x_{k+1}$ for all $k \in \mathbb{N}$. Now, it follows that

$$\begin{aligned} d(x_{k+1}, x_k) &= d(f(x_k), f(x_{k-1})) \\ &\leq \psi(\mathcal{B}(x_k, x_{k-1})) \\ &\leq \psi \left((d(x_k, x_{k-1}))^\gamma \left(\frac{d(x_{k-1}, x_k)d(x_{k-1}, x_{k+1}) + d(x_k, x_{k+1})d(x_k, x_k)}{\max\{d(x_{k-1}, x_{k+1}), d(x_k, x_k)\}} \right)^{1-\gamma} \right) \\ &= \psi \left((d(x_{k-1}, x_k))^\gamma (d(x_{k-1}, x_k))^{1-\gamma} \right), \\ &= \psi(d(x_{k-1}, x_k)), \end{aligned}$$

for all $k \in \mathbb{N}$. Inductively, we obtain

$$d(x_{k+1}, x_k) \leq \psi^k(d(x_1, x_0)).$$

Now, we prove that $\{x_k\}$ is a Cauchy sequence. Let $\varepsilon > 0$ and $n(\varepsilon) \in \mathbb{N}$ in such a way that $\sum_{k \geq n(\varepsilon)} \psi^k(d(x_1, x_0)) < \varepsilon$.

Let $n, m \in \mathbb{N}$ with $m > n > n(\varepsilon)$. By the triangle inequality, we get

$$\begin{aligned} d(x_n, x_m) &\leq \sum_{k=n}^{m-1} d(x_k, x_{k+1}) \leq \sum_{k=n}^{m-1} \psi^k(d(x_1, x_0)) \\ &\leq \sum_{k \geq n(\varepsilon)} \psi^k(d(x_1, x_0)) < \varepsilon. \end{aligned}$$

Hence, we deduce that $\{x_k\}$ is a Cauchy sequence in the complete metric space (X, d) . Thus, there exists $u \in X$ such that $\lim_{k \rightarrow \infty} x_k = u$.

For each $k \in \mathbb{N}$, we have

$$\begin{aligned} 0 \leq d(u, f(u)) &\leq d(f(x_k), u) + d(f(x_k), f(u)) \\ &\leq d(x_{k+1}, u) + \mathcal{B}(f(x_k), f(u)) \\ &\leq d(x_{k+1}, u) + \psi \left(d(x_k, u)^\gamma \left(\frac{d(x_k, f(x_k))d(x_k, f(u)) + d(u, f(u))d(u, f(x_k))}{d(x_k, f(u)) + d(u, f(x_k))} \right)^{1-\gamma} \right). \end{aligned}$$

Letting $k \rightarrow \infty$ in the above inequality and using the right continuity of ψ at 0 and Remark 1.1 we infer that

$$d(u, f(u)) \leq 0.$$

Thus, we have $d(u, f(u)) = 0$, that is $u = f(u)$. Therefore $u \in X$ is a fixed point of f which achieves the proof. \square

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