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Some classes of matrix transforms connected with subspaces of Maddox spaces defined by speeds of convergence

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Abstract.
Let X, Y be two sets of sequences with real or complex entries, and (X, Y) the set of matrices (with real or complex entries) to map X into Y. Let λ and μ be speeds of the convergence, i.e.; monotonically increasing positive sequences. In this paper, we give necessary and sufficient conditions for a matrix $A \in (X, Y)$, if X is the subset of the set of convergent sequences defined by λ, and Y is the subset of the certain Maddox space defined by μ.

1. Introduction

Let X, Y be two sequence spaces and $A = (a_{nk})$ be a matrix with real or complex entries. Throughout this paper we assume that indices and summation indices run from 0 to ∞ unless otherwise specified. If for each $x = (x_k) \in X$ the series

$$A_n x = \sum_k a_{nk} x_k$$

converge and the sequence $Ax = (A_n x)$ belongs to Y, we say that the matrix A transforms X into Y. By (X, Y) we denote the set of all matrices which transform X into Y. Let ω be the set of all real or complex valued sequences. Further we need the following well-known subspaces of ω: c - the space of all convergent sequences, c_0 - the space of all sequences converging to zero, l_∞ - the space of all bounded sequences, and

$$l_1 := \{x = (x_n) \, : \, \sum_n |x_n| < \infty\}.$$

Let throughout this paper $\lambda = (\lambda_k)$ be a positive monotonically increasing sequence, i.e.; the speed of convergence. Following Kangro [11], [12] a convergent sequence $x = (x_k)$ with

$$\lim_k x_k := s \text{ and } v_k = \lambda_k (x_k - s) \tag{1.1}$$

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is called bounded with the speed λ (shortly, λ-bounded) if $v_k = O(1)$ (or $(v_k) \in l_\infty$), and convergent with the speed λ (shortly, λ-convergent) if there the finite limit

$$\lim_k v_k := b$$

exists (or $(v_k) \in c$). Following the authors of the current paper (see [1]), a convergent sequence $x = (x_k)$ with the finite limit s is called absolutely convergent with speed λ (or shortly, absolutely λ-convergent), if $(v_k) \in l_1$. We denote the set of all λ-bounded sequences by l_∞^λ, the set of all λ-convergent sequences by c^λ, and the set of all absolutely λ-convergent sequences by l_1^λ. Moreover, let

$$c_0^\lambda := \{x = (x_k) : x \in c^\lambda \text{ and } \lim_k \lambda_k(x_k - s) = 0\}.$$

It is not difficult to see that

$$l_1^\lambda \subset c_0^\lambda \subset c^\lambda \subset l_\infty^\lambda \subset c.$$

In addition to it, for unbounded sequence λ these inclusions are strict. For $\lambda_k = O(1)$, we get $c^\lambda = l_\infty^\lambda = c$.

Let $e := (1, 1, ...)$, $e^k := (0, ..., 0, 1, 0, ...)$, where 1 is in the k-th position, and $\lambda^{-1} := (1/\lambda_k)$. We note that

$$e, e^k, \lambda^{-1} \in c^\lambda; \quad e, e^k \in l_1^\lambda \cap c_0^\lambda.$$

Let $p := (p_k)$ be a sequence of strictly positive numbers, and let

$$c_0(p) := \{x = (x_k) : \lim_k |x_k|^{p_k} = 0\},$$

$$l_\infty(p) := \{x = (x_k) : |x_k|^{p_k} = O(1)\},$$

$$c(p) := \{x = (x_k) : \lim_k |x_k - l|^{p_k} = 0 \text{ for some } l \in \mathrm{C}\},$$

$$l(p) := \{x = (x_k) : \sum_k |x_k|^{p_k} < \infty\}.$$

The sets $c_0(p)$, $l_\infty(p)$, $c(p)$ and $l(p)$ are known as Maddox spaces (see, for example, [15], [16] and [19]). The Maddox spaces are also the paranormed spaces if p is bounded. Good overview on the paranormed spaces, including the Maddox spaces, has been given, for example, in [9] and [17]. If $p_k \equiv 1$, then

$$c_0(p) = c_0, \ c(p) = c, \ l_\infty(p) = l_\infty, \ l(p) = l_1.$$

Definition 1.1. *We say that a convergent sequence $x = (x_k)$ with the finite limit s is paranormally bounded with speed λ (shortly, paranormally λ-bounded), if $(v_k) \in l_\infty(p)$.*

Definition 1.2. *We say that a convergent sequence $x = (x_k)$ with the finite limit s is paranormally convergent with speed λ (shortly, paranormally λ-convergent), if $(v_k) \in c(p)$.*

Definition 1.3. *We say that a convergent sequence $x = (x_k)$ with the finite limit s is paranormally zero-convergent with speed λ (shortly, paranormally λ–zero–convergent), if $(v_k) \in c_0(p)$.*

Definition 1.4. *We say that a convergent sequence $x = (x_k)$ with the finite limit s is paranormally absolutely convergent with speed λ (shortly, paranormally absolutely λ-convergent), if $(v_k) \in l(p)$.*

We note that Definitions 1.1 - 1.3 first are presented in [18]. The set of all paranormally λ-bounded sequences we denote by $(l_\infty(p))^\lambda$, the set of all paranormally λ-convergent sequences by $(c(p))^\lambda$, the set of all paranormally λ-zero-convergent sequences by $(c_0(p))^\lambda$, and the set of all paranormally absolutely λ-convergent sequences by $(l(p))^\lambda$. It is easy to see that for $p_k \equiv 1$ we have

$$(l_\infty(p))^\lambda = l_\infty^\lambda, \ (c(p))^\lambda = c^\lambda, \ (c_0(p))^\lambda = c_0^\lambda, \ (l(p))^\lambda = l_1^\lambda.$$

Let $\mu := (\mu_k)$ be another speed of convergence, i.e., a monotonically increasing positive sequence. Matrix classes (X, Y), where X is one of the sets l_∞^λ, c^λ, c_0^λ or l_1^λ and Y is one of the sets l_∞^μ, c^μ, c_0^μ or l_1^μ have been characterized by Kangro in [11] and [12], and by the authors of the present work in [1] and [2]. A short overview on the convergence with speed has been presented in [3] and [13].

We note that the results connected with boundedness, convergence and absolute convergence with speed can be used in several applications. For example, in the theoretical physics such results can be used for accelerating the slowly convergent processes, a good overview of such investigations can be found, for example, from the sources [8] and [10]. These results also have several applications in the approximation theory. Besides, in [4] and [5] such results are used for the estimation of the order of approximation of Fourier expansions in Banach spaces.

Necessary and sufficient conditions for a matrix $A \in (X, Y)$, if X is one of the sets l_∞^λ, c^λ or c_0^λ, and Y is one of the sets $(l_\infty(p))^\mu$, $(c(p))^\mu$ or $(c_0(p))^\mu$ have been presented in [18]. The present paper is the continuation of the paper [18]. We give the characterization of matrix classes $(l_1^\lambda, (c_0(p))^\mu)$, $(l_1^\lambda, (c(p))^\mu)$, $(l_1^\lambda, (l_\infty(p))^\mu)$, $(l_1^\lambda, (l(p))^\mu)$, $(c_0^\lambda, (l(p))^\mu)$, $(c^\lambda, (l(p))^\mu)$ and $(l_\infty^\lambda, (l(p))^\mu)$.

2. Auxiliary results

For the proof of the main results we need some auxiliary results.

Lemma 2.1 ([7], p. 44, see also [20], Proposition 12). *A matrix $A = (a_{nk}) \in (c_0, c)$ if and only if*

$$\lim_n a_{nk} := a_k \text{ for all } k, \tag{2.1}$$

$$\sum_k |a_{nk}| = O(1). \tag{2.2}$$

Moreover,

$$\lim_n A_n x = \sum_k a_k x_k \tag{2.3}$$

for every $x = (x_k) \in c_0$.

Lemma 2.2 ([7], p. 51, see also [20], Proposition 10). *The following statements are equivalent:*

(a) $A = (a_{nk}) \in (l_\infty, c)$.

(b) *The conditions* (2.1), (2.2) *are satisfied and*

$$\lim_n \sum_k |a_{nk} - a_k| = 0.$$

(c) *The condition* (2.1) *holds and the series* $\sum_k |a_{nk}|$ *converges uniformly in n.*

Moreover, if one of statements (a)-(c) is satisfied, then the equation (2.3) *holds for every $x = (x_k) \in l_\infty$.*

Lemma 2.3 ([7], p. 46, see also [20], Proposition 11). *A matrix $A = (a_{nk}) \in (c, c)$ if and only if conditions* (2.1), (2.2) *are satisfied and there exists a finite limit*

$$\lim_n \sum_k a_{nk} = \tau.$$

Moreover, if $\lim_k x_k = s$ for $x = (x_k) \in c$, then

$$\lim_n A_n x = s\tau + \sum_k (x_k - s) a_k.$$

Lemma 2.4 ([6], Theorem 3.1 and [20], Propositions 17). *A matrix $A = (a_{nk}) \in (l_1, c)$ if and only if condition* (2.1) *is satisfied and $a_{nk} = O(1)$. Moreover, equation* (2.3) *holds for every $x = (x_k) \in l_1$.*

Lemma 2.5 ([17], Theorem 4.3 or [14], Theorem 5). *Let $p := (p_k)$ be a bounded sequence of strictly positive numbers and $B = (b_{nk})$ a matrix with real or complex entries.*

(a) $B \in (l_1, l_\infty(p))$ if and only if

$$|b_{nk}|^{p_n} = O(1). \tag{2.4}$$

(b) $B \in (l_1, c_0(p))$ if and only if

$$\lim_n |b_{nk}|^{p_n} = 0 \ for\ every\ k, \tag{2.5}$$

$$\lim_{M\to\infty} \sup_n \sup_k \left(\frac{|b_{nk}|}{M}\right)^{p_n} = 0. \tag{2.6}$$

(c) $A \in (l_1, c(p))$ if and only if

$$b_{nk} = O(1) \tag{2.7}$$

and there exists complex numbers $b_1, b_1, ...$ such that

$$\lim_n |b_{nk} - b_k|^{p_n} = 0 \ for\ every\ k, \tag{2.8}$$

$$\lim_{M\to\infty} \sup_n \sup_k \left(\frac{|b_{nk} - b_k|}{M}\right)^{p_n} = 0. \tag{2.9}$$

Lemma 2.6 ([9], Theorem 5.1, 0. or [17], Remark 4.12 (b)). *Let $p := (p_k)$ be a sequence satisfying the condition $p_k \geq 1$ for every k and $B = (b_{nk})$ a matrix with real or complex entries. Then $B \in (l_1, l(p))$* if and only if

$$\sum_n |b_{nk}|^{p_n} = O(1). \tag{2.10}$$

Lemma 2.7 ([17], Theorem 4.1 (b)). *Let $p := (p_k)$ be a sequence satisfying the condition $p_k \geq 1$ for every k, and $B = (b_{nk})$ a matrix with real or complex entries. Then $B \in (l_\infty, l(p)) = (c_0, l(p))$* if and only if

$$\sum_n \left|\sum_{k\in K} b_{nk}\right|^{p_n} = O(1),\ K\ is\ a\ finite\ subset\ of\ N. \tag{2.11}$$

3. Matrix transforms from l_1^λ into $(l_\infty(p))^\mu$, $(c(p))^\mu$, $(c_0(p))^\mu$ and $(l(p))^\mu$

Now we are able to prove the main results of the paper. To formulate these results, further in the present and next section we use the matrix $B = (b_{nk})$ defined by

$$b_{nk} := \frac{\mu_n(a_{nk} - a_k)}{\lambda_k},$$

provided that condition (2.1) holds. Further, throughout the paper we assume that p is a bounded sequence.

Theorem 3.1. *A matrix* $A = (a_{nk}) \in \left(l_1^\lambda, (l_\infty(p))^\mu\right)$ *if and only if condition* (2.4) *holds and*

$$Ae^k \in l_\infty(p))^\mu, \tag{3.1}$$

$$Ae = (A_n e) \in (l_\infty(p))^\mu, \ \tau_n := A_n e = \sum_k a_{nk}, \tag{3.2}$$

$$\frac{a_{nk}}{\lambda_k} = O(1). \tag{3.3}$$

Proof. Necessity. Assume that $A \in \left(l_1^\lambda, (l_\infty(p))^\mu\right)$. As $e \in l_1^\lambda$ and $e^k \in l_1^\lambda$, then conditions (3.1) and (3.2) hold. Besides, (3.1) implies the validity of (2.1). Since, from (1.1) we have

$$x_k = \frac{v_k}{\lambda_k} + s; \ s := \lim_k x_k, \ (v_k) \in l_1$$

for every $x := (x_k) \in l_1^\lambda$, it follows that

$$A_n x = \sum_k \frac{a_{nk}}{\lambda_k} v_k + s\tau_n. \tag{3.4}$$

As $(\tau_n) \in (l_\infty(p))^\mu$ by (3.2), then the finite limit

$$\tau := \lim_n \tau_n \tag{3.5}$$

exists. Hence, from (3.4) we obtain that the matrix

$$A_\lambda := \left(\frac{a_{nk}}{\lambda_k}\right)$$

transforms this sequence $(v_k) \in l_1$ into c. In addition, for every sequence $(v_k) \in l_1$, the sequence $(v_k/\lambda_k) \in c_0$. But, for (v_k/λ_k), there exists a convergent sequence $x := (x_k)$ with $s := \lim_k x_k$, such that $v_k/\lambda_k = x_k - s$. So we have proved that, for every sequence $(v_k) \in l_1$ there exists a sequence $(x_k) \in l_1^\lambda$ such that $v_k = \lambda_k (x_k - s)$. Hence $A_\lambda \in (l_1, c)$. This implies, by Lemma 2.4, that condition (3.3) is satisfied and the finite limit

$$\phi := \lim_n A_n x = \sum_k \frac{a_k}{\lambda_k} v_k + s\tau$$

exists for every $x \in l_1^\lambda$. Writing

$$\mu_n(A_n x - \phi) = \mu_n \sum_k \frac{a_{nk} - a_k}{\lambda_k} v_k + s\mu_n(\tau_n - \tau), \tag{3.6}$$

we conclude, by (3.2) that the matrix $B \in (l_1, l_\infty(p))$. Hence condition (2.4) is satisfied by Lemma 2.5 (a).

Sufficiency. Let condition (2.4) and conditions (3.1) - (3.3) be fulfilled. Then relation (3.4) also holds for every $x \in l_1^\lambda$ and $(\tau_n) \in (l_\infty(p))^\mu$ by (3.2). In addition, $A_\lambda \in (l_1, c)$ and the finite limit ϕ exists for every $x \in l_1^\lambda$ by Lemma 2.4, since (3.1) and (3.3) hold. Hence relation (3.6) holds for every $x \in l_1^\lambda$. As (2.4) is valid, then $B \in (l_1, l_\infty(p))$ by Lemma 2.5 (a). Therefore, due to (3.2), $A \in \left(l_1^\lambda, (l_\infty(p))^\mu\right)$. □

Theorem 3.2. *A matrix* $A = (a_{nk}) \in \left(l_1^\lambda, (c_0(p))^\mu\right)$ *if and only if conditions* (2.5), (2.6), (3.3) *hold and*

$$Ae^k \in (c_0(p))^\mu, \tag{3.7}$$

$$Ae = (A_n e) \in (c_0(p))^\mu, \ \tau_n := A_n e = \sum_k a_{nk}, \tag{3.8}$$

Proof is similar to the proof of Theorem 3.1. The only difference is that now instead of conditions (3.1) and (3.2) are (3.7) and (3.8), and $B \in (l_1, c_0(p))$. Therefore instead of Lemma 2.5 (a) we use Lemma 2.5 (b). □

Theorem 3.3. *A matrix* $A = (a_{nk}) \in \left(l_1^\lambda, c(p))^\mu\right)$ *if and only if conditions* (2.7), (3.3) *are satisfied,*

$$Ae^k \in c(p))^\mu, \tag{3.9}$$

$$Ae = (A_n e) \in c(p))^\mu, \ \tau_n := A_n e = \sum_k a_{nk}, \tag{3.10}$$

and there exists complex numbers $b_1, b_1, \ldots$ *such that conditions* (2.8) *and* (2.9) *hold.*
Proof is similar to the proof of Theorem 3.1. The only difference is that now instead of conditions (3.1) and (3.2) are (3.9) and (3.10), and $B \in (l_1, c(p))$. So instead of Lemma 2.5 (a) we use Lemma 2.5 (c). □

Theorem 3.4. *Let* $p = (p_k)$ *be a sequence satisfying the condition* $p_k \geq 1$. *A matrix* $A = (a_{nk}) \in \left(l_1^\lambda, (l(p))^\mu\right)$ *if and only if conditions* (2.10), (3.3) *are satisfied, and*

$$Ae^k \in (l(p))^\mu, \tag{3.11}$$

$$Ae = (A_n e) \in (l(p))^\mu, \ \tau_n := A_n e = \sum_k a_{nk}. \tag{3.12}$$

Proof is similar to the proof of Theorem 3.1. The only difference is that now instead of conditions (3.1) and (3.2) are (3.11) and (3.12), and $B \in (l_1, (l(p))$. So instead of Lemma 2.5 (a) we use Lemma 2.6. □

4. Matrix transforms from c_0^λ, c^λ and l_∞^λ into $(l(p))^\mu$

In this section we characterize the matrix classes $(c_0^\lambda, (l(p))^\mu)$, $(c^\lambda, (l(p))^\mu)$ and $(l_\infty^\lambda, (l(p))^\mu)$.

Theorem 4.1. *Let* $p = (p_k)$ *be a sequence satisfying the condition* $p_k \geq 1$. *A matrix* $A = (a_{nk}) \in \left(l_\infty^\lambda, (l(p))^\mu\right)$ *if and only if conditions* (2.11), (3.11), (3.12) *are satisfied, and*

$$\sum_k \frac{|a_{nk}|}{\lambda_k} = O(1), \tag{4.1}$$

$$\lim_n \sum_k \frac{|a_{nk} - a_k|}{\lambda_k} = 0. \tag{4.2}$$

Proof is similar to the proof of Theorem 3.1. The only difference is that now instead of $A_\lambda \in (l_1, c)$ we have $A_\lambda \in (l_\infty, c)$, and instead of $B \in (l_1, (l_\infty(p))$ we obtain $B \in (l_\infty, (l(p))$. Therefore instead of Lemma 2.4 we use Lemma 2.2 (a) and (b), and instead of Lemma 2.5 (a) we use Lemma 2.7. □

Remark 4.1. Using Lemma 2.3 (a) and (c) we obtain that conditions (4.1) and (4.2) can be replaced by the condition

$$\text{the series } \sum_k \frac{|a_{nk}|}{\lambda_k} \text{ converges uniformly in } n$$

in Theorem 4.1.

In the following we characterize the matrix class $\left(c^\lambda, (l(p))^\mu\right)$.

Theorem 4.2. *Let $p = (p_k)$ be a sequence satisfying the condition $p_k \geq 1$. A matrix $A = (a_{nk}) \in \left(c^\lambda, (l(p))^\mu\right)$ if and only if conditions* (2.11), (3.11), (3.12), (4.1) *are satisfied, and*

$$A\lambda^{-1} \in (l(p))^\mu. \tag{4.3}$$

Proof. Necessity. Assume that $A \in \left(c^\lambda, (l(p))^\mu\right)$. As $e^k \in c^\lambda$, $e \in c^\lambda$ and $\lambda^{-1} \in c^\lambda$, then conditions (3.11), (3.12) and (4.3) hold. Besides, (3.11) implies the validity of (2.1). As equality (3.4) holds for every $x := (x_k) \in c^\lambda$, and the finite limit (3.5) exists due to $Ae \in (l(p))^\mu$, then A_λ transforms this convergent sequence (v_k) into c. Similar to the proof of the necessity of Theorem 3.1, it is possible to show that, for every sequence $(v_k) \in c$, there exists a sequence $(x_k) \in c^\lambda$ such that $v_k = \lambda_k (x_k - s)$. Hence $A_\lambda \in (c, c)$. This implies by Lemma 2.3 that the finite limits a_k and

$$a^\lambda := \lim_n \sum_k \frac{a_{nk}}{\lambda_k}$$

exist, and that condition (4.1) is satisfied. With the help of (3.4), for every $x \in c^\lambda$, we can write by Lemma 2.3 that

$$\phi := \lim_n A_n x = a^\lambda b + \sum_k \frac{a_k}{\lambda_k} (v_k - b) + \tau s, \tag{4.4}$$

where $s := \lim_k x_k$ and $b := \lim_k v_k$. Now, using (3.4) and (4.4), we obtain

$$\mu_n(A_n x - \phi) = \mu_n \sum_k \frac{a_{nk} - a_k}{\lambda_k} (v_k - b) + \mu_n (\tau_n - \tau) s + \mu_n \left(\sum_k \frac{a_{nk}}{\lambda_k} - a^\lambda \right) b. \tag{4.5}$$

As $Ae \in (l(p))^\mu$ and $A\lambda^{-1} \in (l(p))^\mu$ by (3.12) and (4.3), then $B \in (c_0, l(p))$. Therefore we can conclude by Lemma 2.7 that condition (2.11) holds.

Sufficiency. Suppose that conditions (2.11), (3.11), (3.12), (4.1) and (4.3) are satisfied. We note that relation (3.4) holds for every $x \in c^\lambda$ and the finite limits a_k, τ and a^λ exist correspondingly by (3.11), (3.12) and (4.3). As (4.1) is also satisfied, then $A_\lambda \in (c, c)$ by Lemma 2.3, and therefore relations (4.4) and (4.5) hold for every $x \in c^\lambda$. As condition (2.11) holds, then $B \in (c_0, l_1(p))$ by Lemma 2.7. In addition, $Ae \in (l(p))^\mu$ and $A\lambda^{-1} \in (l(p))^\mu$ correspondingly by (3.12) and (4.3). Thus, $A \in \left(c^\lambda, (l(p))^\mu\right)$. □

Theorem 4.3. *Let $p = (p_k)$ be a sequence satisfying the condition $p_k \geq 1$. A matrix $A = (a_{nk}) \in \left(c_0^\lambda, (l(p))^\mu\right)$ if and only if conditions* (2.11), (3.11), (3.12) *and* (4.1) *are satisfied.*

Proof is similar to the proof of Theorem 4.2. We only note that that in this case λ^{-1} does not belong into c_0^λ. □

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