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Common fixed point theorems for generalized orthogonal contractions

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Abstract. Fixed point theory began with Banach contraction principle in complete metric spaces. Completeness is a major and important condition for fixed point theorems. In this paper, we define orthogonal ($E.A$) property and give a generalized contraction for two mappings. We prove common fixed point theorems for mappings satisfying orthogonal ($E.A$) property without completeness in orthogonal metric spaces.

1. Introduction

Metric spaces are an important tool in modern analysis and were produced by Frechet in 1906. Fixed point theory in complete metric spaces began with the Banach contraction principle. Fixed point hypotheses are important tools for demonstrating the existence and uniqueness of solutions for different problems. In the literature, many different contraction principles have been produced in both metric spaces and generalized metric spaces. Therefore, fixed point and common fixed point theorems have been proven.

The weak conditions of commutativity of a pair of self mappings was initiated by Sessa [25] with the introduction of the notion of weakly commuting pair. Later on, Jungck [11] enlarged the class of weakly commuting mappings by introducing the notion of compatible mappings which was further widened by Jungck with the notion of weakly compatible mappings. Pant [19, 20] initially investigated common fixed points of noncompatible mappings defined on metric spaces. Aamri and El Moutawakil [1] defined ($E.A$) property which generalizes the concept of noncompatible mappings and gave some common fixed point theorems under strict contractive conditions without completeness of metric spaces. Liu et al. [14] defined the notion of common ($E.A$) property, which contains the ($E.A$) property and proved several fixed point theorems under hybrid contractive conditions. Moreover, some researchers proved common fixed point theorems for mappings satisfying ($E.A$) property [2, 5, 10, 16–18, 21, 26].

Recently, the concept of the orthogonal set was introduced in [3, 6]. A generalization of Banach fixed point theorem was proved by Gordji and Habibi [7]. They worked on classical existence and uniqueness theorems of solutions to the first-order differential equation in orthogonal metric spaces. Ramezani and Baghani [22] introduced the concept of the strongly orthogonal set and analyzed the existence of fixed points for generalized contractive operators in strongly metric spaces. Bilgili and Turkoglu [9] presented fixed point theorems on orthogonal metric spaces via altering distance functions. Beg et al. [4] presented the notion of generalized orthogonal F-Suzuki contraction mapping and obtained fixed point theorems on

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orthogonal b-metric spaces. Later, notions of generalized orthogonal F-contraction and orthogonal F-Suzuki contraction mappings were presented by Mani et al. [15]. Thus, many results that were very common in the literature were generalized. Sawangsup et.al. defined F-contraction in orthogonal metric spaces [23]. Senapati et. al. and Bilgili proved some orthogonal fixed point theorems using w-distance function [8, 24]. Kanwal et.al. [12] prove some fixed and periodic point theorems for orthogonal contraction in orthogonal F-metric spaces and apply the results for existence and uniqueness of the solution of nonlinear fractional differential equation. Lael [13] introduced common fixed point theorems in orthogonal modular metric spaces. Uddin et.al. [27] gave the concept of orthogonal m-metric space.

In this work, orthogonal $(E.A)$ property was defined. A generalized contraction via altering distance functions was introduced and common fixed point theorems were proved for two mappings satisfying orthogonal $(E.A)$ property in orthogonal metric spaces.

2. Preliminaries

Definition 2.1. *[6] Let $X \neq \emptyset$ and $\perp \subseteq X \times X$ be a binary relation. If $\perp$ satisfies the following condition*

$$\exists x_0 \in X; (\forall y \in X, y \perp x_0) \vee (\forall y \in X, x_0 \perp y)$$

then $(X, \perp)$ is called an orthogonal set (shortly O-set). And the element x_0 is called an orthogonal element.

Definition 2.2. *[6] Let $(X, \perp)$ be an orthogonal set (O-set). Any two elements $x, y \in X$ are said to be orthogonally related if $x \perp y$.*

Example 2.3. *Let $X = [0, \infty)$ and define $x \perp y$ if $xy \in \{x, y\}$. Then by the setting $x_0 = 0$ or $x_0 = 1$, $(X, \perp)$ is an orthogonal set.*

Definition 2.4. *[6] A sequence $\{x_n\}$ is called orthogonal sequence (shortly O-sequence) if*

$$(\forall n \in N; x_n \perp x_{n+1}) \vee (\forall n \in N; x_{n+1} \perp x_n).$$

Similarly, a Cauchy sequence $\{x_n\}$ is said to be an orthogonally Cauchy sequence (shortly O-Cauchy sequence) if

$$(\forall n \in N; x_n \perp x_{n+1}) \vee (\forall n \in N; x_{n+1} \perp x_n).$$

Definition 2.5. *[6] Let $(X, \perp)$ be an orthogonal set and d be a usual metric on X. Then $(X, \perp, d)$ is called an orthogonal metric space (shortly O-metric space).*

Definition 2.6. *[6] An O-metric space $(X, \perp, d)$ is said to be a complete O-metric space (O-complete) if every O-Cauchy sequence converges in X.*

Definition 2.7. *[6] Let $(X, \perp, d)$ be an O-metric space. A function $f : X \to X$ is said to be orthogonally continuous ($\perp$-continuous) at x if for each O-sequence $\{x_n\}$ converging to x implies $fx_n \to fx$ as $n \to \infty$. Also f is $\perp$-continuous on X if f is $\perp$-continuous at every $x \in X$.*

Definition 2.8. *[6] Let $(X, \perp, d)$ be an O-metric space and $\lambda \in R$ with $0 < \lambda < 1$ where R is the set of real numbers. A function $f : X \to X$ is said to be orthogonal contraction ($\perp$-contraction) with Lipschitz constant λ if*

$$d(fx, fy) \leq \lambda d(x, y)$$

for all $x, y \in X$ whenever $x \perp y$.

Definition 2.9. *[6] Let $(X, \perp, d)$ be an O-metric space. A function $f : X \to X$ is called orthogonal preserving ($\perp$-preserving) if $fx \perp fy$ whenever $x \perp y$.*

Theorem 2.10. *[6] Let $(X, \perp, d)$ be an O-complete metric space and $0 < \lambda < 1$. Let $f : X \to X$ be $\perp$-continuous, $\perp$-contraction (with Lipschitz constant λ) and $\perp$-preserving. Then f has a unique fixed point $x^* \in X$. Therefore Picard operator converges to x^*.*

Definition 2.11. *[11] Let f and g be self mappings on nonempty set X. If $fx = gx$ for some $x \in X$, then x is called coincidence point of f and g. If f and g commute at their coincidence point, then f and g is said to be weakly compatible.*

Definition 2.12. *Let $(X, \perp, d)$ be an O-metric space and $f, g : X \to X$ be self mappings. f and g are said to satisfy the orthogonal $(E.A)$ property (shortly $O - (E.A)$ property) if there exists an O-sequence $\{x_n\}$ such that*

$$lim_{n\to\infty} fx_n = lim_{n\to\infty} gx_n = t$$

for some $t \in X$.

Example 2.13. *Let $X = [0, 1)$. Define $d : X \times X \to [0, \infty)$ such that $d(x, y) = \left|x - y\right|$. Let the binary relation $\perp$ on X such that $x \perp y \Longleftrightarrow x \leq y \leq \frac{1}{2}$ or $x = 0$. Then $(X, \perp)$ is an orthogonal set and $(X, \perp, d)$ is an O-metric space. But is not O-complete. We define O-sequence $(x_n) = 1 - \frac{1}{n}$ and (x_n) is an O-Cauchy and converges to 1. So $(X, \perp, d)$ is not O-complete. Define*

$$f, g : X \to X, \quad f(x) = x - x^2, \quad g(x) = \frac{1 - x}{2}$$

then

$$lim_{n\to\infty} fx_n = lim_{n\to\infty} gx_n = 0.$$

Hence, f and g satisfy $O - (E.A)$ property.

3. Main Results

In this section, we suppose that the set

$$\Psi = \{\psi : [0, \infty) \to [0, \infty) : \psi \text{ is nondecreasing, continuous, sub additive and } \psi(t) = t \text{ for } t = 0\}.$$

Theorem 3.1. *Let $(X, \perp, d)$ be an O-metric space and let $f, g : X \to X$ be mappings such that*

$$\psi(d(fx, fy)) \leq \psi(M(x, y)) - \varphi(M(x, y)) \tag{1}$$

where

$$M(x, y) = \max\left\{d(gx, gy), d(fx, gx), d(fy, gy), \frac{d(fx, gy) + d(gx, fy)}{2}\right\}$$

for each $x, y \in X$ with $x \perp y$, $x \neq y$ and $\psi, \varphi \in \Psi$. Suppose that

(i) $f(X) = g(X)$

(ii) *g is one to one and $g^{-1} \circ f$ is an orthogonal preserving map,*

(iii) *$\{f, g\}$ satisfy $O - (E.A)$ property,*

(iv) *$g(X)$ is closed in X.*

Then f and g have a unique coincidence point. Moreover if f and g are weakly compatible, then f and g have a unique common fixed point.

Proof. Since X is an O-set, there exists $x_0 \in X$ such that

$$x_0 \perp x, \quad \forall x \in X.$$

As $f(X) = g(X)$, there exists $x_1 \in X$ such that $fx_0 = gx_1$. We define the sequence $\{x_n\}$ in X as $fx_n = gx_{n+1}$ for all $n \in \mathbb{N}$. Thus $x_{n+1} = g^{-1} \circ fx_n$. Since $g^{-1} \circ f$ is $\perp$ −preserving and $x_0 \perp x_k$ for all $\forall k \in \mathbb{N}$, we have,

$$\begin{array}{rcl} g^{-1} \circ fx_0 & \perp & g^{-1} \circ fx_k \\ x_1 & \perp & x_{k+1} \\ & & \vdots \\ x_n & \perp & x_{k+n}. \end{array}$$

Hence x_n is an O-sequence.

If the pairs (f, g) satisfies the O-$(E.A)$ property, then there exists an O-sequence $\{x_n\}$ in X satisfying

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = q,$$

for some $q \in X$. Since $g(X)$ is closed subspace of X, there exists a $r \in X$, such that $\lim_{n\to\infty} fx_n = gr = q$.

Now, we prove that $x_n \perp r$. Since $f(X) = g(X)$, we have $g^{-1} \circ f(X) = X$. Moreover $(g^{-1} \circ f \circ ... \circ g^{-1} \circ f)(X) = X$, and there exists $t \in X$ such that $(g^{-1} \circ f \circ ... \circ g^{-1} \circ f)(t) = r$.

Since $x_0 \perp t$ and $g^{-1} \circ f$ is $\perp$-preserving, then

$$\begin{array}{rcl} (g^{-1} \circ f)(x_0) & \perp & \left(g^{-1} \circ f\right)(t) \Longrightarrow x_1 \perp \left(g^{-1} \circ f\right)(t) \\ & & \vdots \\ x_n & \perp & (g^{-1} \circ f \circ ... \circ g^{-1} \circ f)(t). \end{array}$$

Thus we have $x_n \perp r$.

By (1),

$$\psi(d(fx_n, fr)) \leq \psi(M(x_n, r)) - \varphi(M(x_n, r)) \tag{2}$$

where

$$M(x_n, r) = \max\left\{d(gx_n, gr), d(fx_n, gx_n), d(fr, gr), \frac{d(fx_n, gr) + d(gx_n, fr)}{2}\right\}.$$

Letting $n \to \infty$,

$$\begin{aligned} \lim_{n\to\infty} M(x_n, r) &= \max\left\{d(gr, gr), d(gr, gr), d(fr, gr), \frac{d(gr, gr) + d(gr, fr)}{2s}\right\} \\ &= d(fr, gr). \end{aligned}$$

Now, using (2) and definition of ψ and φ, as $n \to \infty$,

$$\psi(d(fr, gr) \leq \psi(d(fr, gr)) - \varphi(d(q, gr))$$

which implies $\varphi(d(fr, gr)) \leq 0$ give $gr = fr$. Thus r is a coincidence point of the pair f and g .

For the uniqueness of coincidence point, we assume on the contrary. Let s be another coincidence point of f and g, i.e. $gs = fs$ with $r \neq s$.

Case 1:

If $r \perp s$ or $s \perp r$, then by (1) we have

$$\psi(d(fr, fs) \leq \psi(M(r, s)) - \varphi(M(r, s))$$

where

$$\begin{aligned} M(r,s) &= \max\left\{ d(gr,gs), d(fr,gs), d(fs,gs), \frac{d(fr,gs)+d(gr,fs)}{2} \right\} \\ &= \max\left\{ d(fr,fs), d(fr,fs), 0, \frac{d(fr,fs)+d(fr,fs)}{2} \right\} \\ &= d(fr,fs). \end{aligned}$$

Hence $fr = fs$.

Case 2:

If not, for the choosen an O-element $x_0 \in X$,

$$x_0 \perp r \text{ and } x_0 \perp s, \text{ or } r \perp x_0 \text{ and } s \perp x_0$$

and since $g^{-1} \circ f$ is orthogonal preserving map,

$$\begin{aligned} (g^{-1} \circ f)(x_0) &\perp (g^{-1} \circ f)(r) \text{ and } (g^{-1} \circ f)(x_0) \perp (g^{-1} \circ f)(s), \text{ or} \\ (g^{-1} \circ f)(r) &\perp (g^{-1} \circ f)(x_0) \text{ and } (g^{-1} \circ f)(s) \perp (g^{-1} \circ f)(x_0), \\ &\vdots \\ x_n &\perp r \text{ and } x_n \perp s \text{ or } r \perp x_n \text{ and } s \perp x_n. \end{aligned}$$

By (1), sub-additivity of ψ and triangular inequality,

$$\begin{aligned} \psi(d(fr,fs)) &\leq \psi(d(fr,fx_n) + d(fx_n,fs)) \\ &\leq \psi(d(fr,fx_n)) + \psi(d(fx_n,fs)) \\ &\leq \psi(M(r,x_n)) - \varphi(M(r,x_n)) + \psi(M(x_n,s)) - \varphi(M(x_n,s)) \end{aligned}$$

where,

$$M(r,x_n) = \max\left\{ d(gr,gx_n), d(fx_n,gx_n), d(fr,gr), \frac{d(fx_n,gr)+d(gx_n,fr)}{2} \right\}$$

and

$$M(x_n,s) = \max\left\{ d(gx_n,gs), d(fx_n,gx_n), d(fs,gs), \frac{d(fx_n,gs)+d(gx_n,fs)}{2} \right\}$$

Letting $n \to \infty$,

$$\lim_{n\to\infty} M(x_n,r) = d(fr,gr) = 0$$

and

$$\lim_{n\to\infty} M(x_n,s) = d(gr,gs).$$

Thus, we get

$$\begin{aligned} \psi(d(fr,fs)) &\leq \psi(d(gr,gs)) - \varphi(d(gr,gs)) \\ &= \psi(d(fr,fs)) - \varphi(d(fr,fs)) \end{aligned}$$

which is a contradiction. Hence f and g have a unique coincidende point. Therefore by weak compatibility of f and g, we get $fgr = gfr = ggr$. So gr is a coincidende point of f and g. By uniqueness of coincidence point, $gr = fr = r$. Hence r is a common fixed point of f and g.

Uniqueness of common fixed point can be shown in the same way with uniqueness of coincidence. □

If we take $\psi(t) = t$ for all $t \in [0,\infty)$ in Theorem 3.1 we have the following result.

Corollary 3.2. *Let $(X, \perp, d)$ be an orthogonal metric space and let $f, g : X \to X$ be mappings such that*

$$d(fx, fy) \leq M(x, y) - \varphi(M(x, y))$$

where

$$M(x, y) = \max\left\{d(gx, gy), d(fx, gx), d(fy, gy), \frac{d(fx, gy) + d(gx, fy)}{2}\right\}$$

for each $x, y \in X$ with $x \perp y$, $x \neq y$ and $\varphi \in \Psi$. Suppose that

(i) $f(X) = g(X)$

(ii) *g one to one and $g^{-1} \circ f$ is $\perp$-preserving map,*

(iii) *$\{f, g\}$ satisfy O-(E.A) property,*

(iv) *$g(X)$ is closed in X.*

Then f and g have a coincidence point. Moreover, if f and g are weakly compatible, then f and g have a unique common fixed point.

If we take $\psi(t) = t$ and $\varphi(t) = (1 - \alpha)t$ for $\alpha \in [0, 1)$ in Theorem 3.1, we have the following corollary.

Corollary 3.3. *Let $(X, \perp, d)$ be an O-metric space and let $f, g : X \to X$ be mappings such that for each $x, y \in X$*

$$d(fx, fy) \leq \alpha \max\left\{d(gx, gy), d(fx, gx), d(fy, gy), \frac{d(fx, gy) + d(gx, fy)}{2}\right\}$$

where $x \perp y$ and $x \neq y$. Suppose that

(i) $f(X) = g(X)$

(ii) *g one to one and $g^{-1} \circ f$ is $\perp$-preserving*

(iii) *$\{f, g\}$ satisfy O-(E.A) property,*

(iv) *$g(X)$ is closed in X.*

Then f and g have a coincidence point. Moreover, if f and g are weakly compatible, then f and g have a unique common fixed point.

Example 3.4. *Let $X = [0, \infty)$. Define $d : X \times X \to [0, \infty)$ such that $d(x, y) = \left|x - y\right|$. Suppose the binary relation $\perp$ on X such that $x \perp y \iff x \leq y \leq 1$ or $x = 0$. Then $(X, \perp)$ is an O-set and $(X, \perp, d)$ is an O-metric space. Let define $x_n = \frac{1}{n}$ then x_n is an O-sequence.*

Suppose

$$\begin{aligned} \psi &: [0, +\infty) \to [0, \infty), \quad \psi(t) = \frac{5t}{4} \\ \phi &: [0, +\infty) \to [0, \infty), \quad \phi(t) = \frac{t}{4}. \end{aligned}$$

And

$$f, g : X \to X$$
$$f(x) = \frac{x}{16}, \; g(x) = \frac{x}{2}$$

Clearly, $g(X)$ is closed and $f(X) = g(X)$ and $lim_{n\to\infty} fx_n = lim_{n\to\infty} gx_n = 0$. Thus $\{f, g\}$ satisy $O - (E.A)$ property.

For any $x, y \in X$, $x \perp y$,

$$\psi(d(fx, fy)) = \frac{5}{64}\left|x - y\right| \leq \frac{1}{2}\left|x - y\right| = \psi(d(gx, gy)) - \varphi(d(gx, yx)) \leq \psi(M(x, y)) - \varphi(M(x, y)).$$

Thus contractive condition (1) and all conditions of Theorem 3.1 are satisfied. Also $x = 0$ is a fixed point of f and g.

4. Application to integral equation

Let consider the following nonlinear Fredholm integral equation

$$x(p) = z(p) + \int_0^1 K(p, r, x(r))\, dr, \tag{3}$$

and $X = C[0,1]$ (the set of all continuous functions from $[0,1]$ to $\mathbb{R}$) equipped with the metric $d(x,y) = \sup_{p\in[0,1]} \left|x(p) - y(p)\right|$.

Theorem 4.1. *Consider the integral equation (3) and suppose*

(i) $K : [0,1] \times [0,1] \times [0,\infty) \to [0,\infty)$ *and* $z : [0,1] \to [0,\infty)$ *are continuous,*

(ii) *for all* $(p,r) \in [0,1] \times [0,1]$ *such that*

$$\left|K(p, r, x(r)) - K(p, r, y(r))\right| \leq \ln\left(\left|x(r) - y(r)\right| + 1\right)$$

Then integral equation (3) has a unique solution.

Proof. We consider the following orthogonality relation on $X = C[0,1]$,

$$x \perp y \Leftrightarrow x(r)\, y(r) \geq 0 \text{ for all } r \in [0,1].$$

Then $(X, \perp)$ is an O-set and $(X, \perp, d)$ is O-metric space. We define the mappings $f, g : X \to X$, by

$$f(x(r)) = z(p) + \int_0^1 K(p, r, x(r))\, dr,$$

for all $x \in X$ and $p \in [0,1]$ and $g(x) = x$. Let $x \perp y$ for $x, y \in X$, then $x(r)\, y(r) \geq 0$ for all $r \in [0,1]$. We have

$$\begin{aligned} f(x(r)) &= z(p) + \int_0^1 K(p, r, x(r))\, dr \geq 0, \\ f(y(r)) &= z(p) + \int_0^1 K(p, r, y(r))\, dr \geq 0. \end{aligned}$$

Since g is identity map,

$$(g^{-1} \circ f)(x(r)).(g^{-1} \circ f)(y(r)) \geq 0 \text{ i.e. } (g^{-1} \circ f)(x(r)) \perp (g^{-1} \circ f)(y(r)).$$

Thus $g^{-1} \circ f$ is orthogonal preserving and f and g satisfy $O-(E.A)$ property.

From (ii),

$$\begin{aligned}
\left|f(x(r)) - f(y(r))\right| &= \left|\int_0^1 K(p,r,x(r))\,dr - \int_0^1 K(p,r,y(r))\,dr\right| \\
&\leq \int_0^1 \left|K(p,r,x(r)) - K(p,r,y(r))\right| dr \\
&\leq \int_0^1 \left|\ln\left(\left|x(r) - y(r)\right| + 1\right)\right| dr \\
&\leq \int_0^1 \ln\left(\sup_{r\in[0,1]} \left|x(r) - y(r)\right| + 1\right) dr \\
&= \int_0^1 \ln(d(x,y) + 1)\,dr \\
&\leq \int_0^1 \ln(M(x,y) + 1)\,dr \\
&\leq \ln(M(x,y) + 1) \\
&= M(x,y) - (M(x,y) - \ln(M(x,y) + 1))
\end{aligned}$$

Thus, we have

$$d(f(x), f(y)) \leq M(x,y) - \varphi(M(x,y)) \tag{4}$$

where $\varphi(t) = t - \ln(t+1)$ and $\psi(t) = t$. All conditions of Theorem 3.1 are satisfied. Thus integral equation (3) has unique solution. □

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