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Topological generalization of minimal structure with medical application

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Abstract. In this paper we used the $MSA-$ space, $\eta-$open sets and $\alpha\eta$-open sets to introduce a new approximation of uncertain sets as a mathematical tool to modify the approximations. Moreover, several important measures such accuracy measure and quality of approximation would be studied. We compared the previous methods with the current one we obtained. We proved that minimal structure is more efficient and accurate in obtaining results than topology. Our new $M\alpha\eta-$approximation space is more accurate than $MSA-$ *space* , $\eta-$approximations and $\alpha\eta-$approximations since by using it the boundary regions decreased by increasing the lower approximations and decreasing the upper approximations. Finally we show the importance of our new approximations with medical science by applying these approximations in corona virus problem.

1. Introduction

The notion of minimal strcture [22](briefly MS) was presented by V.Popa and T.Noiri, also they presented the notion of MS- open sets, MS- closed sets, MS-interior and MS-closure. Furthermore, the concepts of separation axioms by using the concept of minimal structure have been introduced. In [8] Buadong et al. represented the concept of generalized topology and minimal structure spaces (briefly GTMS). They introduced the notion of M-continuous functions on functions between minimal structures. In [26] some generalizations for closed sets in generalized topology and minimal structure spaces were investigated. In [25] Zakaria introduced the notion of $gm-$ continuity, gm-convergent to a point, gmT_2- space,$gm-$ closed graph and strongly $gm-$ closed graph on generalized topology and minimal structure spaces. Shyamapada [15] studied separation axioms in ideal minimal spaces. El- sharkasy [9, 10] introduced a new approximation space by using the notion of minimal structure approximation space and near open sets. Many authors [11, 16, 18, 25] studied the properties and applications of minimal structure.

Rough set theory has been considered as an extension of set theory. Wiweger [24] introduced the concept of topological rough sets. Many authors [1, 2, 12–14, 19, 21, 24] studied the relation between rough set and topology. The relation between rough set and minimal structure were studied in [9]. Abu-donia [2] used the notion of $\alpha\eta$- open sets to generalize rough sets [17]. The concept of Cj-neighborhoods were used to improve rough sets's accuracy measure[5, 6]. Also Al-Shami and others used the concepts of j-adhesion

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neighborhoods and ideals to generate topologies and defined a new rough set model derived from these topologies. These models have been proved to be finar than other topologies[7].

Shain Karg and M.Yucel defined the concepts of anti-topology and neutro-topological spaces. The properties of those spaces were studied. The concept of the anti-topology is defined as a topological structure that has at least one anti-axiom[23]. Since the minimal stucture defined as a topological structure which has two anti-axiom(i.e the closure of arbitrary union and finite intersection). We can consider the minimal structure as a special case of anti-topology.

In this paper, we generalized a minimal structure by using η- open and $\alpha\eta$-open sets. We demonstrated that some of Pawlak's rough set models are a special cases of minimal generalizations. Moreover, the new model measures, such as accuracy and quality of approximation have been investigated. The study about $\alpha\eta$-open sets has specific importance to help the modifications of the approximation space via adding new concepts and facts. We used minimal structure concepts to introduce definitions of $\alpha\eta$-approximations and $\alpha\eta$- boundary regions. We introduced the $\alpha\eta$-boundary regions as different areas of uncertainty. Finally, we used minimal structure concepts to introduce the definitions of $\alpha\eta$-rough and $\alpha\eta$-exact sets. The rest of this article is organized as follows: Section 2 is devoted to recalling some basics and properties of η-open sets, $\alpha\eta$-open sets, and minimal structure. In section 3, we introduced the notion of η-open, $\alpha\eta$-open in minimal structure, furthermore, we introduced the concept of $\alpha\eta$-lower approximations and $\alpha\eta$-upper approximations. The notions were further explored by studying its properties. Section 4 introduced $\alpha\eta$-region of uncertain sets. Section 5 contained comparison between $\alpha\eta$-topological approximation space and $\alpha\eta$-minimal structure approximation space. In section 6, we applied the $\alpha\eta$-approximation space in minimal structure in covid -19 problem.

2. Premliminaries

In this section, we recalled some basic notions that are useful for discusssion in the next section

Definition 2.1 [7] Let W be a non-empty universe set which is finite, and $\check{R}_\eta$ is a general binary relation that can be used to obtain a subbase for a topology $\mp$ on W that generates the class $\eta O(W)$ of all $\eta-$open sets. Then the pair $(W, \check{R}_\eta)$ is called a $\eta-$approximation space.

Definition 2.2 [7] Let $(W,\mp)$ be a topological space and $B \subseteq W$, then B is called $\eta-$open if $B \subseteq cl(int(cl(B)))$.

Definition 2.3 [7] Let $(W, \check{R}_\eta)$ be a $\eta-$approximation space and B is a non-empty subset of W, then the $\eta-$lower (resp $\eta-$upper) approximation of B is defined as:
(1) $\underline{\check{R}_\eta}(B) = \cup\{G \in \eta O(W) : G \subseteq B\}$
(2) $\overline{\overline{\check{R}_\eta}}(B) = \cap\{F \in \eta C(W) : F \supseteq B\}$

Definition 2.4 [7] Let $(W, \check{R}_\eta)$ be a $\eta-$approximation space and $B \subseteq W$. The $\eta-$accuracy measure of B is defined as follows: $\theta_\eta(B) = \frac{|\underline{\check{R}_\eta}(B)|}{|\overline{\overline{\check{R}_\eta}}(B)|}$, where $B \neq \phi$.The $\eta-$ boundary of B is defined as $BND_\eta(B) = \overline{\check{R}_\eta}(B) - \underline{\check{R}_\eta}(B)$.

Definition 2.5 [7] Let W be a non-empty universe set which is finite, and $\check{R}_{\alpha\eta}$ is a general binary relation that can be used to obtain a subbase for a topology $\mp$ on W that generates the class $\alpha\eta O(W)$ of all $\alpha\eta-$open sets. Then the pair $(W, \check{R}_{\alpha\eta})$ is called a $\alpha\eta-$approximation space.

Definition2.6 [7] Let $(W,\mp)$ be a topological space, and $B \subseteq W$. Then B is called $\alpha\eta-$open if $B \subseteq cl(int(cl_\alpha(B)))$.

Definition 2.7 [7] Let $(W, \check{R}_{\alpha\eta})$ be a $\alpha\eta-$approximation space and B is a non-empty subset of W. $\alpha\eta-$lower approximations and $\alpha\eta-$upper approximation of B are defined as follow:

$\underline{\check{R}_{\alpha\eta}}(B) = \cup\{G : G \in \alpha\eta O(W), G \subseteq B\}$
$\overline{\check{R}_{\alpha\eta}}(B) = \cap\{F : F \in \alpha\eta C(W), B \subseteq F\}$

Definition 2.8 [7] Let $(W, \check{R}_{\alpha\eta})$ be $\alpha\eta-$ approximation space and $B \subseteq W$, then the $\alpha\eta$-accuracy measure of B defined as follows $\theta_{\alpha\eta}(B) = \frac{|\underline{R_{\alpha\eta}}(B)|}{|\overline{\check{R}_{\alpha\eta}}(B)|}$ where $B \neq \phi$.The $\alpha\eta-$ boundary of B is defined as $BND_{\alpha\eta}(B) = \overline{\check{R}_{\alpha\eta}}(B) - \underline{\check{R}_{\alpha\eta}}(B)$.

Definition 2.9 [25] A family $M \subseteq P(W)$ is said to be a minimal structure on W if $W, \phi \in M$, in this case (W, M) is called a minimal space. The elements of W are known as points of space and the subsets of W that belong to M are called $M-$open sets. $M-$closed sets are the complement of the subsets of W that belong to M.

Definition 2.10 [25] Let (W, M) be a minimal space and $B \subseteq W$ then the $M-$ closure of B is defined as $cl_M(B) = \cap\{F \subset W, B \subset F$ and F is closed $\}$, so $cl_M(B)$ is the smallest closed subset of W which contains B .

Definition 2.11 [25] Let (W, M) be a minimal space and $B \subseteq W$, then the $M-$ interior of B is defined as $int_M(B) = \cup\{G \subset W, G \subset B,$ and G is open$\}$, so $int_M(B)$ is the union of all open subsets of W which containing in B .

Definition 2.12 [25] Let (W, M) be a minimal space and $B \subseteq W$ then the $M-$ boundary of is given by $BND_M(B) = [\ cl_M(B)] - [\ int_M(B)]$.

Propostion 2.1 [14] Let (W, M) be a minimal space and for any two set B and $C \subseteq W$
1) $int_M(B) \subseteq B$ and $int_M(B) = B$ if B is $M-$open
2) $cl_M(B) \supseteq B$ and $cl_M(B) = B$ if B is $M-$closed
3) $int_M(B) \subseteq\ int_M(C)$ and $cl_M(B) \subseteq cl_M(C)$ if $B \subseteq C$
4) $int_M(B \cap C) = int_M(B) \cap int_M(C)$ and $int_M(B \cup C) \supseteq int_M(B) \cup int_M(C)$
5) $cl_M(B \cap C) \subseteq cl_M(B) \cap cl_M(C)$ and $cl_M(B \cup C) \subseteq cl_M(B) \cup cl_M(C)$
6) $int_M(int_M(B)) = int_M(B)$ and $cl_M(cl_M(B))) = cl_M(B)$
7) $(cl_M(B))^c \subseteq int_M(B^c)$ and $(int_M(B))^c \subseteq cl_M(B^c)$

Definition 2.13 [25] Let B be a subset of minimal space (W, M), B is exact if and only if $BND_M(B) = \phi$ otherwise B is rough. B is exact if and only if $cl_M(B) = int_M(B)$. There are two possibilities in Pawlak approximation space for a subset $B \subseteq W$ which is rough or exact.

Definition 2.14 [11] Let $(W, \check{R})$ be a generalized approximation space where W be a finite nonempty universe set and $\check{R}$ an arbitarary relation on W and $N_x(W) = \{y \in W : x\check{R}y\}$ is the right neighbourhood of x for all $x \in W$, then the class $M(W) = \{\phi, W, N_x(W)\}$is called a minimal structure on $(W, \check{R})$, then $(W, \check{R}, M)$ is called a minimal structure approximation space briefly($MSA-$ *space*).

Definition 2.15 [11] Let $(W, \check{R}, M)$ be $MSA-$ *space*, and $B \subseteq W$. Then B is called
(1) $M-$Regular open if $B = int_M(cl_M(B))$
(2) $M-$Semi open if $B \subseteq cl_M(int_M(B))$
(3) $M - \theta-$open if $B \subseteq int_M(cl_M(int_M(B)))$
(4) $M-$Pre open if $B \subseteq int_M(cl_M(B))$
(5) $M-$Semi- Preopen $(M - \eta-$open) if $B \subseteq cl_M(int_M(cl_M(B)))$

The family of all $M\alpha-$open [resp. $M-$Regular open , $M-$Semiopen, $M\theta-$open, $M-$preopen, $M\eta-$open] sets of W is denoted by $M\alpha O(W)$ [$M\check{R}O(W)$, $MSO(W)$, $M\theta O(W)$, $MPO(W)$, $M\eta O(W)$]. The complement of $M\alpha-$open [resp. $M-$Regularopen, $M-$Semiopen, $M\theta-$open, $M-$preopen, $M\eta-$open] sets of W is called $M\alpha-$closed [resp. $M-$Regular closed, $M-$Semi closed, $M\theta-$closed, $M-$preclosed, $M\eta-$closed] is denoted by $M\alpha C(W)$ [$M\check{R}C(W)$, $MSC(W)$, $M\theta C(W)$, $MPC(W)$, $M\eta C(W)$].

Definition 2.16 [11] Let$(W, \check{R}, M)$ be a minimal structure approximation space ($MSA-$ *space*) and $B \subseteq W$. Then a minmal lower approximation of B ($\underline{M}(B)$) is defined as $\underline{M}(B) = \cup\{G : G \in M(W), G \subset B\}$, and a minimal upper aproximation of B ($\overline{M}(B)$) is defined as $\overline{M}(B) = \cap\{F : F \in MS^c(W), B \subset F\}$. The $M-$ boundary of is given by $BND_M(B) = \overline{M}(B) - \underline{M}(B)$.

Definiton2.17 Let $(W, \check{R}, M)$ be $MSA-$ *space*, $B \subseteq W$, then the accuracy measure of B defined as follows $\theta_{MS}(B) = \frac{|\underline{MS}(B)|}{|\overline{\overline{MS}}(B)|}$ where $B \neq \phi$.

Definition 2.18 Let $(W, \check{R}, M)$ be a $MSA-$ *space* and $B \subseteq W$. Then B is said to be

(1) Roughly M- definable if $\underline{M}(B) \neq \phi$ and $\overline{M}(B) \neq W$

(2) Internally M- undefinable if $\underline{M}(B) = \phi$ and $\overline{M}(B) \neq W$

(3) Externally M- undefinable if $\underline{M}(B) \neq \phi$ and $\overline{M}(B) = W$

(4) Totally M- undefinable if $\underline{M}(B) = \phi$ and $\overline{M}(B) = W$

We denote the set of all roughly$M-$definable (resp. internally$M-$ undefinable , externally$M-$undefinable and totally$M-$undefinable) sets by $MD(W)$ (resp. $IMWD(W)$, $EMWD(W)$ and $TMWD(W)$).

3. $\alpha\eta-$ open sets on minimal structure:

In this section, we introduced the notion of $\alpha\eta-$open set in minimal structure and its properties.

Definition 3.1 Let W be a non-empty universe set which is finite and $\check{R}$ be a general relation used to get a minimal structure M on W, which is used to generates the class $M\eta O(W)$ of all $M\eta-$open sets. Then $(W, \check{R}, M)$ is $MSA-$ *space*.

Definition 3.2 Let $(W, \check{R}, M)$ be a $MSA-$ *space* and $B \subseteq W$. Then $\eta-$lower (resp $\eta-$upper) approximation of B is defined as: $\underline{M_\eta}(B) = \cup\{G \in M\eta O(W), G \subseteq B\}$and $\overline{M_\eta}(B) = \cap\{F \in M\eta C(W), F \supseteq B\}$.

Definition3.3 Let $(W, \check{R}, M)$ be a $MSA-$ *space* and $B \subseteq W$, then the η-accuracy measure of B defined as follows $\theta_{M_\eta}(B) = \frac{|\underline{M_\eta}(B)|}{|\overline{\overline{M_\eta}}(B)|}$ where $B \neq \phi$.

Definition 3.4 Let $(W, \check{R}, M)$ be a $MSA-$ *space* then the subset $B \subseteq W$ is called

(1) Roughly M_η- definable if $\underline{M_\eta}(B) \neq \phi$ and $\overline{M_\eta}(B) \neq W$

(2) Internally M_η- undefinable if $\underline{M_\eta}(B) = \phi$ and $\overline{M_\eta}(B) \neq W$

(3) Externally M_η- undefinable if $\underline{M_\eta}(B) \neq \phi$ and $\overline{M_\eta}(B) = W$

(4) Totally M_η- undefinable if $\underline{M_\eta}(B) = \phi$ and $\overline{M_\eta}(B) = W$

We defined the set of all roughly M_η- definable (resp. internally M_η- undefinable , externally M_η- undefinable and totally M_η- undefinable) sets by $\eta MD(W)$ (resp. $\eta MIWD(W)$, $\eta MEWD(W)$ and $\eta MTWD(W)$).

Definition3.5 Let $(W, \check{R}, M)$ be $MSA-$ *space* and $B \subset W$, then there are memberships($\underline{\in}$, $\overline{\in}$, $\underline{\in}_\eta$ and $\overline{\in}_\eta$) say strong, weak , $\eta-$strong and $\eta-$weak membership resp.which is defined as

(1) $y \,\underline{\in}\, B$ iff $y \in \underline{M}(B)$

(2) $y \,\overline{\in} B$ iff $y \in \overline{M}(B)$

(3) $y \,\underline{\in}_\eta\, B$ iff $y \in \underline{M}_\eta(B)$

(4) $y \,\overline{\in}_{\eta} B$ iff $y \in \overline{M_{\eta}}(B)$

Definition 3.6 Let $(W, \check{R}, M)$ be $MSA-$ *space* and $B \subseteq W$ the $M\alpha-$closure of B is defined by $cl_{M\alpha}(B) = \{w \in W : B \cap int_M(cl_M(G)) \neq \phi, G \in M$ and $W \in G\}$. A set B is called $M\alpha-$closed if $B = cl_{M\alpha}(B)$, and $int_{M\alpha}(B) = W/[cl_{M\alpha}(W/B)]$.

Definition 3.7 Let $(W, \check{R}, M)$ be $MSA-$ *space* and $B \subseteq W$, then B is called $M-\alpha\eta-$open if $B \subseteq cl_M(int_M(cl_{M\alpha}(B)))$.

The family of all $M\alpha\eta-$open sets of W is defined by $M\alpha\eta O(W)$, The complement of $M\alpha\eta-$open sets of W is called $M\alpha\eta-$closed is defined by $M\alpha\eta C(W)$.

Remark 3.1 $M - \alpha\eta$ open sets are stronger than any near open sets such as ($M\alpha-$ open, $M-$*Regular* open, $M-$*semi* open, $M\theta-$open, $M-$Pr*e*open and $M\eta-$ open) sets as shown in Figure (1)

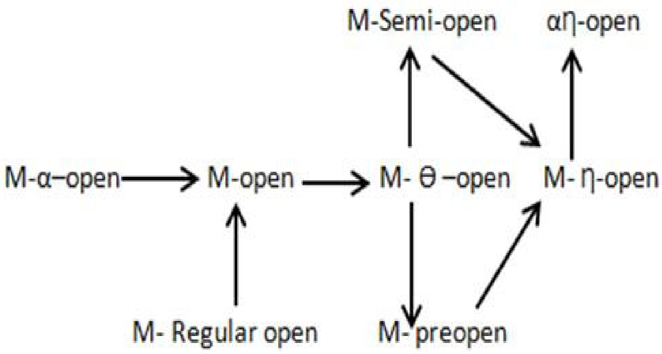

Figure 1:

The next example illustrates this idea

Example 3.1 Let $W = \{l, m, n, o, w\}$, $\check{R} = \{(l,l), (m,m), (n,l)(n,m), (o,l), (o,n), (o,o)\}$ a general relation defined on W, then the minimal structure associated with this relation is $MS(W) = \{\phi, W, \{l\}, \{m\}, \{l,m\}, \{l,n,o\}\}$. We have $\{l\} \in M\alpha\eta O(W)$ but $\{l\} \notin M\alpha O(W)$ and $\{l\} \notin M\check{R}O(W)$.Also $\{m,n\} \in M\alpha\eta O(W)$ but $\{m,n\} \notin M\theta O(W)$, $\{m,n\} \notin MSO(W)$ and $\{m,n\} \notin MPO(W)$ and $\{m,n\} \notin M\eta O(W)$.

Definition 3.8 Let $(W, \check{R}, M)$ be $MSA-$ *space* where M be a minimal structure generated by a general relation $\check{R}$ on W which is used to generates the class $M\alpha\eta O(W)$of all $M\alpha\eta-$open sets.

Example 3.2 From Example 3.1 let $W = \{l, m, n, o, w\}$ be a universe set and $\check{R} = \{(l,l), (m,m), (n,l)(n,m), (o,l), (o,n), (o,o)\}$ a general relation defined on W, the minimal structure is $MS(W) = \{\phi, W, \{l\}, \{m\}, \{l,m\}, \{l,n,o\}\}$. Then $M\alpha\eta O(W) = \{\phi, W, \{l\}, \{m\}, \{n\}, \{o\}, \{l,m\}, \{l,n\}\{l.o\}, \{l,w\}, \{m,n\}, \{m,o\}, \{m,w\}, \{n,o\}, \{n,w\}, \{o,w\}, \{l,m,n\}, \{l,m,o\}, \{l,m,w\}, \{l,n,$ $\{m,n,o\}, \{m,n,w\}, \{m,o,w\}, \{l,m,n,o\}, \{l,m,n,w\}, \{l,n,o,w\}, \{l,m,o,w\}, \{m,n,o,w\}\}$.

Definition 3.9 Let $(W, \check{R}, M)$ be a $MSA-$ *space* and $B \subseteq W$, then the $\alpha\eta-$ lower and $\alpha\eta-$upper approximation of B is defined as follows

$\underline{M_{\alpha\eta}}(B) = \cup\{G : G \in M\alpha\eta O(W), G \subseteq B\}$

$\overline{\overline{M_{\alpha\eta}}}(B) = \cap\{F : F \in M\alpha\eta C(W), B \subseteq F\}$

Theorem3.1: Let $(W, \check{R}, M)$ be $MSA-$ *space* and $B \subseteq W$, we have $\underline{M}(B) \subseteq \underline{M}_\eta(B) \subseteq \underline{M}_{\alpha\eta}(B) \subseteq B \subseteq \overline{M}_{\alpha\eta}(B) \subseteq \overline{M}_\eta(B) \subseteq \overline{M}(B)$

proof:
$\underline{M}(B) = \cup\{G : G \in M(W), G \subset B\} \subseteq \cup\{G : G \in M\eta O(W), G \subseteq B\} \subseteq \cup\{G : G \in M\alpha\eta O(W), G \subseteq B\} \subset B, i, e :$ $\underline{M}(B) \subseteq \underline{M}_\eta(B) \subseteq \underline{M}_{\alpha\eta}(B) \subseteq B$, also $\overline{M}(B) = \cap\{F : F \in M^c(W), B \subseteq F\} \supseteq \cap\{F : F \in M\eta C(W), B \subseteq F\} \supseteq \cap\{F : F \in M\alpha\eta C(W), B \subseteq F\} \supseteq B$, i.e. $\overline{M}(B) \supseteq \overline{M}_\eta(B) \supseteq \overline{M}_{\alpha\eta}(B) \supseteq B$. Consequently $\underline{M}(B) \subseteq \underline{M}_\eta(B) \subseteq \underline{M}_{\alpha\eta}(B) \subseteq B \subseteq \overline{M}_{\alpha\eta}(B) \subseteq \overline{M}_\eta(B) \subseteq \overline{M}(B)$ as shown in Figure (2)

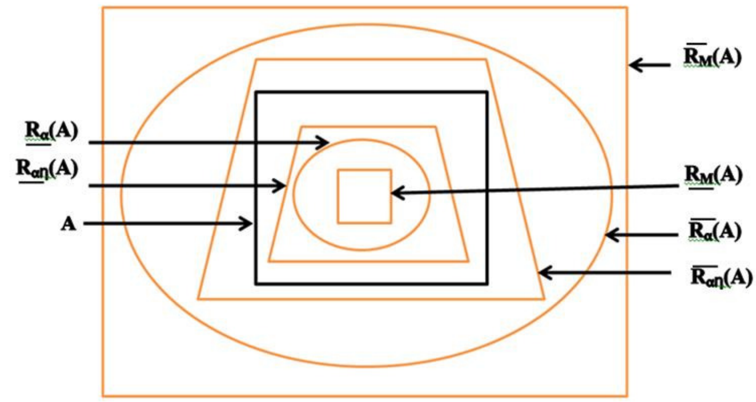

Figure 2:

Definition 3.10 Let $(W, \check{R}, M)$ be a $MSA-$ *space*, and $B \subseteq W$. Then there are memberships $\underline{\in}_{\alpha\eta}$ and $\overline{\in}_{\alpha\eta}$ say $\alpha\eta-$ strong , $\alpha\eta-$weak memberships resp. which are defined by

(1) $x \underline{\in}_{\alpha\eta} B$ iff $x \in \underline{M}_{\alpha\eta}(B)$

(2) $x \overline{\in}_{\alpha\eta} B$ iff $x \in \overline{M}_{\alpha\eta}(B)$

Remark 3.2 According to definition 3.11 $\alpha\eta-$lower and $\alpha\eta-$upper approximation of subset can be written as:

(1) $\underline{M}_{\alpha\eta}(B) = \{x \in B : x \underline{\in}_{\alpha\eta} B\}$

(2) $\overline{\overline{M}}_{\alpha\eta}(B) = \{x \in B : x \overline{\in}_{\alpha\eta} B\}$

Remark 3.3 Let $(W, \check{R}, M)$ be a $MSA-$ *space*, and $B \subseteq W$. Then we have

(1) $x \underline{\in} B \Rightarrow x \underline{\in}_\eta B \Rightarrow x \underline{\in}_{\alpha\eta} B$

(2) $x \overline{\in}_{\alpha\eta} B \Rightarrow x \overline{\in}_\eta B \Rightarrow x \overline{\in} B$

The opposite of Remark 3.3 are not generally true as seen in the next example

Example 3.3 From Example 3.2 we can see that
if $B = \{m, n, w\}$ we have $n \underline{\in}_{\alpha\eta} B$ but $n \underline{\notin}_\eta B$ and $w \in_\eta B$ but $w \notin B$ Also, if $C = \{l, n\}$ we have $w \overline{\in} C$ but $w \overline{\notin}_\eta C$ and $o \overline{\in}_\eta C$ but $o \overline{\notin}_{\alpha\eta} C$.

We identify the degree of completeness by $M\alpha\eta-$accuracy measure as follows

$\theta_{M_{\alpha\eta}}(B) = \frac{|\underline{M_{\alpha\eta}}(B)|}{|\overline{M_{\alpha\eta}}(B)|}$ where $B \neq \phi$.

Example 3.4 In Example 3.2 we show in Table (1) the degree of accuracy measure $\theta_M(B)$, $M\eta-$ accuracy measure $\theta_{M_\eta}(B)$and $M\alpha\eta-$ accuracy measure $\theta_{M_{\alpha\eta}}(B)$ for some subsets of W

B	$\theta_{M_{\alpha\eta}}(B)$	$\theta_{M_\eta}(B)$	$\theta_M(B)$
$\{l\}$	1	1	$\frac{1}{4}$
$\{m\}$	1	1	$\frac{1}{2}$
$\{o\}$	1	0	0
$\{l,n\}$	1	$\frac{2}{3}$	$\frac{1}{4}$
$\{l,w\}$	1	$\frac{1}{2}$	$\frac{1}{4}$
$\{l,m,n\}$	1	$\frac{3}{5}$	$\frac{2}{5}$
$\{l,m,n,w\}$	1	$\frac{4}{5}$	$\frac{2}{5}$

Table (1)

As we see if $B = \{l, w\}$ the accuracy measure of Pawlak is $\frac{1}{4}$, the $M\eta-$accuracy measure is $\frac{1}{2}$ and the $M\alpha\eta-$ accuracy measure is 1.

Definition 3.11 Let $(W, \check{R}, M)$ be $MSA-$ *space*, and $B, C \subseteq W$ then B and C are
(1) $\alpha\eta-$roughly bottom equals ($B \simeq C$) if $\underline{M_{\alpha\eta}}(B) = \underline{M_{\alpha\eta}}(C)$
(2) $\alpha\eta-$roughly top equals ($B \eqsim C$) if $\overline{M_{\alpha\eta}}(B) = \overline{M_{\alpha\eta}}(C)$
(3) $\alpha\eta-$roughly equals ($B \approx C$) if ($B \simeq C$) and ($B \eqsim C$)

Example 3.5 Let $W = \{l, m, n, o\}$, $\check{R} = \{(m, l), (m, m), (n, n), (o, m), (o, n), (o, o)\}$, then $M = \{\phi, W, \{l, m\}, \{n\}, \{m, n, o\}\}$ we find $\alpha\eta-$roughly bottom equals $\phi, \{o\}$ and $\alpha\eta-$roughly top equals $\{l, m, n\}$, W

Definition 3.12 Let $(W, \check{R}, M)$ be $MSA-$ *space*, and $B, C \subseteq W$ is called
(1) B is $\alpha\eta-$roughly bottom included in C ($B \underset{\sim}{\subset}_{\alpha\eta} C$) if $\underline{M}_{\alpha\eta}(B) \subseteq \underline{M}_{\alpha\eta}(C)$
(2) B is $\alpha\eta-$roughly top included in C ($B \tilde{\subset}_{\alpha\eta} C$) if $\overline{M}_{\alpha\eta}(B) \subseteq \overline{M}_{\alpha\eta}(C)$
(3) B is $\alpha\eta-$roughly included in C ($B \underset{\sim}{\tilde{\subset}}_{\alpha\eta} C$) if ($B \underset{\sim}{\subset} C$) and ($B \tilde{\subset} C$)

Example 3.6 From Example 3.5 ,
if $B = \{m\}$and $C = \{m, n\}$ then $B \underset{\sim}{\subset}_{\alpha\eta} C$
if $B = \{m, o\}$ and $C = \{l, m, o\}$ then $B \tilde{\subset}_{\alpha\eta} C$
if $B = \{l\}$ and $C = \{l, m\}$ then $B \underset{\sim\alpha\eta}{\tilde{\subset}} C$

Lemma 3.1 Let $(W, \check{R}, M)$ be $MSA-$ *space*, and for all $x, y \in W$. The condition $x \in \overline{M}_{\alpha\eta}(\{y\})$ and $y \in \overline{M}_{\alpha\eta}(\{x\})$ implies $\overline{M}_{\alpha\eta}(\{x\}) = \overline{M}_{\alpha\eta}(\{y\})$

Proof Since $\eta cl_\alpha(y)$ is a closed set and $x \in \eta cl_\alpha(y)$, while $\eta cl_\alpha(\{x\})$ is the smallest closed set containing x, thus $\eta cl_\alpha(\{x\}) \subseteq \eta cl_\alpha(\{y\})$.Hence $\overline{M_{\alpha\eta}}(\{x\}) \subseteq \overline{M_{\alpha\eta}}(\{y\})$. The opposite inclusion follows by symmetry $\eta cl_\alpha(\{y\}) \subseteq \eta cl_\alpha(\{x\})$ Thus $\overline{M_{\alpha\eta}}(\{x\}) \subseteq \overline{M_{\alpha\eta}}(\{y\})$.Consequently $\overline{M_{\alpha\eta}}(\{x\}) = \overline{M_{\alpha\eta}}(\{y\})$.

Lemma 3.2: Let $(W, \check{R}, M)$ be $MSA-$ *space*, which verified that every $\alpha\eta$-open set $A \subseteq W$ is $\alpha\eta$-closed, then $y \in \overline{M}_{\alpha\eta}(\{x\})$ implies $x \in \overline{M}_{\alpha\eta}(\{y\})$ for all $x, y \in W$.

proof: Assume that $x \notin \overline{M_{\alpha\eta}}(\{y\})$, then $x \notin \cap\{f \in \alpha\eta C(W), f \supseteq \{y\}\}$. So, $x \in \cup\{G \in \alpha\eta O(W), G^c \subseteq \{y\}^c\}$, then $x \in \cup\{G \in \alpha\eta O(W), y \notin G\}$ and thus there exist $G \subseteq \alpha\eta O(W)$ such that $x \in G$ and
$G \cap \{y] = \phi$. i.e. $\{y\} \subseteq (W \setminus G)$. From assumption $(W \setminus G)$ is $\alpha\eta-$open
and $y \in (W \setminus G)$, so $(W \setminus G) \cap \{W\} = \phi$. Hence $y \notin \overline{M_{\alpha\eta}}(\{x\})$.

Proposition 3.1: Let $(W, \check{R}, M)$ be $MSA-$ *space* and every $\alpha\eta-$open subset A of W is $\alpha\eta-$closed, then the family of set $\{\overline{M}_{\alpha\eta}(\{x\}) : x \in A\}$ is a partition of the set W

Proof: Let $x, y, z \in A$ and $z \in \overline{M}_{\alpha\eta}(\{x\}) \cap \overline{M}_{\alpha\eta}(\{y\})$, then $z \in \overline{M}_{\alpha\eta}(\{x\})$ and $z \in \overline{M}_{\alpha\eta}(\{y\})$. From lemma 3.2 $x \in \overline{M}_{\alpha\eta}(\{z\})$ $y \in \overline{M}_{\alpha\eta}(\{z\})$.From lemma 4.1 since $\overline{M}_{\alpha\eta}(\{x\}) = \overline{M}_{\alpha\eta}(\{z\})$ and $\overline{M}_{\alpha\eta}(\{y\}) = \overline{M}_{\alpha\eta}(\{z\})$, then $\overline{\check{R}}_{\alpha\eta}(\{x\}) = \overline{M}_{\alpha\eta}(\{y\}) = \overline{M}_{\alpha\eta}(\{z\})$. Hence either $\overline{M}_{\alpha\eta}(\{x\}) = \overline{M}_{\alpha\eta}(\{y\})$ or $\overline{M}_{\alpha\eta}(\{x\}) \cap \overline{M}_{\alpha\eta}(\{y\}) = \phi$

Proposition 3.2: Let $(W, \check{R}, M)$ be $MSA-$ *space*, and $B, C \subseteq W$, then
(1) $\underline{M_{\alpha\eta}}(B) \subseteq B \subseteq \overline{M_{\alpha\eta}}(B)$
(2) $\underline{M_{\alpha\eta}}(\phi) = \phi = \overline{M_{\alpha\eta}}(\phi), \underline{M_{\alpha\eta}}(W) = W = \overline{M_{\alpha\eta}}(W)$
(3) If $B \subseteq C$ then $\underline{M_{\alpha\eta}}(B) \subseteq \underline{M_{\alpha\eta}}(C)$ and $\overline{M_{\alpha\eta}}(B) \subseteq \overline{M_{\alpha\eta}}(C)$

proof: (1) Let $x \in M_{\alpha\eta}(B)$ then $x \in \cup\{G \in \alpha\eta O(W), G \subseteq B\}$, then there exist $G_0 \in \alpha\eta O(W)$ such that $x \in G_0 \subset B$. So $x \in B$ and $\underline{M_{\alpha\eta}}(B) \subseteq B$. Let $x \in B$, then by definition of
$\overline{M_{\alpha\eta}}(B)$, $x \in \overline{M_{\alpha\eta}}(B)$ and hence $B \subseteq \overline{M_{\alpha\eta}}(B)$
(2) $\underline{M_{\alpha\eta}}(\phi) = \cup\{G \in \alpha\eta O(W), G \subseteq \phi\} = \phi = \cap\{F \in \alpha\eta c(W), F \supseteq \phi\}$
$\overline{M_{\alpha\eta}}(W) = \cup\{G \in \alpha\eta O(W), G \subseteq W\} = W = \cap\{F \in \alpha\eta c(W), F \supseteq W\}$
(3) $\underline{M_{\alpha\eta}}(B) = \cup\{G \in \alpha\eta O(W), G \subseteq B\}$ and $B \subseteq C$
$\subseteq \cup\{G \in \alpha\eta O(W), G \subseteq C\} = \underline{M_{\alpha\eta}}(C)$
$\overline{M_{\alpha\eta}}(B) = \cap\{F \in \alpha\eta c(W), F \supseteq B\}$ and $B \supseteq A$
$\supseteq \cap\{F \in \alpha\eta c(W), F \supseteq A\} = \overline{M_{\alpha\eta}}(A)$

Proposition 3.3: Let $(W, \check{R}, M)$ be a $MSA-$ *space*, and $B, C \subseteq W$ then
(1) $\underline{M_{\alpha\eta}}(W \setminus B) = W\setminus \overline{M_{\alpha\eta}}(B)$
(2)$\overline{M_{\alpha\eta}}(W \setminus B) = W\setminus \underline{M_{\alpha\eta}}(B)$
(3) $\underline{M_{\alpha\eta}}(\underline{M_{\alpha\eta}}(B)) = \underline{M_{\alpha\eta}}(B)$
(4)$\overline{M_{\alpha\eta}}(\overline{M_{\alpha\eta}}(B)) = \overline{M_{\alpha\eta}}(B)$
(5) $\underline{M_{\alpha\eta}}(\underline{M_{\alpha\eta}}(B)) \subseteq \overline{M_{\alpha\eta}}(\underline{M_{\alpha\eta}}(B))$
(6) $\underline{M_{\alpha\eta}}(\overline{M_{\alpha\eta}}(B)) \subseteq \overline{M_{\alpha\eta}}(\overline{M_{\alpha\eta}}(B))$

Proof: (1) Let $x \in \underline{M_{\alpha\eta}}(W \setminus B) = \cup\{G \in \alpha\eta O(W), G \subseteq W \setminus B\}$, so there exist $G_0 \in \alpha\eta O(W)$ such that $x \in G_0 \subseteq (W \setminus B)$, then there exist G_0^c such that $B \subseteq G_0^c$and $x \notin G_0^c$, $G_0^c \in \alpha\eta C(W)$.Thus $x \notin G_0^c \in \alpha\eta C(W)$ and $B \subseteq G_0^c$. Then $\underline{M_{\alpha\eta}}(W \setminus B) \subseteq W\setminus \overline{M_{\alpha\eta}}(B)$. Similarly $W\setminus \overline{M_{\alpha\eta}}(B) \subseteq \underline{M_{\alpha\eta}}(W \setminus B)$. So, $\underline{M_{\alpha\eta}}(x \setminus B) = W\setminus \overline{M_{\alpha\eta}}(B)$.

(2) Let $x \in \overline{M_{\alpha\eta}}(W \setminus B) = \cap\{F \in \alpha\eta C(W), F \supseteq (W \setminus B)\}$, so there exist $F_0 \in \alpha\eta C(W)$such that $x \in (W \setminus B) \subset F_0$,then $x \notin B \supset F_0^c, x \notin F_0^c \in \alpha\eta O(W)$, thus there exist $F_0^c \in \alpha\eta O(W)$and $x \notin F_0^c$,then $x \notin \underline{M_{\alpha\eta}}(B)$, then $x \in (W \setminus \underline{M_{\alpha\eta}}(B))$, then $\overline{M_{\alpha\eta}}(W \setminus B) \subseteq W\setminus \underline{M_{\alpha\eta}}(B)$. Similarly $W\setminus \underline{M_{\alpha\eta}}(B) \subseteq \overline{M_{\alpha\eta}}(W \setminus B)$. So $\overline{M_{\alpha\eta}}(W \setminus B) = W\setminus \underline{R_{\alpha\eta}}(B)$.

(3) Since $\underline{M_{\alpha\eta}}(B) = \cup\{G \in \alpha\eta O(W), G \subseteq B\}$, then $\underline{M_{\alpha\eta}}(\underline{M_{\alpha\eta}}(B)) = \cup\{G \in \alpha\eta O(W), G \subseteq \underline{M_{\alpha\eta}}(B) \subseteq B\} = \cup\{G \in \alpha\eta O(W), G \subseteq B\}=\underline{M_{\alpha\eta}}(B)$.

(4) Since $\overline{M_{\alpha\eta}}(B) = W \setminus \underline{M_{\alpha\eta}}(B)$, From (2) then $\overline{M_{\alpha\eta}}(W \setminus \underline{M_{\alpha\eta}}(B)) = W \setminus \underline{M_{\alpha\eta}}(W \setminus (W \setminus \underline{M_{\alpha\eta}}(W \setminus B))) = W \setminus \underline{M_{\alpha\eta}}(\underline{M_{\alpha\eta}}(W \setminus B)) = W \setminus \overline{M_{\alpha\eta}}(W \setminus B) = \underline{M_{\alpha\eta}}(B)$.

(5) From (3) $\underline{M_{\alpha\eta}}(\underline{M_{\alpha\eta}}(B)) = \underline{M_{\alpha\eta}}(B) \subseteq \overline{M_{\alpha\eta}}(\underline{M_{\alpha\eta}}(B))$, then $\underline{M_{\alpha\eta}}(\underline{M_{\alpha\eta}}(B)) \subseteq \overline{M_{\alpha\eta}}(\underline{M_{\alpha\eta}}(B))$.

(6) $\underline{M_{\alpha\eta}}(\overline{M_{\alpha\eta}}(B)) \subseteq \overline{M_{\alpha\eta}}(B) = \overline{M_{\alpha\eta}}(\overline{M_{\alpha\eta}}(B))$. From(4), $\underline{M_{\alpha\eta}}(\overline{M_{\alpha\eta}}(B)) \subseteq \overline{M_{\alpha\eta}}(\overline{M_{\alpha\eta}}(B))$.

Proposition 3.4: Let $(W, \check{R}, M)$ be a $MSA-$ *space*, and $B, C \subseteq W$, then
(1) $\underline{M_{\alpha\eta}}(B \cup C) \supseteq \underline{M_{\alpha\eta}}(B) \cup \underline{M_{\alpha\eta}}(C)$
(2) $\overline{M_{\alpha\eta}}(B \cup C) \supseteq \overline{M_{\alpha\eta}}(B) \cup \overline{M_{\alpha\eta}}(C)$
(3)$\underline{M_{\alpha\eta}}(B \cap C) \subseteq \underline{M_{\alpha\eta}}(B) \cap \underline{M_{\alpha\eta}}(C)$
(4) $\overline{M_{\alpha\eta}}(B \cap C) \subseteq \overline{M_{\alpha\eta}}(B) \cap \overline{M_{\alpha\eta}}(C)$

Proof (1) $B \subset B \cup C$ and $C \subset B \cup C$ implies $M_{\alpha\eta}(B) \subset \underline{M_{\alpha\eta}}(B \cup C)$ and $\underline{M_{\alpha\eta}}(C) \subset \underline{M_{\alpha\eta}}(B \cup C).\underline{M_{\alpha\eta}}$. So $(B \cup C) \supseteq \underline{M_{\alpha\eta}}(B) \cup \underline{M_{\alpha\eta}}(C)$.

(2) $B \subset B \cup C$ and $C \subset B \cup C$ implies $\overline{M_{\alpha\eta}}(B \cup C) \subset \overline{M_{\alpha\eta}}(B \cup C)$ and $\overline{M_{\alpha\eta}}(C) \subset \overline{M_{\alpha\eta}}(B \cup C)$. So $\overline{M_{\alpha\eta}}(B \cup C) \supseteq \overline{M_{\alpha\eta}}(B) \cup \overline{M_{\alpha\eta}}(C)$
(3) and (4) Similar as (1).and (2).

Theorem 3.2: Let $(W, \check{R}, M)$ be a $MSA-$ *space*, and $B, C \subseteq W$ if B is $\alpha\eta-$definable, then the following are hold
(1) $\underline{M_{\alpha\eta}}(B \cup C) = \underline{M_{\alpha\eta}}(B) \cup \underline{M_{\alpha\eta}}(C)$
(2) $\overline{M_{\alpha\eta}}(B \cap C) = \overline{M_{\alpha\eta}}(B) \cap \overline{M_{\alpha\eta}}(C)$

Proof: (1) It's clear that $\underline{M_{\alpha\eta}}(B) \cup \underline{M_{\alpha\eta}}(C) \subseteq \underline{M_{\alpha\eta}}(B \cup C)$. To prove that $\underline{M_{\alpha\eta}}(B \cup C) \subset \underline{M_{\alpha\eta}}(B) \cup \underline{M_{\alpha\eta}}(C)$. Let $x \in \underline{M_{\alpha\eta}}(B \cup C)$, then $x \in \cup\{G \in \alpha\eta O(W), G \subseteq (B \cup C)\}$, then there exist $G_0 \in \alpha\eta O(W)$ such that $x \in G_0 \subseteq (B \cup C)$. We have three cases

case(1) if $G_0 \subseteq B$ and $x \in G_0$,then $G_0 \in \alpha\eta O(W)$, then $x \in \underline{M_{\alpha\eta}}(B)$.

case(2) if $G_0 \cap B = \phi$, then $G_0 \subseteq C$,$x \in G_0$, then $x \in \underline{M_{\alpha\eta}}(C)$.

case(3) if $G_0 \cap B \neq \phi$. Since $x \in G_0$ and G_0 is $\alpha\eta-$open set, then $x \in \alpha\eta cl(B)$, for every G_0 which has the above condition. Thus $x \in \overline{M_{\alpha\eta}}(B)$ and then $x \in \underline{M_{\alpha\eta}}(B)$ because B is $M_{\alpha\eta}-$definable in this three cases Therefore, $x \in \underline{M_{\alpha\eta}}(B) \cup \underline{M_{\alpha\eta}}(C)$

(2) It's clear that $\overline{M_{\alpha\eta}}(B \cap C) \subseteq \overline{M_{\alpha\eta}}(B) \cap \overline{M_{\alpha\eta}}(C)$. To prove the converse inclusion, let $x \in \overline{M_{\alpha\eta}}(B) \cap \overline{M_{\alpha\eta}}(C)$, then $x \in \overline{M_{\alpha\eta}}(B)$ implies $x \in \underline{M_{\alpha\eta}}(B)$ and $x \in G \subseteq B$ where G is $\alpha\eta-$openset and for all $G \in \alpha\eta O(W)$,$G \cap C \neq \phi$. Therefore $G \cap (B \cap C) = (G \cap B) \cap C = G \cap Y \neq \phi$ and hence $x \in \overline{M_{\alpha\eta}}(B \cap C)$.

4. $\alpha\eta-$ regions of uncertain concepts.

In this section, we split the universe set into regions and found the relations between these regions in different ways in $\alpha\eta-$approximation space.

Definition 4.1: let $(W, \check{R}, M)$ be a $MSA-$ *space*, and $B \subseteq W$. According to relation $\underline{M}(B) \subseteq \underline{M_\eta}(B) \subseteq \underline{M_{\alpha\eta}}(B) \subseteq B \subseteq \overline{M_{\alpha\eta}}(B) \subseteq \overline{M_\eta}(B) \subseteq \overline{M}(B)$. The universe W can be divided to regions with respect to any $B \subseteq W$ as follows:

(1) The $M-$internal edge of B $\underline{Edg}_M(B) = B \setminus \underline{M}(B)$

(2) The $M-\eta$-internal edge of$B$ $\underline{Edg}_{M_\eta}(B) = B \setminus \underline{M_\eta}(B)$
(3) The $M-\alpha\eta-$internal edge of B $\underline{Edg}_{M_{\alpha\eta}}(B) = B \setminus \underline{M_{\alpha\eta}}(B)$
(4) The $M-$external edge of B $\overline{Edg_M}(B) = \overline{M}(B) \setminus B$
(5) The $M-\eta-$ external edge of B $\overline{Edg_{M_{\alpha\eta}}}(B) = \overline{M_\eta}(B) \setminus B$
(6) The $M-\alpha\eta-$external ege of B $\overline{Edg_{M_{\alpha\eta}}}(B) = \overline{M_{\alpha\eta}}(B) \setminus B$
(7) The $M-$boundary of B $BND_M(B) = \overline{M}(B) \setminus \underline{M}(B)$
(8) The $M-\eta-$boundary of B $BND_{M_\eta}(B) = \overline{M_\eta}(B) \setminus \underline{M}_\eta(B)$
(9) The $M-\alpha\eta-$ boundary of B $BND_{M_{\alpha\eta}}(B) = \overline{M_{\alpha\eta}}(B) \setminus \underline{M_{\alpha\eta}}(B)$
(10) The $M-$exterior of B $ext_M(B) = W \setminus \overline{M}(B)$
(11) The $M-\eta-$exterior of B $ext_{M_\eta}(B) = W \setminus \overline{M}_\eta(B)$
(12) The $M-\alpha\eta-$exterior of B $ext_{M_{\alpha\eta}}(B) = W \setminus \overline{M}_{\alpha\eta}(B)$
(13) $\overline{M}(B) \setminus \underline{M_\eta}(B)$
(14) $\overline{M}(B) \setminus \underline{M_{\alpha\eta}}(B)$
(15) $\overline{M_\eta}(B) \setminus \underline{M}(B)$
(16) $\overline{M_{\alpha\eta}}(B) \setminus \underline{R}(B)$
(17) $\overline{M_\eta}(B) \setminus \underline{M_{\alpha\eta}}(B)$
(18) $\overline{M_\eta}(B) \setminus \underline{M}(B)$
(19) $\overline{M_{\alpha\eta}}(B) \setminus \overline{M_\eta}(B)$
(20) $\underline{M_\eta}(B) \setminus \overline{\underline{M}}(B)$
(21) $\overline{M_{\alpha\eta}}(B) \setminus \overline{\underline{M}}(B)$
(22) $\overline{M_{\alpha\eta}}(B) \setminus \underline{M_\eta}(B)$
(23) $\overline{\overline{M}}(B) \setminus \overline{M_\eta}(B)$
(24) $\overline{M}(B) \setminus \overline{M}_{\alpha\eta}(B)$

Remark 4.1 The study of $M-\alpha\eta-$approximation space in minimal structure is a generalization of $MSA-$ *space*. The elements of $\{\overline{M_{\alpha\eta}}(B) \setminus \underline{R}(B)\}$ region will be defined well in B, while those elements were undefinable in $MSA-$ *space* [8]. Also, The elements of $\{\overline{M}(B) \setminus \overline{M}_{\alpha\eta}(B)\}$ region do not belong to B, while these elements were not well defined in Pawlak's approximation spaces. In our study, $M-\alpha\eta-$boundary of B is used to reduce the boundary region of B in $MSA-$ *space*. Also, extending the exterior of A which contains the elements that don't belong to B by $M-\alpha\eta-$exterior of B

Proposition 4.1 Let $(W, \check{R}, M)$ be $MSA-$ *space* and $B \subseteq W$, then we have
(1) $BND_M(B) = \overline{Edg}_M(B) \cup \underline{Edg}_M(B)$.
(2) $BND_{M_{\alpha\eta}}(B) = \underline{Edg}_{M_{\alpha\eta}}(B) \cup \overline{Edg}_{M_{\alpha\eta\eta}}(B)$.

Proof:
(1) $BND_M(B) = \overline{M}(B) \setminus \underline{M}(B) = (\overline{M}(B) \setminus B) \cup (B \setminus \underline{M}(B)) = \overline{Edg}_M(B) \cup \underline{Edg_M}(B)$.
(2) $BND_{M_{\alpha\eta}}(B) = \overline{M_{\alpha\eta}}(B) \setminus \underline{M_{\alpha\eta}}(B) = (\overline{M_{\alpha\eta}}(B) \setminus B) \cup (B \setminus \underline{M_{\alpha\eta}}(B)) = \eta\overline{Edg_\alpha}(B) \cup \eta\underline{Edg}_\alpha(B)$.

Proposition 4.2 Let $(W, \check{R}, M)$ be $MSA-$ *space* and $B \subseteq W$, then we have
(1) $\overline{M}(B) \setminus \underline{M}_{\alpha\eta}(B) = \overline{Edg}_M(B) \cup \underline{Edg_{M_{\alpha\eta}}}(B)$.
(2) $\overline{M_{\alpha\eta}}(B) \setminus \underline{M}(B) = \overline{Edg_{M_{\alpha\eta}}}(B) \cup \underline{Edg}_M(B)$.

Proof
(1) $\overline{Edg}_M(B) \cup \underline{Edg_{M_{\alpha\eta}}}(B) = (\overline{M}(B) \setminus B) \cup (B \setminus \ \underline{M_{\alpha\eta}}(B) = \overline{M}(B) \setminus \underline{M}_{\alpha\eta}(B)$.
(2) $\overline{Edg_{M_{\alpha\eta}}}(B) \cup \underline{Edg}_M(B) = (\overline{M_{\alpha\eta}}(B) \ \setminus B) \cup (B \setminus \underline{M}(B)) = \overline{M_{\alpha\eta}}(B) \setminus \underline{M}(B)$.

Proposition 4.3 Let $(W, \check{R}, M)$ be $MSA-$ *space* and $B \subseteq W$, then we have
(1) $\underline{Edg_M}(B) = \underline{Edg}_{M_{\alpha\eta}}(B) \cup (\underline{M_{\alpha\eta}}(B) \setminus \underline{M}(B))$.
(2) $\overline{Edg}_M(B) = \overline{Edg}_{M_{\alpha\eta}}(B) \cup (\ \overline{M}(B) \setminus \overline{M}_{\alpha\eta}(B))$.

proof (1) R.H.S= $\underline{Edg}_{M_{\alpha\eta}}(B) \cup (\underline{M_{\alpha\eta}}(B) \setminus \underline{M}(B)) = (B \setminus \underline{M_{\alpha\eta}}(B)) \cup (\underline{M_{\alpha\eta}}(B) \setminus \underline{M}(B)) = B \setminus \underline{M}(B) = \underline{Edg}_M(B)$.
(2) R.H.S= $\overline{Edg}_{M_{\alpha\eta}}(B) \cup (\ \overline{M}(B) \setminus \overline{M}_{\alpha\eta}(B)) = (\overline{M_{\alpha\eta}}(B) \setminus B) \cup (\ \overline{M}(B) \setminus \overline{M}_{\alpha\eta}(B)) = \overline{M}(B) \setminus B = \overline{Edg}_M(B)$.

Definition 4.2 Let $(W, \check{R}, M)$ be $MSA-$ *space*, and a subset B of W is called
(1) $\alpha\eta-$definable ($\alpha\eta-$exact) if $\underline{M_{\alpha\eta}}(B) = \overline{M_{\alpha\eta}}(B)$ or $BND_{M_{\alpha\eta}}(B) = \phi$.
(2) $\alpha\eta-$Rough if $\underline{M_{\alpha\eta}}(B) \neq \overline{M_{\alpha\eta}}(B)$.

Example 4.1 From Example 3.5 $B = \{l\}, \{l, m\}, \{n\}$, then B is $\alpha\eta-$exact , if $C = \{l, m, n\}$ then C is $\alpha\eta-$rough.

This is a flow chart of determining $\alpha\eta-$exact and $\alpha\eta-$rough set

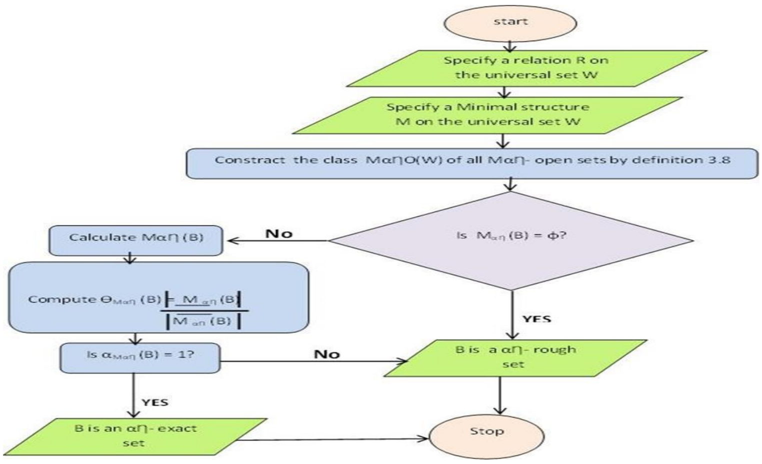

Figure 3:

Algorithm 1 This is an algorithm to determine the $\alpha\eta-$exact and the $\alpha\eta-$rough set in $(W, \check{R}, M)$.

Input: A $MSA-$ space $(W, \check{R}, M)$;
Output: Clasification aset in $MSA-$ space $(W, \check{R}, M)$ into two categories $\alpha\eta-$exact and $\alpha\eta-$rough set;
Specify a relation $\check{R}$ over the universal set W ;
Specify a minimal structure M over W;
Build the class $M\alpha\eta O(W)$of all $M\alpha\eta-$open sets using definition 3.8;
end
for each nonempty subset B of W;

Calculate its lower approximation $\underline{M_{\alpha\eta}}(B)$;
If $\underline{M_{\alpha\eta}}(B) \neq \phi$ then
return B is a $\alpha\eta-$rough set;
else
Calculate its upper approximation $\overline{M_{\alpha\eta}}(B)$;
Compute $\theta_{M_{\alpha\eta}}(B) = \frac{|\underline{M_{\alpha\eta}}(B)|}{|\overline{M_{\alpha\eta}}(B)|}$;
If $\theta_{M_{\alpha\eta}}(B) = 1$ then;
return B is a $\alpha\eta-$exact set;
else
return B is a $\alpha\eta-$rough set;
end
end
end

According to Theorem 3.1, we define the following important definition.

Definition 4.3 Let $(W, \check{R}, M)$ be $MSA-$ *space*, and $B \subseteq W$ is called
(1) Roughly $M_{\alpha\eta}-$definable if $\underline{M_{\alpha\eta}}(B) \neq \phi$ and $\overline{M_{\alpha\eta}}(B) \neq W$
(2) Internally $M_{\alpha\eta}-$undefinable if $\underline{M_{\alpha\eta}}(B) = \phi$ and $\overline{M_{\alpha\eta}}(B) \neq W$
(3) Externally $M_{\alpha\eta}-$undefinable if $\underline{M_{\alpha\eta}}(B) \neq \phi$ and $\overline{M_{\alpha\eta}}(B) = W$
(4) Totally $M_{\alpha\eta}-$undefinable if $\underline{M_{\alpha\eta}}(B) = \phi$ and $\overline{M_{\alpha\eta}}(B) = W$

The intuitive meaning of this classification is as shown:
(1) If X is roughly $M_{\alpha\eta}-$definable, this means that for some elements of W we are able to decide which belong to X and which is belong to W^c by using $\alpha\eta-$approximation space.
(2) If X is internally $M_{\alpha\eta}-$undefinable, this means that we are able to decide for some elements of W that they belong to W^c, but we are unable to decide for any elements of W that belong to W by using $\alpha\eta-$approximation space.
(3) If X is externally $M_{\alpha\eta}-$undefinable, this means that we are able to decide for some elements of W that belong to W, but we are unable to decide for any elements of W that belong to W^c by using $\alpha\eta-$approximation space
(4) If X is totally $M_{\alpha\eta}-$undefinable, this means that we are unable to decide for some elements of W whether they belong to W or W^c by using $\alpha\eta-$approximation space.

Example 4.2 From Example 3.3 we can see that the subsets $\{o\}$, $\{n\}, \{o, w\}$ are internally M- undefinable and they are internally M_η- undefinable but they are roughly $M_{\alpha\eta}$-definable. Also the subsets $\{l, m\}, \{l, m, n\}, \{l, m, o\}$ are externally M- undefinable and externally M_η- undefinable but they are roughly $M_{\alpha\eta}$- definable. So, $\alpha\eta-$approximation space is a refinement.

5. Comparison and Discussion:

The minimal structure is more accurate than the topology as seen in the following example

Example 5.1 Let $W = \{l, m, n, o\}$ and the minimal structure on it is definied as $M(W) = \{W, \phi, \{l\}, \{m\}, \{n, o\}\ \}$ we consider this minimal as subbase for the topology $\text{Ŧ}= \{W, \phi, \{l\}, \{m\}, \{l, m\}, \{l, n, o\}, \{m, n, o\}\}$, So. $MS^c(W) = \{W, \phi, \{m, n, o\}, \{l, n, o\}, \{l, m\}\}$. And, $\text{Ŧ}^c = \{\phi, W, \{m, n, o\}, \{l, n, o\}, \{n, o\}, \{m\}, \{l\}\}$ then $\alpha\eta_{\text{Ŧ}}O(W) = \{W, \phi, \{l\}, \{m\}, \{l, m\}, \{l, n\}, \{l, o\}, \{m, n\}, \{m, o\}, \{l, m, n\}, \{l, m, o\}, \{l, n, o\}, \{m, n, o\}\}$, $\alpha\eta_{\text{Ŧ}}C(W) = \{W, \phi, \{m, n, o\}, \{l, n, o\},$

$\{n,o\}$, $\{m,o\}$, $\{m,n\}$, $\{l,o\}$, $\{l,n\}$, $\{o\}$, $\{n\}$, $\{m\}$, $\{l\}\}$ $M\alpha\eta O(W) = \{W, \phi, \{l\}, \{m\}, \{n\}, \{o\}, \{l,m\}, \{l,n\}, \{l,o\}, \{m,n\}, \{m,o\}, \{n,o\}, \{l,m,n\}, \{l,m,o\}, \{l,n,o\}, \{m,n,o\}\}$ and $M\alpha\eta C(W) = \{W, \phi, \{m,n,o\}, \{l,n,o\}, \{l,m,n\}, \{n,o\}, \{m,o\}, \{m,n\}, \{l,o\}, \{l,n\}, \{l,m\}, \{o\}, \{n\}, \{m\}, \{l\}\}$.

M (current method) Definition 3.10				
B	$\underline{M_{\alpha\eta}}(B)$	$\overline{M_{\alpha\eta}}(B)$	$BND_{M_{\alpha\eta}}(B)$	$\theta_{M_{\alpha\eta}}(B)$
$\{l\}$	$\{l\}$	$\{l\}$	ϕ	1
$\{m\}$	$\{m\}$	$\{m\}$	ϕ	1
$\{n\}$	$\{n\}$	$\{n\}$	ϕ	1
$\{o\}$	$\{o\}$	$\{o\}$	ϕ	1
$\{l,m\}$	$\{l,m\}$	$\{l,m\}$	ϕ	1
$\{l,n\}$	$\{l,n\}$	$\{l,n\}$	ϕ	1
$\{l,o\}$	$\{l,o\}$	$\{l,o\}$	ϕ	1
$\{m,n\}$	$\{m,n\}$	$\{m,n\}$	ϕ	1
$\{m,o\}$	$\{m,o\}$	$\{m,o\}$	ϕ	1
$\{n,o\}$	$\{n,o\}$	$\{n,o\}$	ϕ	1
$\{l,m,n\}$	$\{l,m,n\}$	$\{l,m,n\}$	ϕ	1
$\{l,m,o\}$	$\{l,m,o\}$	$\{l,m,o\}$	ϕ	1
$\{l,n,o\}$	$\{l,n,o\}$	$\{l,n,o\}$	ϕ	1
$\{m,n,o\}$	$\{m,n,o\}$	$\{m,n,o\}$	ϕ	1
W	W	W	ϕ	1
ϕ	ϕ	ϕ	ϕ	0

Table (2)

Ŧ Abu-Donia [3] Definition 2.6				
B	$\underline{R_{\alpha\eta}}(B)$	$\overline{\check{R}_{\alpha\eta}}(B)$	$BND_{\alpha\eta}(B)$	$\theta_{\alpha\eta}(B)$
$\{l\}$	$\{l\}$	$\{l\}$	ϕ	1
$\{m\}$	$\{m\}$	$\{m\}$	ϕ	1
$\{n\}$	ϕ	$\{n\}$	$\{n\}$	0
$\{o\}$	ϕ	$\{o\}$	$\{o\}$	0
$\{l,m\}$	$\{l,m\}$	W	$\{n,o\}$	1/2
$\{l,n\}$	$\{l,n\}$	$\{l,n\}$	ϕ	1
$\{l,o\}$	$\{l,o\}$	$\{l,o\}$	ϕ	1
$\{m,n\}$	$\{m,n\}$	$\{m,n\}$	ϕ	1
$\{m,o\}$	$\{m,o\}$	$\{m,o\}$	ϕ	1
$\{n,o\}$	ϕ	$\{n,o\}$	$\{n,o\}$	0
$\{l,m,n\}$	$\{l,m,n\}$	W	$\{o\}$	3/4
$\{l,m,o\}$	$\{l,m,o\}$	W	$\{n\}$	1
$\{l,n,o\}$	$\{l,n,o\}$	$\{l,n,o\}$	ϕ	1
$\{m,n,o\}$	$\{m,n,o\}$	$\{m,n,o\}$	ϕ	1
W	W	W	ϕ	1
ϕ	ϕ	ϕ	ϕ	0

Table(3)

If we take $B = \{l,m\}$ then the boundary and accuracy of B are ϕ and 1 respectively by the present method in Definition 3.12 whereas the boundary and accuracy of B are$\{n,o\}$ and $\frac{1}{2}$ respectively by using abu-Donia method [3]. There are different methods to approximate the sets . Our new method is the best of them since the boundary regions decreased by increasing the lower approximations and decreasing the upper approximations. Moreover the accuracy of $\theta_{\alpha\eta}(B)$ in ($MSA-$ *space*) is more accurate than the other accuracy measures such as [3**?**] as shown in Table(2) and Table (3)

Example 5.2 From Example 3.2 we can see that

Current method in Definition 3.9				
B	$\underline{M_{\alpha\eta}}(B)$	$\overline{M_{\alpha\eta}}(B)$	$BND_{M_{\alpha\eta}}(B)$	$\theta_{M_{\alpha\eta}}(B)$
$\{l\}$	$\{l\}$	$\{l\}$	ϕ	1
$\{m\}$	$\{m\}$	$\{m\}$	ϕ	1
$\{n\}$	$\{n\}$	$\{m\}$	ϕ	1
$\{o\}$	$\{o\}$	$\{o\}$	ϕ	1
$\{w\}$	ϕ	$\{w\}$	$\{w\}$	0
$\{l,m\}$	$\{l,m\}$	$\{l,m\}$	ϕ	1
$\{l,n\}$	$\{l,n\}$	$\{l,n\}$	ϕ	1
$\{l,o\}$	$\{l,o\}$	$\{l,o\}$	ϕ	1
$\{m,n\}$	$\{m,n\}$	$\{m,n\}$	ϕ	1
$\{m,o\}$	$\{m,o\}$	$\{m,o\}$	ϕ	1
$\{n,o\}$	$\{n,o\}$	$\{n,o\}$	ϕ	1
$\{l,m,n\}$	$\{l,m,n\}$	$\{l,m,n\}$	ϕ	1
$\{l,m,o\}$	$\{l,m,o\}$	$\{l,m,o\}$	ϕ	1
$\{l,n,w\}$	$\{l,n,w\}$	$\{l,n,w\}$	ϕ	1
$\{l,m,n,o\}$	$\{m,n,o\}$	W	$\{w\}$	4/5
W	W	W	ϕ	1
ϕ	ϕ	ϕ	ϕ	0

Table (4)

Current method in Definition 3.2			
$\underline{M_{\eta}}(B)$	$\overline{M_{\eta}}(B)$	$BND_{\eta}(B)$	$\theta_{M_{\eta}}(B)$
$\{l\}$	$\{l\}$	ϕ	1
$\{m\}$	$\{m\}$	ϕ	1
ϕ	$\{n\}$	$\{n\}$	0
ϕ	$\{o\}$	$\{o\}$	0
ϕ	$\{w\}$	$\{w\}$	0
$\{l,m\}$	$\{l,m\}$	ϕ	1
$\{l,n\}$	$\{l,n,o\}$	$\{o\}$	2/3
$\{l,o\}$	$\{l,n,o\}$	$\{n\}$	2/3
$\{m\}$	$\{m,n\}$	$\{n\}$	1/2
$\{m,o\}$	$\{m,o\}$	ϕ	1
ϕ	$\{n,o\}$	$\{n,o\}$	0
$\{l,m,n\}$	W	$\{o,w\}$	3/5
$\{l,m,o\}$	W	$\{n,w\}$	3/5
$\{l,n,w\}$	$\{l,n,o,w\}$	$\{o\}$	3/4
$\{l,m,n,o\}$	W	$\{w\}$	4/5
W	W	ϕ	1
ϕ	ϕ	ϕ	0

Table (5)

El-Sharkasy method in Definition 2.16 [11]			
$\underline{M}(A)$	$\overline{M}(A)$	$BN_M(A)$	$\theta_M(A)$
$\{l\}$	$\{l,n,o,w\}$	$\{n,o,w\}$	1/4
$\{m\}$	$\{m,w\}$	$\{w\}$	1/2
ϕ	$\{n,o,w\}$	$\{n,o,w\}$	0
ϕ	$\{n,o,w\}$	$\{n,o,w\}$	0
ϕ	$\{n,o,w\}$	$\{n,o,w\}$	0
$\{l,m\}$	W	$\{n,o,w\}$	2/5
$\{l\}$	$\{l,n,o,w\}$	$\{n,o,w\}$	1/4
$\{l\}$	$\{l,n,o,w\}$	$\{n,o,w\}$	1/4
$\{m\}$	$\{m,n,o,w\}$	$\{n,o,w\}$	1/4
$\{m\}$	$\{m,n,o,n\}$	$\{n,o,w\}$	1/4
ϕ	$\{n,o,w\}$	$\{n,o,w\}$	0
$\{l,m\}$	W	$\{n,o,w\}$	2/5
$\{l,m\}$	W	$\{n,o,w\}$	2/5
$\{l\}$	$\{l,n,o,w\}$	$\{n,o,w\}$	1/4
$\{l,m\}$	W	$\{n,o,w\}$	2/5
W	W	ϕ	1
ϕ	ϕ	ϕ	0

Table (6)

From Table(4) , Table (5) and Table (6), we can see that there are many subsets such as $\{o\},\{n\},\{o,w\}$ which are internally $\check{R}$- undefinable and internally M_η- undefinable but they are roughly $M_{\alpha\eta}$- definable. Also the subsets $\{l,m\},\{l,m,n\},\{l,m,o\}$ are externally M- undefinable and externally M_η- undefinable but they are roughly $M_{\alpha\eta}$- definable So, $M-\alpha\eta-$approximation space is a refinement We can say that the proposed approach are useful in removing the impreciseness of rough sets.

6. $\alpha\eta-$approximation space in minimal structure in Covid-19

Coronavirus disease (Covid-19) is an infectious disease caused by the Coronavirus, which was recently discovered. The majority of people who contract COVID-19 experience only mild to moderate symptoms and recover without the need for medication. The virus that causes Covid-19 disease is spread primarily by droplets that an infected person exhales while coughing, sneezing, or inhaling. Since the droplets are too large to stay suspended in the air, they easily fall to the ground or other surfaces. If you are very close to a person with Covid-19 disease or touch a contaminated surface and then touch your eyes, nose, or mouth, you may become infected by breathing. The most common signs and symptoms are: Fever, dry cough, Exhaustion. Less common symptoms: Pains and aches, sore throat, diarrhea, conjunctivitis, a headache, loss of sense of taste or smell, and a rash or change in the color of the fingers or toes

we showed that the importance of $\alpha\eta-$approximation space with a minimal structure in medical science. We applied this approximation in Coronavirus problem with seven symptoms for five patients. Table (7) represents the problem of Coronavirus, the columns represent the symptoms which (yes means that the patient has the symptoms and no means that the patient has not the symptoms). The condition attributes where $Sy_1$is the fever on admission, Sy_2 is cough, $Sy_3$is Dyspnoea, Sy_4 is a sore throat, $Sy_5$is diarrhea, Sy_6 is chest pain and Sy_7 is malaise, the attribute D is the decision of the covid-19 in which y means the patient has the virus and n means the patient has no virus, the rows are the patients $P=\{pa_1,pa_2,pa_3,pa_4,pa_5\}$

	pa_1	pa_2	pa_3	pa_4	pa_5
Sy_1	yes	yes	yes	yes	yes
Sy_2	no	yes	no	no	no
Sy_3	no	on	no	no	no
Sy_4	no	yes	yes	no	no
Sy_5	no	no	no	no	no
Sy_6	no	no	no	no	no
Sy_7	yes	yes	no	no	yes
D	yes	yes	no	no	yes

Table (7)

	pa_1	pa_2	pa_3	pa_4	pa_5
pa_1	1	5/7	5/7	6/7	6/7
pa_2	5/7	1	5/7	4/7	4/7
pa_3	5/7	5/7	1	6/7	6/7
pa_4	6/7	4/7	6/7	1	1
pa_5	6/7	4/7	6/7	1	1

Table (8)

Table (8) represents similarities between symptoms patients where the degree of similarity $\mu(x,y)$ is defined as $\mu(x,y) = \frac{\sum_{i=1}^{n}(a_i(x)=a_i(y))}{n}$ where n is the number of symptoms we define the relationship in each issue according to the expert's requirement, in this case, a$\mathfrak{R}$ b if $\mu(a,b) > 0.8$

$M = \{P, \phi, \{pa_1, pa_4, pa_5\}, \{pa_2\}, \{pa_3, pa_4, pa_5\}, \{pa_1, pa_3, pa_4, pa_5\}\}$, $2^P \setminus M = \{\phi, P, \{pa_2, pa_3\}, \{pa_1, pa_3, pa_4, pa_5\}, \{pa_1, pa_2\}, \{pa_2\}\}$ and B_1 (patients have corona) = $\{pa_1, pa_2, pa_5\}$, $\underline{\check{R}}(B_1) = \{pa_2\}$, $\overline{\check{R}}(B_1) = P$, So. $\theta(B_1) = 1/5$, and B_2 (patients have not corona) = $\{pa_3, pa_4\}$, $\underline{\check{R}}(B_2) = \phi$, $\overline{\check{R}}(B_2) = \{pa_1, pa_3, pa_4, pa_5\}$ and $\theta(B_2) = 0$, So. $M\alpha\eta O(P) = \{\ \phi, P, \{pa_1\}, \{pa_2\}, \{pa_3\}, \{pa_4\}, \{pa_5\}, \{pa_1, pa_2\}, \{pa_1, pa_3\}, \{pa_1, pa_4\}, \{pa_1, pa_5\}, \{pa_2, pa_3\}, \{pa_2, pa_4\}, \{pa_3, pa_4\}, \{pa_2, pa_5\}, \{pa_4, pa_5\}, \{pa_1, pa_2, pa_3\}, \{pa_1, pa_2, pa_4\}, \{pa_1, pa_2, pa_5\}, \{pa_1, pa_3, pa_5\}, \{pa_1, pa_3, pa_4\}\ , \{pa_1, pa_4, pa_5\}, \{pa_1, pa_2, pa_5\}, \{pa_2, pa_3, pa_4\}, \{pa_2, pa_3, pa_5\}, \{pa_2, pa_4, pa_5\}, \{pa_3, pa_5, pa_5\}, \{pa_1, pa_2, pa_3.pa_4\}, \{pa_1, pa_2, pa_3.pa_5\}, \{pa_1, pa_3.pa_4, pa_5\}, \{pa_1, pa_2, pa_4, pa_5\}, \{pa_2, pa_3.pa_4, pa_5\}\}$. We notice that $\underline{\check{R}}(B_1) = \{pa_1, pa_2, pa_5\}$, $\overline{\check{R}}(B_1) = \{pa_1, pa_2, pa_5\}$, $\theta(B_1) = 1$ and $\underline{\check{R}}(B_2) = \{pa_2, pa_4\}$, $\overline{\check{R}}(B_2) = \{pa_2, pa_4\}$, $\theta(B_2) = 1$

In this example we got the minimal structure by a general relation $\check{R}$ and we calculated the accuracy measure which is 1/5 for the patients has corona. When we use the $\alpha\eta-$approximation space, the accuracy measure becomes 1 for the same group of patients.

7. Conclusions

We used minimal structure concepts to introduce the definitions of $\alpha\eta-$rough and $\alpha\eta-$exact sets. we introduced the notion of η-open, $\alpha\eta$-open in minimal structure, furthermore, we introduced the concept of $\alpha\eta$-lowerapproximation and $\alpha\eta$-upper approximation. The notion is further explored by studying their properties. We introduced $\alpha\eta$-regions of uncertain sets. We compared topological approximation space and $\alpha\eta$-minimal structure approximation space. We applied the $\alpha\eta$-approximation space in minimal structure in covid -19 problem. We got that the minimal structure is more efficient and accurate in obtaining results than Topology. Minimal structure increase the accuracy of decision making and help us to remove the impreciseness of rough sets.

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