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Quasi-type frame and quasi-type osculating curves in Myller configuration

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Abstract. In this article, we introduce a quasi-type frame and quasi-type osculating curves in Myller configuration. First, we obtain a quasi-type frame by using Frenet-type frame in Myller configuration. Then, we introduce quasi-type osculating curves. We obtain some relations for a curve to be a quasi-type osculating curve relative to the quasi-type frame in the Myller configuration. Then, we examine the relationships between the curvatures and invariants of the quasi-type osculating curves. Finally, we give some classifications according to the special cases of quasi curvatures. We shown that these classifications are a generalization of work done in both Myller configuration and Euclidean space.

1. Introduction

Moving frames built on a curve have an important place in differential geometry. Frames such as Frenet-Serret frame, Bishop frame for a regular curve, Darboux frame for curves on a surface and Sabban frame for spherical curves are quite popular in the literature. The quasi frame or q-frame for a regular curve in Euclidean space is one of the most recent works [4]. Quasi frame has some advantages over Frenet-Serret frame. A Frenet-Serret frame cannot be built at zero curvature points, but a quasi frame eliminates this situation. A projection vector is defined for the quasi frame, and a quasi frame is created using projection vectors in the x-axis, y-axis, or z-axis direction [4]. Special curves created with frame elements in differential geometry have a wide place in the literature. If the position vector of a curve is always in the rectifying plane, in the osculating plane, or in the normal plane, it is called the rectifying curve, the osculating curve, and the normal curve, respectively [2]. Special plane curves according to the classical Frenet-Serret frame in Euclidean space are introduced in [2]-[8]. In [6], the rectifying curves with respect to the quasi frame were examined.

A Myller configuration is a concept with interesting consequences associated with versor fields and plane fields. A couple $\{(C,\xi),(C,\pi)\}$ where $\xi \in \pi$ is called a Myller configuration and denoted by $\mathcal{M}(C,\overline{\xi},\pi)$ where (C,ξ) is a versor field and (C,π) is a plane field. If the planes π are tangent to C, it is called a tangent Myller configuration $\mathcal{M}_t(C,\overline{\xi},\pi)$ [14]. A investigate of the geometry of Myller configurations $\mathcal{M}(C,\overline{\xi},\pi)$ and $\mathcal{M}_t(C,\overline{\xi},\pi)$ is given [3, 14, 15]. In [14], the Frenet-type frame is introduced in Myller configuration and the curvatures associated with Frenet-type frame are obtained. It is emphasized that the geometric

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interpretation of curvatures is the same as classical curvatures in Euclidean space. Also, the derivative of the position vector $\overline{r}(s)$ of the curve C is formed as a linear combination of Frenet-type frame elements and the obtained functions are named as invariants of the frame. Therefore, as a result of some special cases of the obtained invariants, the Frenet-type frame corresponds to the classical Frenet frame in Euclidean space. Also, rectifying-type curves are introduced in Myller configuration in [12]. In addition, Bertrand curves are examined in Myller configuration in [13]. In [11], rotating minimizing frame is introduced in Myller configuration. On the other hand, in [9, 10], osculating-type curves are given for Frenet-type frame with Myller configuration in three dimension and four dimension.

In this study, firstly, quasi-type frame is introduced in Myller configuration. It is stated that the Myller configuration quasi-type frame corresponds to six different frames under special conditions. Then, as a generalization, quasi-type osculating curves are introduced in Myller configuration. After some characterizations of quasi-type osculating curves are given, some relations between curvature and invariants of quasi-type osculating curves are explained. Finally, the cases of quasi-type osculating curves were examined according to the special conditions of the curvatures.

2. Quasi-type frame with Myller configuration

In this section we introduce a quasi-type frame using Frenet-type frame in Myller Configuration. Let us first recall the Frenet-type frame:
Let $(C, \overline{\xi})$ be a versor field. With the assumption of $\overline{\xi} = \overline{\xi}_1$, the Frenet type frame was

$$\begin{pmatrix} \overline{\xi}_1'(s) \\ \overline{\xi}_2'(s) \\ \overline{\xi}_3'(s) \end{pmatrix} = \begin{pmatrix} 0 & K_1(s) & 0 \\ -K_1(s) & 0 & K_2(s) \\ 0 & -K_2(s) & 0 \end{pmatrix} \begin{pmatrix} \overline{\xi}_1(s) \\ \overline{\xi}_2(s) \\ \overline{\xi}_3(s) \end{pmatrix} \tag{1}$$

and

$$\frac{d\overline{r}(s)}{ds} = a_1(s)\overline{\xi}_1(s) + a_2(s)\overline{\xi}_2(s) + a_3(s)\overline{\xi}_3(s) \tag{2}$$

where $\overline{r}(s)$ is a position vector of C in Myller configuration. Obviously, if $a_1 = 1, a_2 = 0$ and $a_3 = 0$, the classic Frenet frame is obtained in Euclidean space.

Let us define a new versor field with $\overline{\xi}_1 = \xi_1^* = \overline{\xi}$. Since $\langle \xi_1^*, \xi_1^{*\prime} \rangle = 0$, we can define versor fields as

$$\xi_1^* = \overline{\xi}_1, \quad \xi_2^* = \frac{\xi_1^* \wedge \sigma}{\|\xi_1^* \wedge \sigma\|}, \quad \xi_3^* = \xi_1^* \wedge \xi_2^* \tag{3}$$

in order not to show the versor field in the direction of $\xi_1^{*\prime}$ only with ξ_2^*. Here σ is the projection vector of the form $\sigma = (1, 0, 0)$, $\sigma = (0, 1, 0)$ or $\sigma = (0, 0, 1)$. It is clear that for a Frenet-type frame in Myller configuration, the frame cannot be installed when the curvature of K_1 is zero. But we in Myller configuration eliminate this situation with the help of quasi-type frame. From equation (3), if ξ_1^* and σ are parallel, the quasi-type frame becomes singular. Therefore, the projection vector can be chosen to eliminate this situation. If selection $\sigma = (1, 0, 0)$ is used, it is called a quasi-type frame in the x-axis direction. If selection $\sigma = (0, 1, 0)$ is used, it is called a quasi-type frame in the y-axis direction. If selection $\sigma = (0, 0, 1)$ is used, it is called a quasi-type frame in the z-axis direction.

Let us now give the theorem describing the relationships between Frenet-type frame and quasi-type frame in Myller configuration:

Theorem 2.1. *Let $(C, \overline{\xi})$ be a versor field. Assuming $\overline{\xi}_1 = \xi_1^* = \overline{\xi}$, the derivative formulas for the quasi-type frame $\{\xi_1^*, \xi_2^*, \xi_3^*\}$ are*

$$\begin{pmatrix} \xi_1^{*\prime}(s) \\ \xi_2^{*\prime}(s) \\ \xi_3^{*\prime}(s) \end{pmatrix} = \begin{pmatrix} 0 & K_1^*(s) & K_2^*(s) \\ -K_1^*(s) & 0 & K_3^*(s) \\ -K_2^*(s) & -K_3^*(s) & 0 \end{pmatrix} \begin{pmatrix} \xi_1^*(s) \\ \xi_2^*(s) \\ \xi_3^*(s) \end{pmatrix} \tag{4}$$

and

$$\frac{d\overline{r}}{ds} = a_1^*(s)\xi_1^*(s) + a_2^*(s)\xi_2^*(s) + a_3^*(s)\xi_3^*(s) \tag{5}$$

where

$$\begin{aligned}
K_1^*(s) &= K_1(s)\cos\theta(s),\\
K_2^*(s) &= -K_1(s)\sin\theta(s),\\
K_3^*(s) &= K_2(s) + \theta^{'}(s),\\
a_1^*(s) &= a_1(s),\\
a_2^*(s) &= a_2(s)\cos\theta(s) + a_3(s)\sin\theta(s),\\
a_3^*(s) &= -a_2(s)\sin\theta(s) + a_3(s)\cos\theta(s).
\end{aligned}$$

Proof. Since $\xi_1^* = \overline{\xi}_1$, we can write

$$\xi_2^*(s) = \cos\theta(s)\overline{\xi}_2(s) + \sin\theta(s)\overline{\xi}_3(s), \tag{6}$$

$$\xi_3^*(s) = -\sin\theta(s)\overline{\xi}_2(s) + \cos\theta(s)\overline{\xi}_3(s) \tag{7}$$

By differentiating equation (6), we have

$$\begin{aligned}
\xi_2^{*'}(s) &= -K_1(s)\cos\theta(s)\overline{\xi}_1(s) + (K_2(s) + \theta^{'}(s))(-\sin\theta(s)\overline{\xi}_2(s) + \cos\theta(s)\overline{\xi}_3(s),\\
\xi_2^{*'}(s) &= -K_1^*(s)\xi_1^*(s) + K_3^*(s)\xi_3^*(s)
\end{aligned}$$

where

$$\begin{aligned}
K_1^*(s) &= K_1(s)\cos\theta(s),\\
K_3^*(s) &= K_2(s) + \theta^{'}(s).
\end{aligned}$$

By differentiating equation (7), we get

$$\begin{aligned}
\xi_3^{*'}(s) &= K_1(s)\sin\theta(s)\overline{\xi}_1(s) - (K_2(s) + \theta^{'}(s))(\cos\theta(s)\overline{\xi}_2(s) + \sin\theta(s)\overline{\xi}_3(s),\\
\xi_3^{*'}(s) &= -K_2^*(s)\xi_1^*(s) - K_3^*(s)\xi_2^*(s)
\end{aligned}$$

where

$$K_2^*(s) = -K_1(s)\sin\theta(s).$$

By using Frenet-type frame, we can write

$$\begin{aligned}
\xi_1^{*'}(s) &= K_1(s)\overline{\xi}_2(s),\\
&= K_1(s)(\cos\theta(s)\xi_2^*(s) + \sin\theta(s)\xi_3^*(s)),\\
&= K_1(s)\cos\theta(s)\xi_2^*(s) + K_1(s)\sin\theta(s)\xi_3^*(s),\\
&= K_1^*(s)\xi_2^*(s) + K_2^*(s)\xi_3^*(s).
\end{aligned}$$

On the other hand, we get

$$\begin{aligned}
\frac{d\overline{r}(s)}{ds} &= a_1(s)\overline{\xi}_1(s) + a_2(s)\overline{\xi}_2(s) + a_3(s)\overline{\xi}_3(s),\\
&= a_1(s)\xi_1^*(s) + (a_2(s)\cos\theta(s) + a_3(s)\sin\theta(s))\xi_2^*(s) + (-a_2(s)\sin\theta(s) + a_3(s)\cos\theta(s))\xi_3^*(s),\\
&= a_1^*(s)\xi_1^*(s) + a_2^*(s)\xi_2^*(s) + a_3^*(s)\xi_3^*(s)
\end{aligned}$$

where

$$\begin{aligned} a_1^*(s) &= a_1(s), \\ a_2^*(s) &= a_2(s)\cos\theta(s) + a_3(s)\sin\theta(s), \\ a_3^*(s) &= -a_2(s)\sin\theta(s) + a_3(s)\cos\theta(s). \end{aligned}$$

□

Corollary 2.2.

1. *If $a_1^* = 1$, $a_2^* = 0$ and $a_3^* = 0$, classical quasi frame is obtained in Euclidean space [4].*
2. *If $K_3^*(s) = K_2(s) + \theta'(s) = 0$, the Bishop-type frame is obtained in Myller configuration [11].*
3. *If $K_3^*(s) = 0$ and $a_1^* = 1$, $a_2^* = 0$, $a_3^* = 0$, classical Bishop frame is obtained in Euclidean space [1].*
4. *If $K_2^*(s) = 0$ and $K_1^*(s) = K_1(s)$, $K_3^*(s) = K_2(s)$, the Frenet-type frame is obtained in Myller configuration [14].*
5. *If $K_2^*(s) = 0$ and $K_1^*(s) = K_1(s)$, $K_3^*(s) = K_2(s)$ and $a_1^* = 1$, $a_2^* = 0$, $a_3^* = 0$, classical Frenet frame is obtained in Euclidean space [5].*

Example 2.3. Let us consider versor fields

$$\begin{aligned} \overline{\xi}_1(s) &= \left(-\frac{\sin s}{\sqrt{10}}, \frac{\cos s}{\sqrt{10}}, \frac{3}{\sqrt{10}}\right), \\ \overline{\xi}_2(s) &= (-\cos s, -\sin s, 0), \\ \overline{\xi}_3(s) &= \left(\frac{3\sin s}{\sqrt{10}}, -\frac{3\cos s}{\sqrt{10}}, \frac{1}{\sqrt{10}}\right). \end{aligned}$$

Suppose that $a_1 = \frac{1}{2}$, $a_2 = \frac{1}{2}$ and $a_3 = \frac{1}{\sqrt{2}}$. Then, we have

$$\begin{aligned} \frac{d\overline{r}(s)}{ds} &= a_1(s)\overline{\xi}_1(s) + a_2(s)\overline{\xi}_2(s) + a_3(s)\overline{\xi}_3(s), \\ \overline{r}(s) &= \left(\left(\frac{1}{2\sqrt{10}} - \frac{3}{\sqrt{20}}\right)\cos s - \frac{\sin s}{2}, \left(\frac{1}{2\sqrt{10}} - \frac{3}{\sqrt{20}}\right)\sin s + \frac{\cos s}{2}, \frac{3s}{2\sqrt{10}} + \frac{s}{\sqrt{20}}\right). \end{aligned}$$

Then, $\{\overline{\xi}_1, \overline{\xi}_2, \overline{\xi}_3\}$ is a Frenet-type frame in Myller configuration. Also, we get

$$K_1 = \frac{1}{10}, \quad K_2 = \frac{3}{10}.$$

Now let's find the elements of the quasi-type frame. Assume that $\overline{\xi}_1 = \xi_1^*$. Then, we get

$$\begin{aligned} \xi_1^*(s) &= \left(-\frac{\sin s}{\sqrt{10}}, \frac{\cos s}{\sqrt{10}}, \frac{3}{\sqrt{10}}\right), \\ \xi_2^*(s) &= \frac{\xi_1^* \wedge \sigma}{\|\xi_1^* \wedge \sigma\|} = (\cos s, \sin s, 0), \\ \xi_3^*(s) &= \xi_1^* \wedge \xi_2^* = \left(-\frac{3\sin s}{\sqrt{10}}, \frac{3\cos s}{\sqrt{10}}, -\frac{1}{\sqrt{10}}\right). \end{aligned}$$

where $\sigma = (0, 0, 1)$. Therefore, we have

$$\langle \xi_2^*(s), \overline{\xi}_2(s) \rangle = \cos\theta(s) = -1,$$
$$\langle \xi_2^*(s), \overline{\xi}_3(s) \rangle = \sin\theta(s) = 0$$

and

$$K_1^* = -\frac{1}{10}, \quad K_2^* = 0, \quad K_3^* = \frac{3}{10}, \quad a_1^* = \frac{1}{2}, \quad a_2^* = -\frac{1}{2}, \quad a_3^* = -\frac{1}{\sqrt{2}}.$$

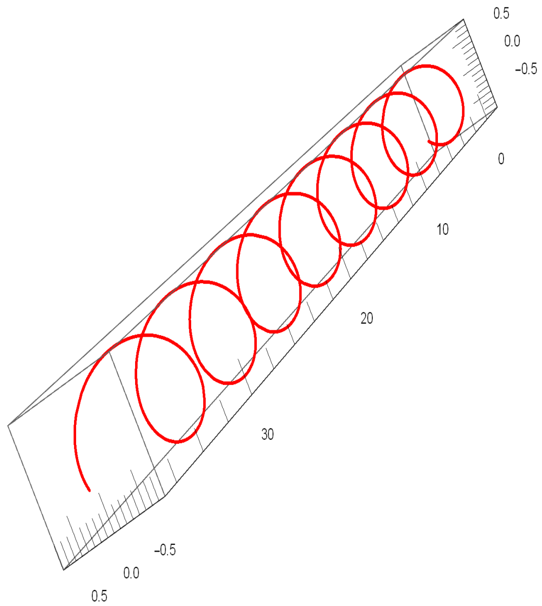

Figure 1: The curve $\bar{r}(s)$ in Myller configuration

3. Quasi-type osculating curves in Myller configuration

In this section, we introduce quasi-type osculating curves in Myller configuration. For a curve to be a quasi-type osculating curve, we obtain some relations between its curvatures and invariants.

Definition 3.1. Let $\bar{r}(s)$ is a curve with quasi-type frame $\{\xi_1^*, \xi_2^*, \xi_3^*\}$. The $\bar{r}(s)$ is called a quasi-type osculating curves if

$$\bar{r}(s) = \lambda(s)\xi_1^*(s) + \mu(s)\xi_2^*(s) \tag{8}$$

where λ, μ are smooth functions.

Theorem 3.2. *Let $\bar{r}(s)$ is a curve with quasi-type frame $\{\xi_1^*, \xi_2^*, \xi_3^*\}$. The $\bar{r}(s)$ is quasi-type osculating curve if*

$$\lambda^2(s) + \mu^2(s) = 2\int(\lambda(s)a_1^*(s) + \mu(s)a_2^*(s))ds \tag{9}$$

and

$$\lambda(s)K_2^*(s) + \mu(s)K_3^*(s) = a_3^*(s). \tag{10}$$

Conversely, the curve $\bar{r}(s)$ that satisfies, equations (9) and (10) is either a quasi-type osculating curve or a curve with $a_3^* = 0$.

Proof. Let us suppose that $\bar{r}(s)$ is quasi-type osculating curve. Then, by differentiating equation (8), we get

$$\begin{aligned} \lambda'(s) - \mu(s)K_1^*(s) &= a_1^*(s), && (11)\\ \mu'(s) + \lambda(s)K_1^*(s) &= a_2^*(s), && (12)\\ \lambda(s)K_2^*(s) + \mu(s)K_3^*(s) &= a_3^*(s). && (13) \end{aligned}$$

If the equation (11) is multiplied by $\lambda(s)$ and the equation (12) is multiplied by $\mu(s)$ and by adding, we get

$$\lambda(s)\lambda'(s) + \mu(s)\mu'(s) = \lambda(s)a_1^*(s) + \mu(s)a_2^*(s)$$

and

$$\lambda^2(s) + \mu^2(s) = 2\int(\lambda(s)a_1^*(s) + \mu(s)a_2^*(s))ds.$$

From equation (13), we have

$$\lambda(s)K_2^*(s) + \mu(s)K_3^*(s) = a_3^*(s).$$

Conversely, assume that the curve $\bar{r}(s)$ that satisfies, equations (9) and (10). By differentiating equation (9), we get

$$\lambda(s)\lambda'(s) + \mu(s)\mu'(s) = \lambda(s)a_1^*(s) + \mu(s)a_2^*(s). \tag{14}$$

Since

$$\lambda(s) = \langle\bar{r}(s), \xi_1^*(s)\rangle, \quad \lambda'(s) = a_1^*(s) + \langle\bar{r}(s), K_1^*(s)\xi_2^*(s) + K_2^*(s)\xi_3^*(s)\rangle, \tag{15}$$

$$\mu(s) = \langle\bar{r}(s), \xi_2^*(s)\rangle, \quad \mu'(s) = a_2^*(s) + \langle\bar{r}(s), -K_1^*(s)\xi_1^*(s) + K_3^*(s)\xi_3^*(s)\rangle. \tag{16}$$

By using equation (14), we have

$$\langle\bar{r}(s), \xi_3^*(s)\rangle(K_2^*(s)\langle\bar{r}(s), \xi_1^*(s)\rangle + K_3^*(s)\langle\bar{r}(s), \xi_2^*(s)\rangle) = 0.$$

Therefore, $\langle\bar{r}(s), \xi_3^*(s)\rangle = 0$ or $K_2^*(s)\langle\bar{r}(s), \xi_1^*(s)\rangle + K_3^*(s)\langle\bar{r}(s), \xi_2^*(s)\rangle = 0$. If $\langle\bar{r}(s), \xi_3^*(s)\rangle = 0$, $\bar{r}(s)$ is a quasi-type osculating curve. If $K_2^*(s)\langle\bar{r}(s), \xi_1^*(s)\rangle + K_3^*(s)\langle\bar{r}(s), \xi_2^*(s)\rangle = 0$, since $\lambda(s)K_2^*(s) + \mu(s)K_3^*(s) = a_3^*(s)$, we get $a_3^* = 0$. □

Corollary 3.3. *Let $\bar{r}(s)$ is a curve with quasi-type frame $\{\xi_1^*, \xi_2^*, \xi_3^*\}$. If $\bar{r}(s)$ is a quasi-type osculating curve, there is a*

$$K_1^*(s) = \frac{\lambda'(s)}{\lambda(s)\lambda'(s) + \mu(s)\mu'(s)}a_2^*(s) - \frac{\mu'(s)}{\lambda(s)\lambda'(s) + \mu(s)\mu'(s)}a_1^*(s),$$
$$\lambda(s)K_2^*(s) + \mu(s)K_3^*(s) = a_3^*(s)$$

relationship between its curvatures.

Now we give some theorems according to special cases of curvature of quasi-type osculating curves:

Theorem 3.4. *Let $\bar{r}(s)$ is a curve with quasi-type frame $\{\xi_1^*, \xi_2^*, \xi_3^*\}$ and $K_1^* = 0, K_2^* \neq 0, K_3^* \neq 0$. $\bar{r}(s)$ is a quasi-type osculating curve if and only if*

$$K_2^*(s)\int a_1^*(s)ds + K_3^*(s)\int a_2^*(s)ds = a_3^*(s). \tag{17}$$

Proof. Let us suppose that $\bar{r}(s)$ is a quasi-type osculating curve with $K_1^* = 0, K_2^* \neq 0, K_3^* \neq 0$. Then, from equation (15), we get

$$\lambda(s) = \int a_1^*(s)ds, \quad \mu(s) = \int a_2^*(s)ds.$$

If the last equation of equation (15) is used, the desired is obtained. Conversely, let the equation (17) be provided when $K_1^* = 0, K_2^* \neq 0, K_3^* \neq 0$. Let us consider the expression

$$\frac{d}{ds}\left[\bar{r}(s) - \int a_1^*(s)ds\xi_1^*(s) - \int a_2^*(s)ds\xi_2^*(s)\right].$$

If the equation (17) is used, it is seen that the result is 0. Hence, $\bar{r}(s)$ is a quasi-type osculating curve. □

Corollary 3.5. *If $a_1^* = 1$, $a_2^* = a_3^* = 0$, from equation (17), we get*

$$\frac{K_3^*(s)}{K_2^*(s)} = -\frac{s + c_1}{c_2}$$

where c_1 and c_2 are constants. This is the characterization of the osculating curves according to the quasi frame in Euclidean space when $K_1^* = 0$.

Theorem 3.6. *Let $\bar{r}(s)$ is a curve with quasi-type frame $\{\xi_1^*, \xi_2^*, \xi_3^*\}$ and $K_2^* = 0, K_1^* \neq 0, K_3^* \neq 0$. $\bar{r}(s)$ is a quasi-type osculating curve if and only if*

$$\left(\frac{a_2^*(s)}{K_1^*(s)} - \frac{1}{K_1^*(s)}\left(\frac{a_2^*(s)}{K_1^*(s)}\right)'\right)' - \frac{a_3^*(s)K_1^*(s)}{K_3^*(s)} = a_1^*(s). \tag{18}$$

Proof. Suppose that that $\bar{r}(s)$ is a quasi-type osculating curve with $K_2^* = 0, K_1^* \neq 0, K_3^* \neq 0$. By according to (15), we have

$$\lambda(s) = \frac{a_2^*(s)}{K_1^*(s)} - \frac{1}{K_1^*(s)}\left(\frac{a_2^*(s)}{K_1^*(s)}\right)', \quad \mu(s) = \frac{a_3^*(s)}{K_3^*(s)}. \tag{19}$$

If the first equation of equation (15) is used, we have equation (18). Conversely, let the equation (17) be provided with $K_2^* = 0, K_1^* \neq 0, K_3^* \neq 0$. By using equation (18), we get

$$\frac{d}{ds}\left[\bar{r}(s) - \left(\frac{a_2^*(s)}{K_1^*(s)} - \frac{1}{K_1^*(s)}\left(\frac{a_2^*(s)}{K_1^*(s)}\right)'\right)\xi_1^*(s) - \frac{a_3^*(s)}{K_3^*(s)}\xi_2^*(s)\right] = 0.$$

Therefore, $\bar{r}(s)$ is a quasi-type osculating curve. □

Corollary 3.7. *Since $K_2^* = 0$, the equation (18) is a characterization of osculating-type curves in Myller configuration with Frenet-type frame where $K_1^*(s) = K_1(s) \neq 0$ and $K_3^*(s) = K_2(s) \neq 0$ [9].*

Corollary 3.8. *If $a_1^* = 1$, $a_2^* = a_3^* = 0$, a contradiction is obtained from the equation (18). Therefore, (18) is not a characterization of osculating curves according to the classical Frenet frame in Euclidean space when $K_3^* \neq 0$. However, if $a_1^* = 1$, $a_2^* = a_3^* = 0$, from equation (11), (12) and (13), we have $K_3^* = K_2 = 0$. This shows that in Euclidean space the curve with respect to the Frenet frame is planar [7].*

Theorem 3.9. *Let $\bar{r}(s)$ is a curve with quasi-type frame $\{\xi_1^*, \xi_2^*, \xi_3^*\}$ and $K_3^* = 0, K_1^* \neq 0, K_2^* \neq 0$. $\bar{r}(s)$ is a quasi-type osculating curve if and only if*

$$\left(-\frac{a_1^*(s)}{K_1^*(s)} + \frac{1}{K_1^*(s)}\left(\frac{a_3^*(s)}{K_2^*(s)}\right)'\right)' + \frac{a_3^*(s)K_1^*(s)}{K_2^*(s)} = a_2^*(s). \tag{20}$$

Proof. Assume that that $\bar{r}(s)$ is a quasi-type osculating curve with $K_3^* = 0, K_1^* \neq 0, K_2^* \neq 0$. By according to (15), we have

$$\mu(s) = -\frac{a_1^*(s)}{K_1^*(s)} + \frac{1}{K_1^*(s)}\left(\frac{a_3^*(s)}{K_2^*(s)}\right)', \quad \lambda(s) = \frac{a_3^*(s)}{K_2^*(s)}.$$

If the second equation of equation (15) is used, we get equation (20). Conversely, the equation (20) be satisfied with $K_3^* = 0, K_1^* \neq 0, K_2^* \neq 0$. Then, by using equation (20), we get

$$\frac{d}{ds}\left[\overline{r}(s) - \frac{a_3^*(s)}{K_2^*(s)}\xi_1^*(s) - \left(-\frac{a_1^*(s)}{K_1^*(s)} + \frac{1}{K_1^*(s)}\left(\frac{a_3^*(s)}{K_2^*(s)}\right)'\right)\xi_2^*(s)\right] = 0.$$

Consequently, $\overline{r}(s)$ is a quasi-type osculating curve. □

Corollary 3.10. *Since $K_3^* = 0$, the equation (20) is a characterization of osculating curves in Myller configuration with Bishop-type frame.*

Corollary 3.11. *If $a_1^* = 1$, $a_2^* = a_3^* = 0$ with $K_3^* = 0$, from equation (20), we get K_1^* =constant. This is a characterization of osculating curves in Euclidean space according to the classical Bishop frame [1].*

4. Conclusion

In this study, quasi-type frame is constructed in Myller configuration. It is emphasized that quasi-type frame consists of many frames. In addition, osculating curves, which are a very suitable curve type for Myller configuration, are introduced. Due to the scarcity of Myller configuration studies in the literature, we believe that our article will be useful to the readers. In the future, it is planned to consider other types of curves and surfaces in the Myller configuration, both relative to the quasi-type frame and to other frames.

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