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Concircular curvature tensor of nearly cosymplectic manifolds in terms of the generalized Tanaka-Webster connection

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Abstract. In this paper, we study concircular curvature tensor of nearly cosymplectic manifolds in terms of the generalized Tanaka-Webster connection and then, we emphasized the properties that concircular curvature tensor of nearly cosymplectic manifolds in terms of the generalized Tanaka-Webster connection provides in case of flatness, ξ-concircularly flatness, ϕ-concircularly semisymmetric.

1. Introduction

The study of contact geometry has became most familiar from past few decades due to its capability to resolve the many issues of basic sciences, medical sciences and technology; although it has broad applications in geometric optics, geometric quantization, control theory, thermodynamics, integrable systems and to the classical mechanics.In 1958, Boothby and Wang [22] considered an odd dimensional diferentiable manifold equipped with the contact and almost contact structures and studied their properties from topological approach. Sasaki [21], in 1960, characterized the properties of an odd dimensional differentiable manifold equipped with the contact structures using tensor calculus. They called such manifolds as contact manifolds. Since then many researchers characterized several classes of the contact manifolds and studied its properties.

In 1940, Yano [14] defined a transformation, named as concircular transformation, which preserve the geodesic circles. Like Riemannian curvature tensor,concircular curvature tensor plays a significant role in the development of modern differential geometry (see, [28–30]).

Throughout this work, the expression "generalized Tanaka-Webster connection" will be denoted as "$g-TW-c$" for short. The aim of this study is to research concircular curvature tensor of nearly cosymplectic manifolds in terms of the $g-TW-c$. With this study, we have focused on the important curvature properties of nearly cosymplectic manifolds equipped with $g-TW-c$. Also, based on these curvature properties, we have defined the concircular curvature tensor of nearly cosymplectic manifolds in terms of the $g-TW-c$. Then, we emphasized the properties that concircular curvature tensor of nearly cosymplectic manifolds in terms of the $g-TW-c$ provides in case of flatness, ξ-concircularly flatness, ϕ-concircularly semisymmetric.

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The canonical affine connection defined on a non-degenerate pseudo-Hermition $CR-$manifold is the $g-TW-c$ [1, 2]. Tanno was the first to investigate the $g-TW-c$ for contact metric manifolds via the canonical connection [3]. If the associated $CR-$structure is integrable, this connection corresponds to the $g-TW-c$.

$g-TW-c$ have been studied by many mathematicians. If we mention some of them, D. G. Prakasha has introduced some curvature tensors on Kenmotsu manifolds with $g-TW-c$. Ünal have showned some curvature properties of $N(\kappa)-$ contact metric manifolds with $g-TW-c$. U. C. De has studied $g-TW-c$ on Sasakian manifolds. K. Kumar, D. L., Nagaraja, H. G., and Kumari, D. have researched concircular curvature tensor of Kenmotsu manifolds admitting $g-TW-c$. And also Ayar and Cavusoglu have studied conharmonic curvature tensor on nearly cosymplectic manifolds with $g-TW-c$. etc [9–11, 15, 16].

The cosymplectic and nearly cosymplectic manifolds are very important classes of almost contact metric manifolds. For more details, see ([23]-[27]). A nearly cosymplectic structure is an almost contact metric structure $\left(\phi, \xi, \eta, g\right)$ satisfying $\left(\nabla_X \phi\right) X = 0$ [13]. Nearly cosymplectic manifolds have been studied by various authors, such as Endo, Nicola et al., Ayar, Aktan et al., and many studies have been done on nearly structures recently [5–8, 17–20]. However, research on nearly structures in terms of the $g-TW-c$ is much more recent.

In this paper, based on these previous studies, both the basic properties provided by nearly cosymplectic manifolds equipped with $g-TW-c$ and the special cases provided by the concircular curvature tensor on nearly cosymplectic manifolds equipped with $g-TW-c$ are discussed.

This study is organized as follows: After a brief review of nearly cosymplectic manifolds, we have studied concircular curvature tensor of nearly a cosymplectic manifold in terms of the $g-TW-c$. Next, we have studied concircularly flat, ξ-concircularly flat, ϕ-concircularly semisymmetric nearly cosymplectic manifolds in terms of the $g-TW-c$.

2. Preliminaries

A smooth $(2n+1)-$dimensional manifold M is said to be an almost contact metric manifold if it admits an almost contact metric structure (ϕ, ξ, η, g) consisting of a $(1,1)$ tensor field ϕ, a vector field ξ, a 1-form η , and a Riemannian metric g compatible with (ϕ, ξ, η) satisfying

$$\phi^2 X = -X + \eta(X)\xi, \phi\xi = 0, g(X, \xi) = \eta(X), \eta(\xi) = 1, \eta \circ \phi = 0, H\xi = 0 \tag{1}$$

and

$$g\left(\phi X, \phi Y\right) = g(X, Y) - \eta(X)\eta(Y). \tag{2}$$

If

$$(\nabla_X \phi)Y + (\nabla_Y \phi)X = 0 \tag{3}$$

for all vector fields X, Y, an almost contact metric manifold is said to be a nearly cosymplectic manifold [13].

Clearly, this condition is equivalent to $(\nabla_X \phi)X = 0$, where ∇ denotes the Riemannian connection of g.

On a nearly cosymplectic manifold the following relations hold [5]:

$$\nabla_X \xi = HX \tag{4}$$

$$(\nabla_X \eta)Y = g(\nabla_X \xi, Y) \tag{5}$$

$$R(\xi, X, Y, Z) = -g((\nabla_X H)Y, Z) = \eta(Y)g(H^2X, Z) - \eta(Z)g(H^2X, Y) \tag{6}$$

$$\eta(R(Y, Z)X) = g((\nabla_X H)Y, Z) \tag{7}$$

$$S(X, \xi) = -\eta(X)tr(H^2) \tag{8}$$

$$Q\xi = -tr(H^2)\xi \tag{9}$$

$$S(\phi X, \phi Y) = S(X, Y) + (trH^2)\eta(X)\eta(Y) \tag{10}$$

that R denotes the curvature tensor of type $(1, 3)$, S is the Ricci tensor, H is the skew-symmetric tensor field and Q is the Ricci operator on M for any vector fields X, Y,and Z on M.

3. The concircular curvature tensor C^* in terms of the generalized Tanaka-Webster connection

Remark 3.1. *The notion of semi-symmetric metric $\xi-$connection ($\widetilde{\nabla}\xi = 0$) on Riemannian manifold has been defined in [31] , and it has been further studied in ([32]-[34]). The condition (13) is equivalent to the generalized Tanaka-Webster $\xi-$connection.*

Througout this paper we associate * with the quantities in terms of the $g - TW - c$. The $g - TW - c$, ∇^* , associated to the Levi-Civita connection ∇ is given by ([4, 12])

$$\nabla^*_X Y = \nabla_X Y - \eta(Y)\nabla_X\xi + (\nabla_X\eta)(Y)\xi - \eta(X)\phi Y \tag{11}$$

for any vector fields X, Y on M.
Using (4) and (5), the above equation yields

$$\nabla^*_X Y = \nabla_X Y - \eta(Y)HX + g(\nabla_X\xi, Y)\xi - \eta(X)\phi Y. \tag{12}$$

By taking $Y = \xi$ in (12) and using (1) and (4) we obtain

$$\nabla^*_X\xi = 0. \tag{13}$$

We now calculate the Riemannian curvature tensor R^* using (12) as follows:

$$\begin{aligned} R^*(X, Y)Z &= R(X, Y)Z + \eta(X)(\nabla_Y\phi)Z - \eta(Y)(\nabla_X\phi)Z + \eta(Z)\eta(Y)H^2X \\ &-\eta(Z)\eta(X)H^2Y + \eta(X)\eta(Z)\phi HY - \eta(Y)\eta(Z)\phi HX - g(Z, HX)HY \\ &+g(Z, HY)HX - 2g(Y, HX)\phi Z + g(H^2Y, Z)\eta(X)\xi - g(H^2X, Z)\eta(Y)\xi \\ &+\eta(X)g(HY, \phi Z)\xi - \eta(Y)g(HX, \phi Z)\xi. \end{aligned} \tag{14}$$

Taking $Z = \xi$ in (14), we get

$$R^*(X,Y)\xi = R(X,Y)\xi - \eta(X)H^2Y + \eta(Y)H^2X. \tag{15}$$

On contracting (14), we obtain the Ricci tensor S^* of a nearly cosymplectic manifold in terms of the $g-TW-c$, ∇^*, as

$$\begin{aligned} &S^*(Y,Z) = S(Y,Z) - \eta(Y)(div\phi)(Z) + \eta(Z)\eta(Y)tr(H^2) \\ &-\eta(Y)\eta(Z)tr(\phi H) + g(Z,HY)tr(H) + 2g(HY,\phi Z). \end{aligned} \tag{16}$$

This gives

$$\begin{aligned} S^*(Y,\xi) &= S(Y,\xi) + \eta(Y)tr(H^2) \\ Q^*Y &= QY + tr(H^2)Y. \end{aligned} \tag{17}$$

After we take the inner product of the above equation with e_i and take $Y = e_i$, on contracting the equation, we get (18).

$$r^* = r + tr(H^2)(2n+1), \tag{18}$$

that the scalar curvatures in terms of $g-TW-c$, ∇^*, and the Levi-Civita connection ∇, respectively, are r^* and r.

The concircular curvature tensor is an interesting invariant of a concircular transformation. In terms of the $g-TW-c$, ∇^*, the concircular curvature tensor $C^*$is as ([14])

$$C^*(X,Y)Z = R^*(X,Y)Z - \frac{r^*}{2n(2n+1)}\{g(Y,Z)X - g(X,Z)Y\} \tag{19}$$

on M for all vector fields X, Y, Z.

By interchanging X and Y in (19), we have

$$C^*(Y,X)Z = R^*(Y,X)Z - \frac{r^*}{2n(2n+1)}\{g(X,Z)Y - g(Y,Z)X\}. \tag{20}$$

On adding (19) and (20) and using the fact that $R^*(X,Y)Z + R^*(Y,X)Z = 0$, we get

$$C^*(X,Y)Z + C^*(Y,X)Z = 0. \tag{21}$$

From the first Bianchi identity $R(X,Y)Z + R(Y,Z)X + R(Z,X)Y = 0$, (14) and (19) with respect to ∇, we obtain

$$\begin{aligned} &C^*(X,Y)Z + C^*(Y,Z)X + C^*(Z,X)Y = 2\eta(X)(\nabla_Y\phi)Z + 2\eta(Y)(\nabla_Z\phi)X + 2\eta(Z)(\nabla_X\phi)Y \\ &-2g(Z,HX)HY + 2g(Z,HY)HX - 2g(X,HY)HZ - 2g(Z,HY)\phi X - 2g(X,HZ)\phi Y \\ &-2g(Y,HX)\phi Z - 2\eta(X)g(\phi HY,Z)\xi + 2\eta(Y)g(\phi HX,Z)\xi - 2\eta(Z)g(\phi HX,Y)\xi. \end{aligned} \tag{22}$$

Hence, from (21), it is shown that concircular curvature tensor in terms of the $g-TW-c$ on a nearly cosymplectic manifold is skew-symmetric.

Next, we assume that the manifold M in terms of the $g-TW-c$ is concircularly flat, that is, $C^*(X,Y)Z = 0$.

Then, from (19), it follows that

$$R^*(X,Y)Z = \frac{r^*}{2n(2n+1)}\{g(Y,Z)X - g(X,Z)Y\}. \tag{23}$$

Taking inner product of the above equation with ξ, we have

$$g(R^*(X,Y)Z,\xi) = \frac{r^*}{2n(2n+1)}\{g(Y,Z)\eta(X) - g(X,Z)\eta(Y)\}. \tag{24}$$

Using (1), (7), and (15) in (24), we get

$$g((\nabla_Z H)X,Y) + \left[\eta(X)g(H^2Y,Z) - \eta(Y)g(H^2X,Z)\right] = \frac{r^*}{2n(2n+1)}\{g(Y,Z)\eta(X) - g(X,Z)\eta(Y)\}. \tag{25}$$

Taking $X=\xi$ in (25), yields

$$g((\nabla_Z H)\xi,Y) + g(H^2Y,Z) = \frac{r^*}{2n(2n+1)}\{g(Y,Z) - \eta(Z)\eta(Y)\}. \tag{26}$$

Using (4) in (26), we get

$$\frac{r^*}{2n(2n+1)}\{g(Y,Z) - \eta(Y)\eta(Z)\} = 0. \tag{27}$$

Replacing Y by QY in (27), we get

$$\frac{r^*}{2n(2n+1)}\{g(QY,Z) - \eta(QY)\eta(Z)\} = 0. \tag{28}$$

Using (8) and (18) in (28), we get

$$\frac{r + tr(H^2)(2n+1)}{2n(2n+1)}\left\{S(Y,Z) + \eta(Z)\eta(Y)tr(H^2)\right\} = 0. \tag{29}$$

This implies either the scalar curvature of M is $-(2n+1)tr(H^2)$ or

$$S(Y,Z) = -\eta(Z)\eta(Y)tr(H^2). \tag{30}$$

As a result, the following theorem can be stated:

Theorem 3.2. *Either the scalar curvature is $-(2n+1)tr(H^2)$ or the manifold is a special form of $\eta-$Einstein manifold for a concircularly flat nearly cosymplectic manifold in terms of the $g-TW-c$.*

Definition 3.3. *A nearly cosymplectic manifold in terms of the $g-TW-c$, ∇^* , is said to be ξ-concircularly flat if $C^*(X,Y)\xi = 0$.*

Taking $Z = \xi$ and $C^*(X, Y)\xi = 0$ in (19), we have

$$R^*(X, Y)\xi - \frac{r^*}{2n(2n+1)}\{\eta(Y)X - \eta(X)Y\} = 0. \tag{31}$$

Using (15) in (31) and taking the inner product of the equation with Z,

$$R(X, Y, \xi, Z) - \eta(X)g(H^2Y, Z) + \eta(Y)g(H^2X, Z) - \frac{r^*}{2n(2n+1)}\{\eta(Y)g(X, Z) - \eta(X)g(Y, Z)\} = 0. \tag{32}$$

Using (6) in (32),

$$\frac{r^*}{2n(2n+1)}\{\eta(Y)g(X, Z) - \eta(X)g(Y, Z)\} = 0. \tag{33}$$

Taking $Y = \xi$ in (33) and using (1), we get

$$\frac{r^*}{2n(2n+1)}\{g(X, Z) - \eta(X)\eta(Z)\} = 0. \tag{34}$$

Replacing X by QX in (34), we get

$$\frac{r^*}{2n(2n+1)}\{g(QX, Z) - \eta(QX)\eta(Z)\} = 0. \tag{35}$$

Using (8) and (18) in (35), we get

$$\frac{r + tr(H^2)(2n+1)}{2n(2n+1)}\left\{S(X, Z) + tr(H^2)\eta(X)\eta(Z)\right\} = 0. \tag{36}$$

This implies either the scalar curvature of M is $-(2n+1)tr(H^2)$ or

$$S(X, Z) = -tr(H^2)\eta(X)\eta(Z). \tag{37}$$

Theorem 3.4. *Either the scalar curvature is $-(2n+1)tr(H^2)$ or the manifold is a special type of $\eta-$Einstein manifold for a ξ-concircularly flat nearly cosymplectic manifold in terms of the $g - TW - c$.*

Definition 3.5. *A manifold is said to be $\phi-$concircularly semisymmetric in terms of the $g - TW - c$, ∇^* , if $C^*(X, Y).\phi = 0$ holds on M.*

Now, we consider ϕ-concircularly semisymmetric nearly cosymplectic manifolds in terms of the $g - TW - c$. Then

$$\left(C^*(X, Y).\phi\right)Z = C^*(X, Y)\phi Z - \phi C^*(X, Y)Z = 0 \tag{38}$$

for all X, Y, Z on M.

Taking $Z = \xi$ in (38), we get

$$\phi\left(C^{*}(X,Y)\xi\right) = 0. \tag{39}$$

Using (15) , (6) and (39) in (19), we get

$$\frac{r^{*}}{2n(2n+1)}\{\eta(Y)\phi X - \eta(X)\phi Y\} = 0. \tag{40}$$

Replace Y by ξ and X by ϕX in (40) and using (1), we get

$$\frac{r^{*}}{2n(2n+1)}\{-X + \eta(X)\xi\} = 0. \tag{41}$$

Taking inner product of the above equation with U, we get

$$\frac{r^{*}}{2n(2n+1)}\{-g(X,U) + \eta(X)\eta(U)\} = 0. \tag{42}$$

Now, replacing X by QX in (42), we obtain

$$\frac{r^{*}}{2n(2n+1)}\{g(QX,U) + \eta(QX)\eta(U)\} = 0. \tag{43}$$

Using (8) and (18) in (43), we get

$$\frac{r + tr(H^2)(2n+1)}{2n(2n+1)}\left\{-S(X,U) - tr(H^2)\eta(X)\eta(U)\right\} = 0. \tag{44}$$

This implies either the scalar curvature of M is -$tr(H^2)(2n+1)$ or

$$S(X,U) = -tr(H^2)\eta(X)\eta(U). \tag{45}$$

Theorem 3.6. *In terms of the $g-TW-c$, the scalar curvature of a ϕ-concircularly semisymmetric nearly cosymplectic manifold is either $tr(H^2)(2n+1)$ or the manifold is a special form of η-Einstein manifold.*

Example 3.7. *Considering $M = \{(x,y,z) \in R^3\}$with $(x,y,z) \in R^3$ standard coordinates,*

$$E_1 = e^{-z}\frac{\partial}{\partial x}, E_2 = e^{-z}\frac{\partial}{\partial y}, E_3 = \frac{\partial}{\partial z}$$

there are linear independent vector fields at every point of M. Riemannian metric g is defined as

$$\begin{aligned} g(E_1,E_1) &= g(E_2,E_2) = g(E_3,E_3) = 1 \\ g(E_1,E_2) &= g(E_1,E_3) = g(E_2,E_3) = 0 \end{aligned}$$

In this case, the g metric is

$$g = \frac{dx^2 + dy^2 + dz^2}{z^2}$$

Let 1−form η be defined as $\eta(Z) = g(Z, E_3)$ for $\forall Z \in \chi(M)$ and ϕ be a tensor field of type $(1, 1)$ defined as $\phi(E_1) = -E_2, \phi(E_2) = E_1, \phi(E_3) = 0$.

For $\forall Z, W \in \chi(M)$, using the linearity of ϕ and g,

$$\eta(E_3) = 1, \phi^2 Z = -Z + \eta(Z)E_3, g(\phi Z, \phi W) = g(Z, W) - \eta(Z)\eta(W).$$

So for $E_3 = \xi$, the structure (ϕ, ξ, η, g) defines an almost contact metric structure on M.

Let's ∇ be the Levi Civita connection with the metric g. In this case, we get

$$[E_1, E_2] = 0, [E_1, E_3] = E_1, [E_2, E_3] = E_2$$

In the Koszul formula, the ∇ Riemannian connection of the g metric is given as

$$2g(\nabla_X Y, Z) = Xg(Y, Z) + Yg(Z, X) - Zg(X, Y) - g(X, [Y, Z]) - g(Y, [X, Z]) + g(Z, [X, Y])$$

Using this formula, the following equations are obtained.

$$\begin{aligned}
\nabla_{E_1} E_1 &= 0, \nabla_{E_1} E_2 = 0, \nabla_{E_1} E_3 = HE_1 \\
\nabla_{E_2} E_1 &= 0, \nabla_{E_2} E_2 = 0, \nabla_{E_2} E_3 = HE_2 \\
\nabla_{E_3} E_1 &= 0, \nabla_{E_3} E_2 = 0, \nabla_{E_3} E_3 = 0
\end{aligned}$$

In this case, with $E_3 = \xi$, M provides the relation $\nabla_X \xi = HX$. In addition, the following equations are obtained:

$$\begin{aligned}
\nabla^*_{E_1} E_1 &= g(HE_1, E_1)E_3, \nabla^*_{E_1} E_2 = 0, \nabla^*_{E_1} E_3 = 0 \\
\nabla^*_{E_2} E_1 &= 0, \nabla^*_{E_2} E_2 = g(HE_2, E_2)E_3, \nabla^*_{E_2} E_3 = 0 \\
\nabla^*_{E_3} E_1 &= E_2, \nabla^*_{E_3} E_2 = -E_1, \nabla^*_{E_3} E_3 = 0
\end{aligned}$$

Again, in this case, M provides the relation $\nabla^*_X \xi = 0$, with $E_3 = \xi$. So the following equations are reached:

$$\begin{aligned}
R(E_1, E_1)E_1 &= R(E_2, E_2)E_2 = R(E_3, E_3)E_3 = 0 \\
R(E_1, E_1)E_2 &= R(E_1, E_1)E_3 = R(E_1, E_2)E_3 = 0 \\
R(E_1, E_3)E_1 &= R(E_1, E_3)E_2 = R(E_2, E_1)E_3 = 0 \\
R(E_2, E_2)E_1 &= R(E_2, E_2)E_3 = R(E_2, E_3)E_1 = 0 \\
R(E_2, E_3)E_2 &= R(E_3, E_1)E_1 = R(E_3, E_2)E_2 = 0 \\
R(E_3, E_1)E_2 &= R(E_3, E_2)E_1 = R(E_3, E_3)E_1 = 0 \\
R(E_3, E_3)E_2 &= 0 \\
R(E_2, E_1)E_2 &= R(E_3, E_1)E_3 = HE_1 \\
R(E_1, E_2)E_2 &= R(E_1, E_3)E_3 = -HE_1 \\
R(E_1, E_2)E_1 &= R(E_3, E_2)E_3 = HE_2 \\
R(E_2, E_1)E_1 &= R(E_2, E_3)E_3 = -HE_2
\end{aligned}$$

4. Conclusion

With this study, we have demonstrated the important curvature properties of nearly cosymplectic manifolds in terms of the Tanaka-Webster connection. Also, based on these curvature properties, we defined the concircular curvature tensor of a nearly cosymplectic manifold in terms of the $g-TW-c$. Then, we emphasized the properties that concircular curvature tensor provides in case of flatness, $\xi-$concircularly flatness, ϕ-concircularly semisymmetric. In addition to this work, some symmetri conditions as $C^{*}.\phi = 0, C^{*}.S^{*} = 0,\ R^{*}.C^{*} = R^{*}.R^{*}$. and ϕ-concircularly flatness and pseudo-concircularly flatness can be examined.

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